UDC 517.9

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ON SIGN OF SOLUTIONS OF SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

ПРО ЗНАК РОЗВ'ЯЗКІВ СИСТЕМ ЗВИЧАЙНИХ ДИФЕРЕНЦІАЛЬНИХ РІВНЯНЬ

We show that Theorems 1 and 3 in A. G. Gricai's paper "Monotonicity properties of solutions of systems of nonlinear differential equations", which was published in the collection of works "Approximate and qualitative methods in the theory of differential and functional-differential-equations" (Institute of Mathematics, Ukrainian Academy of Sciences, Kiev, 1979), are incorrect in the represented form.

Показано, що теореми 1 і 3 роботи А. Г. Грицай „О свойствах монотонности решений систем нелинейных дифференциальных уравнений", опублікованої у збірнику праць Інституту математики АН України „Приближенные и качественные методы теории дифференциальных и дифференциально-функциональных уравнений" (1979 р.), у наведеному вигляді неправильні.

1. Introduction. The linear homogeneous system of ordinary differential equations with variable coefficients

$$x'(t) = A(t)x(t) \quad \text{for} \quad t \in [t_0, +\infty), \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $A(t) \in \mathbb{R}^{n\times n}$, with the initial condition

$$x_i(t_0) = x_{i0} > 0 \quad \text{for} \quad i = 1, 2, \dots, n, \tag{2}$$

where x_{i0} for $i = 1, 2, \dots, n$ are given, was investigated in the papers [1] (in the case $n = 3$) and [2], under assumption, that $A(t)$ is a Metzler matrix for every $t \in [t_0, +\infty)$. The authors have obtained some results on sign and monotonicity of solutions of initial value problems (1), (2).

These results were generalized in [3] to the case of nonlinear system of ordinary differential equations of the form

$$x_i'(t) = \sum_{j=1}^{n} a_{ij}(t)G_{ij}(x_j(t)) \quad \text{for} \quad t \in [t_0, T) \tag{3}$$

for $i, j = 1, 2, \dots, n$ with initial condition (2).

In [4] the following nonlinear system of ordinary differential equations

$$x_i'(t) = F_i(t, x_1(t), x_2(t), \dots, x_n(t)) \quad \text{for} \quad t \in [t_0, T) \tag{4}$$

for $i = 1, 2, \dots, n$ with initial condition (2), was investigated in the paper [4]. The author formulated and proved sufficient conditions on sign and monotonicity of solutions of the initial value problem (4), (2).

2. Main result. Among others, in [4] the following theorem has been presented.

Theorem 1 (cf. [4], Theorem 1). *Let us assume that*

$$F_i(t, x_1, \dots, x_{i-1}, 0, x_{i+1}, \dots, x_n) \geq 0 \quad \textit{for all} \quad x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n > 0. \tag{5}$$

Then all components $x_i(t)$, $i = 1, 2, \dots, n$, *of solution* $x(t)$ *of system of equations* (4) *which fulfill the initial value condition* (2), *are positive on the interval* $[t_0, T)$ *under assumption of existence of the solution.*

If additionally we assume that the inequalities

$$\frac{\partial F_i}{\partial x_j}(x_1, x_2, \dots, x_n) \geq 0 \quad \textit{for all} \quad x_1, x_2, \dots, x_n > 0 \tag{6}$$

are fulfilled for $i, j = 1, 2, \ldots, n$, *then*

$$x_i'(t) \geq 0 \quad \text{for} \quad t \in [t_0, T) \quad \textit{for} \quad i = 1, 2, \ldots, n.$$

The proof of the result is not correct and cannot be improved as is shown by the following counterexample to the statement. The same counterexample show that Theorem 3 from [4] is also incorrect.

Example. Fix an $n \in \mathbb{N}$ and let us consider the system of differential equations on the interval $[0, 2)$ of the form

$$\begin{aligned} x_1'(t) &= -(n+2)\sqrt[3]{x_1^2(t)} + \sqrt[3]{x_2^2(t)} + \sqrt[3]{x_3^2(t)} + \ldots + \sqrt[3]{x_n^2(t)}, \\ x_2'(t) &= \sqrt[3]{x_1^2(t)} - (n+2)\sqrt[3]{x_2^2(t)} + \sqrt[3]{x_3^2(t)} + \ldots + \sqrt[3]{x_n^2(t)}, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_n'(t) &= \sqrt[3]{x_1^2(t)} + \sqrt[3]{x_2^2(t)} + \sqrt[3]{x_3^2(t)} + \ldots -(n+2)\sqrt[3]{x_n^2(t)}, \end{aligned} \tag{7}$$

with the initial conditions

$$x_i(0) = 1 \quad \text{for} \quad i = 1, 2, \ldots, n. \tag{8}$$

Assumptions of the Theorems 1 and 3 are fulfilled. From (7) we infer that the functions

$$F_i(t, x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n) =$$
$$= \sqrt[3]{x_1^2} + \sqrt[3]{x_2^2} + \ldots + \sqrt[3]{x_{i-1}^2} - (n+2)\sqrt[3]{x_1^2} + \sqrt[3]{x_{i+1}^2} + \ldots + \sqrt[3]{x_n^2},$$
$$i = 1, 2, \ldots, n,$$

fulfill the assumption (5).

Let us observe, that the functions $x_i(t) = (1-t)^3$ for $i = 1, 2, \ldots, n$ are the solutions of the initial value problem (7), (8) in the interval $[0, 2)$ and $x_i(3/2) < 0$. Thus the components x_i, $i = 1, 2, \ldots, n$, of the solution x are not positive.

Remark. Let us note that the paper [4] is not a generalization of [3]. Indeed, in [3] the author assumes only continuity of functions G_{ij}, $i, j = 1, 2, \ldots, n$, appearing in (3) while it is clear from (6) that in [4] existence of all partial derivatives are required.

1. *Butlewski Z.* Sur les intégrales non oscilatoires d'un systéme de trois équations différentielles linéaires // Ann. pol. math. – 1969. – **18**. – P. 95 – 106.
2. *Ratajczak Z.* On monotonic solutions of system of n linear homogeneous differential equations (in Polish) // Zeszyty Naukowe Politechn. Poznańskiej. Ser. Mat. – 1967. – **42**, № 3. – P. 59 – 65.
3. *Przybycin Z.* On some property nonlinear system differential equations (in Polish) // Fasc. math. – 1975. – № 9. – P. 23 – 32.
4. *Gricai A. G.* Monotonicity properties of solutions of systems of nonlinear differential equations (in Russian) // Approximate and Qualitative Methods in the Theory of Differential and Functional-Differential Equations. – Kiev: Inst. Math. Acad. Sci. USSR, 1979. – P. 13 – 19.

Received 26.02.2002