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M. Et, A. Gökhan, H. Altinok (Fırat Univ., Turkey)

ON STATISTICAL CONVERGENCE OF VECTOR-VALUED SEQUENCES ASSOCIATED WITH MULTIPLIER SEQUENCES

ПРО СТАТИСТИЧНУ ЗБІЖНІСТЬ ВЕКТОРНОЗНАЧНИХ ПОСЛІДОВНОСТЕЙ, ЩО ПОВ'ЯЗАНІ З КОЕФІЦІЄНТНИМИ ПОСЛІДОВНОСТЯМИ

In this paper we introduce the vector-valued sequence spaces $w_\infty(F,Q,p,u)$, $w_1(F,Q,p,u)$, $w_0(F,Q,p,u)$, S_u^q, and S_{0u}^q using a sequence of modulus functions and the multiplier sequence $u=(u_k)$ of nonzero complex numbers. We give some relations related to these sequence spaces. It is also shown that if a sequence is strongly u_q-Cesàro summable with respect to the modulus function then it is u_q-statistically convergent.

Введено простори векторнозначних послідовностей $w_\infty(F,Q,p,u)$, $w_1(F,Q,p,u)$, $w_0(F,Q,p,u)$, S_u^q та S_{0u}^q з використанням послідовності модуль-функцій і коефіцієнтної послідовності $u=(u_k)$ ненульових комплексних чисел. Наведено деякі співвідношення, що стосуються цих просторів послідовностей. Також показано, що якщо послідовність сильно u_q-Чезаро-сумовна по відношенню до модуль-функції, то вона u_q-статистично збіжна.

1. Introduction. Let w be the set of all sequences of real or complex numbers and ℓ_∞, c, and c_0 be, respectively, the Banach spaces of *bounded*, *convergent*, and *null sequences* $x=(x_k)$ with the usual norm $\|x\|=\sup|x_k|$, where $k\in\mathbb{N}=\{1,2,\ldots\}$ is the set of positive integers.

Studies on vector-valued sequence spaces were carried out by Rath and Srivastava [1], Das and Choudhary [2], Leonard [3], Srivastava and Srivastava [4], Tripathy and Sen [5], Tripathy and Mahanta [6], and many others.

Throughout the article, for all $k\in\mathbb{N}$ E_k are seminormed spaces seminormed by q_k and X is a seminormed space seminormed by q. If what follows, $w(E_k)$, $c(E_k)$, $\ell_\infty(E_k)$, and $\ell_p(E_k)$ denote the spaces of *all, convergent, bounded*, and *p-absolutely summable* E_k-valued sequences, respectively. In the case where $E_k=\mathbb{C}$ (the field of complex numbers) for all $k\in\mathbb{N}$, one has the corresponding scalar-valued sequence spaces. The zero elements of E_k are denoted by θ_k. The zero sequence is denoted by $\bar\theta=(\theta_k)$.

Let $u=(u_k)$ be a sequence of nonzero scalar. Then for a sequence space E, the multiplier sequence space $E(u)$ associated with the multiplier sequence u is defined as

$$E(u)=\{(x_k)\in w:(u_kx_k)\in E\}.$$

Studies on the multiplier sequence spaces were carried out by Çolak [7], Çolak et al. [8], Srivastava and Srivastava [4], Tripathy and Mahanta [6], and many others.

The notion of a modulus was introduced by Nakano [9]. We recall that a modulus f is a function from $[0,\infty)$ to $[0,\infty)$ such that:

i) $f(x)=0$ if and only if $x=0$,

ii) $f(x+y)\le f(x)+f(y)$ for $x,y\ge0$,

iii) f is increasing,

iv) f is continuous from the right at 0.

It follows that f must be continuous everwhere on $[0,\infty)$. A modulus may be unbounded or bounded. Ruckle [10], Maddox [11] used a modulus f to construct some sequence spaces.

2. Main results. In this section, we prove some results involving the sequence spaces $w_0(F,Q,p,u)$, $w_1(F,Q,p,u)$, and $w_\infty(F,Q,p,u)$.

Definition 1. *Let $p = (p_k)$ be a sequence of strictly positive real numbers, let $F = (f_k)$ be a sequence of modulus functions, and let $u = (u_k)$ be any fixed sequence of nonzero complex numbers u_k. We define the following sequence spaces:*

$$w_0(F, Q, p, u) = \left\{ x_k \in E_k : \frac{1}{n}\sum_{k=1}^{n} \left[f_k\left(q_k(u_k x_k)\right)\right]^{p_k} \to 0, \ \text{as } n \to \infty \right\},$$

$$w_1(F, Q, p, u) = \left\{ \begin{array}{c} x_k \in E_k : \dfrac{1}{n}\displaystyle\sum_{k=1}^{n} \left[f_k\left(q_k\left(u_k x_k - \ell\right)\right)\right]^{p_k} \to 0, \\ \text{as } n \to \infty \quad \text{and} \quad \ell \in E_k \end{array} \right\},$$

$$w_\infty(F, Q, p, u) = \left\{ x_k \in E_k : \sup_n \frac{1}{n}\sum_{k=1}^{n} \left[f_k\left(q_k(u_k x_k)\right)\right]^{p_k} < \infty \right\}.$$

In the case where $f_k = f$ and $q_k = q$ for all $k \in \mathbb{N}$, we shall write $w_0(f, q, p, u)$, $w_1(f, q, p, u)$, and $w_\infty(f, q, p, u)$ instead of $w_0(F, Q, p, u)$, $w_1(F, Q, p, u)$, and $w_\infty(F, Q, p, u)$, respectively.

Throughout the paper, Z will denote any one of the notation 0, 1, or ∞.

If $x \in w_1(f, q, p, u)$, we say that x is strongly u_q-Cesàro summable with respect to the modulus function f and we will write $x_k \to \ell(w_1(f, q, p, u))$; ℓ will be called u_q-limit of x with respect to the modulus f.

The proofs of the following theorems are obtained by using the known standard techniques, therefore we give them without proofs.

Theorem 1. *Let the sequence (p_k) be bounded. Then the spaces $w_Z(F, Q, p, u)$ are linear spaces.*

Theorem 2. *Let f be a modulus function and the sequence (p_k) be bounded, then*

$$w_0(f, q, p, u) \subset w_1(f, q, p, u) \subset w_\infty(f, q, p, u)$$

and the inclusions are strict.

Theorem 3. *$w_0(F, Q, p, u)$ is a paranormed (need not total paranorm) space with*

$$g(x) = \sup_n \left(\frac{1}{n}\sum_{k=1}^{n} \left[f_k\left(q_k(u_k x_k)\right)\right]^{p_k} \right)^{\frac{1}{M}}, \qquad (1)$$

where $M = \max(1, \sup p_k)$.

Theorem 4. *Let $F = (f_k)$ and $G = (g_k)$ be any two sequences of modulus functions. For any bounded sequences $p = (p_k)$ and $t = (t_k)$ of strictly positive real numbers and for any two sequences of seminorms $q = (q_k)$ and $r = (r_k)$, we have:*

i) $w_Z(f, Q, u) \subset w_Z(f \circ g, Q, u)$,

ii) $w_Z(F, Q, p, u) \cap w_Z(F, R, p, u) \subset w_Z(F, Q + R, p, u)$,

iii) $w_Z(F, Q, p, u) \cap w_Z(G, Q, p, u) \subset w_Z(F + G, Q, p, u)$,

iv) *if q is stronger than r, then $w_Z(F, Q, p, u) \subset w_Z(F, R, p, u)$,*

v) *if q is equivalent to r, then $w_Z(F, Q, p, u) = w_Z(F, R, p, u)$,*

vi) $w_Z(F, Q, p, u) \cap w_Z(F, R, p, u) \neq \varnothing$.

Proof. i) We shall only prove i) for $Z = 0$, and the other cases can be proved by using similar arguments. Let $\varepsilon > 0$. We choose δ, $0 < \delta < 1$, such that $f(t) < \varepsilon$ for $0 \leq t \leq \delta$ and all $k \in \mathbb{N}$. Write $y_k = g\left(q_k(u_k x_k)\right)$ and consider

$$\sum_{k=1}^{n}[f(y_k)] = \sum_{1}[f(y_k)] + \sum_{2}[f(y_k)],$$

where the first summation is over $y_k \leq \delta$ and second summation is over $y_k > \delta$. Since f is continuous, we have

$$\sum_{1}[f(y_k)] < n\varepsilon. \tag{2}$$

By the definition of f, we have the following relation for $y_k > \delta$:

$$f(y_k) < 2f(1)\frac{y_k}{\delta}.$$

Hence,

$$\frac{1}{n}\sum_{2}[f(y_k)] \leq 2\delta^{-1}f(1)\frac{1}{n}\sum_{k=1}^{n}y_k. \tag{3}$$

It follows from (2) and (3) that $w_0(f,Q,u) \subset w_0(f\circ g,Q,u)$.

The following result is a consequence of Theorem 4 (i).

Proposition 1. *Let fbe a modulus function. Then* $w_Z(Q,u) \subset w_Z(f,Q,u)$.

Theorem 5. *Let* E_k *be a complete seminormed space for each* $k \in \mathbb{N}$. *Then the sequence space* $w_0(F,Q,p,u)$ *is complete and seminormed by* (1).

Proof. Let $(x^i t)$ be a Cauchy sequence in $w_0(F,Q,p,u)$, where $x^i = (x_k^i)_{k=1}^{\infty}$. Then

$$g(x^i - x^j) \to 0, \quad \text{as} \quad i,j \to \infty. \tag{4}$$

Hence, for each fixed k, we have

$$\left[f_k\left(q_k\left(u_k\left(x_k^i - x_k^j\right)\right)\right)\right]^{p_k} \to 0 \quad \text{as} \quad i,j\to\infty.$$

By continuity of f_k for all $k\in\mathbb{N}$, we have

$$\lim_{i,j\to\infty}\left[f_k\left(q_k\left(u_k\left(x_k^i - x_k^j\right)\right)\right)\right]^{p_k} = \left[f_k\left(\lim_{i,j\to\infty}q_k\left(u_kx_k^i - u_kx_k^j\right)\right)\right]^{p_k} = 0.$$

Since f_k is a modulus for all $k\in\mathbb{N}$,

$$\lim_{i,j\to\infty}q_k\left(u_kx_k^i - u_kx_k^j\right) = 0.$$

Let $y_k^i = u_kx_k^i$ for all $k\in\mathbb{N}$. Then $\left(y_k^i\right)_{i=1}^{\infty}$ is a Cauchy sequence in E_k for each $k\in\mathbb{N}$. Since E_k are complete, there exists $y_k\in E_k$ such that $y_k^i\to y_k$ as $i\to\infty$ for all $k\in\mathbb{N}$. Since E_k are linear, we can express y_k as $y_k = u_kx_k$, where $k\in\mathbb{N}$.

Since g is continuous, taking $j\to\infty$ in (4), we have $g(x^i - x) < \varepsilon$ for all $i\geq n_0$. Hence,

$$g(x^i - x)\in w_0(F,Q,p,u) \quad \text{for all} \quad i\geq n_0.$$

Since $(x^i - x)$, $(x^it)\in w_0(F,Q,p,u)$, and the space $w_0(F,Q,p,u)$ is linear, we have $x = x^i - (x^i - x)\in w_0(F,Q,p,u)$. Hence $w_0(F,Q,p,u)$ is complete.

Theorem 6. *Let* $0 < p_k \leq t_k$ *and let* $\left(\dfrac{t_k}{p_k}\right)$ *be bounded. Then* $w_Z(F,Q,t,u) \subset$ $\subset w_Z(F,Q,p,u)$.

Proof. By taking $w_k = [f_k(q_k(u_k x_k))]^{t_k}$ for all k and using the same technique as in Theorem 5 of Maddox [12], one can easily prove the theorem.

Theorem 7. *Let f be a modulus function. If* $\lim_{t\to\infty} \frac{f(t)}{t} = \beta > 0$, *then* $w_1(Q,p,u) = w_1(f,q,p,u)$.

Proof. Omitted.

3. u_q-Statistical convergence. The notion of statistical convergence was introduced by Fast [13] and Schoenberg [14] independently. Over the years and under different names, statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory, number theory. Later on, it was further investigated from sequence space point of view and linked with summability theory by Fridy [15], Connor [16], Šalát [17], Mursaleen [18], Işık [19], Savaş [20], Malkowsky and Savaş [21], Kolk [22], Maddox [23], Tripathy and Sen [24], and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also connected with subsets of the Stone – Čech compactification of the natural numbers. Moreover, statistical convergence is closely related to the concept of convergence in probability. The notion depends on the density of subsets of the set $\mathbb{N}$ of natural numbers.

A subset E of $\mathbb{N}$ is said to have density positive integers is defined by $\delta(E)$ if

$$\delta(E) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \chi_E(k)$$

exists, where χ_E is the characteristic function of E. It is clear that any finite subset of $\mathbb{N}$ have zero natural density and $\delta(E^c) = 1 - \delta(E)$.

In this section, we introduce u_q-statistically convergent sequences and give some inclusion relations between u_q-statistically convergent sequences and $w_1(f,q,p,u)$-summable sequences.

Definition 2. *A sequence $x = (x_k)$ is said to be u_q-statistically convergent to ℓ if, for every $\varepsilon > 0$,*

$$\delta\Big(\{k \in \mathbb{N} : q(u_k x_k - \ell) \geq \varepsilon\}\Big) = 0.$$

In this case, we write $x_k \to \ell\,(S_u^q)$. The set of all u_q-statistically convergent sequences is denoted by S_u^q.

By S, we denote the set of all statistically convergent sequences. If $q(x) = |x|$ and $u_k = 1$ for all $k \in \mathbb{N}$, then S_u^q is the same as S. In the case $\ell = 0$, we shall write S_{0u}^q instead of S_u^q.

Theorem 8. *Let f be a modulus function. Then*:

i) *if $x_k \to \ell\,(w_1(Q,u))$, then $x_k \to \ell\,(S_u^q)$,*

ii) *if $x \in \ell_\infty(u_q)$ and $x_k \to \ell\,(S_u^q)$, then $x_k \to \ell\,(w_1(Q,u))$,*

iii) $S_u^q \cap \ell_\infty(u_q) = w_1(Q,u) \cap \ell_\infty(u_q)$,

where $\ell_\infty(u_q) = \{x \in w(X) : \sup_k q(u_k x_k) < \infty\}$.

Proof. Omitted.

In the following theorems, we shall assume that the sequence $p = (p_k)$ is bounded and $0 < h = \inf_k p_k \leq p_k \leq \sup_k p_k = H < \infty$.

Theorem 9. *Let f be a modulus function. Then* $w_1(f,q,p,u) \subset S_u^q$.

Proof. Let $x \in w_1(f,q,p,u)$ and let $\varepsilon > 0$ be given. Let $\sum_1$ and $\sum_2$ denote the sums over $k \le n$ with $q(u_kx_k - \ell) \ge \varepsilon$ and $q(u_kx_k - \ell) < \varepsilon$, respectively. Then

$$\frac{1}{n}\sum_{k=1}^{n}[f(q(u_kx_k-\ell))]^{p_k} \ge$$

$$\ge \frac{1}{n}\sum_1 [f(q(u_kx_k-\ell))]^{p_k} \ge \frac{1}{n}\sum_1 [f(\varepsilon)]^{p_k} \ge$$

$$\ge \frac{1}{n}\sum_1 \min\left([f(\varepsilon)]^h, [f(\varepsilon)]^H\right) \ge$$

$$\ge \frac{1}{n}\Big|\big\{k \le n : q(u_kx_k-\ell) \ge \varepsilon\big\}\Big| \min\left([f(\varepsilon)]^h, [f(\varepsilon)]^H\right).$$

Hence, $x \in S_u^q$.

Theorem 10. *Let fbe bounded. Then* $S_u^q \subset w_1(f,q,p,u)$.

Proof. Suppose that f is bounded. Let $\varepsilon > 0$ and let $\sum_1$ and $\sum_2$ be the sums introduced in previous theorem. Since f is bounded, there exists an integer K such that $f(x) < K$ for all $x \ge 0$. Then

$$\frac{1}{n}\sum_{k=1}^{n}\Big[f(q(u_kx_k-\ell))\Big]^{p_k} \le$$

$$\le \frac{1}{n}\left(\sum_1 \Big[f(q(u_kx_k-\ell))\Big]^{p_k} + \sum_2 \Big[f(q(u_kx_k-\ell))\Big]^{p_k}\right) \le$$

$$\le \frac{1}{n}\sum_1 \max(K^h, K^H) + \frac{1}{n}\sum_2 \big[f(\varepsilon)\big]^{p_k} \le$$

$$\le \max(K^h, K^H)\frac{1}{n}\Big|\big\{k \le n : q(u_kx_k-\ell) \ge \varepsilon\big\}\Big| +$$

$$+ \max\big(f(\varepsilon)^h,\ f(\varepsilon)^H\big).$$

Hence, $x \in w_1(f,q,p,u)$.

Theorem 11. $S_u^q = w_1(f,q,p,u)$ *if and only if* f *is bounded.*

Proof. Let f be bounded. By Theorems 9 and 10, we have $S_u^q = w_1(f,q,p,u)$.

Conversely, suppose that f is unbounded. Then there exists a sequence (t_k) of positive numbers with $f(t_k) = k^2$ for $k = 1, 2, \ldots$. If we choose

$$u_ix_i = \begin{cases} t_k, & i = k^2, \quad k = 1, 2, \ldots, \\ 0, & \text{otherwise,} \end{cases}$$

then we have

$$\frac{1}{n}\Big|\Big\{k \le n : \Big|u_kx_k\Big| \ge \varepsilon\Big\}\Big| \le \frac{\sqrt{n}}{n}$$

for all n and so $x \in S_u^q$, but $x \notin w_1(f,q,p,u)$ for $X = \mathbb{C}$, $q(x) = |x|$ and $p_k = 1$ for all $k \in \mathbb{N}$. This contradicts to $S_u^q = w_1(f,q,p,u)$.

4. Special cases. Firstly, we note that $w_\infty(F,Q,p,u)$ and $w_\infty(F,Q,p)$ overlap but neither one contains the other. For example, $p_k = 1$, $f_k(x) = x$, and $q_k(x) = |x|$ for all $k \in \mathbb{N}$. If we choose $x = (1)$ and $u = (k)$, then $x \in w_\infty(F,Q,p)$, but

$x \notin w_\infty(F,Q,p,u)$, conversely, if we choose $x=(k)$ and $u=\left(\dfrac{1}{k}\right)$, then $x \notin$ $\notin w_\infty(F,Q,p)$, but $x \in w_\infty(F,Q,p,u)$. Similarly:

i) $w_0(F,Q,p,u)$ and $w_0(F,Q,p)$,

ii) $w_1(F,Q,p,u)$ and $w_1(F,Q,p)$,

iii) S_u^q and S^q,

iv) S_{0u}^q and S_0^q

overlap but neither one contains the other.

The definition of v-invariance of a sequence spaces E was given by Çolak [7] and the v-invariantness of the sequence spaces ℓ_∞, c, c_0, and ℓ_p was examined.

Definition 3. *Let X be any sequence space and $u=(u_k)$ be any sequence of nonzero complex numbers. We say that the sequence space X is u_q-invariant if $X_u^q = X^q$.*

By $E[u]$, we denote one of the sequence spaces $w_\infty(F,Q,p,u)$, $w_1(F,Q,p,u)$, $w_0(F,Q,p,u)$, S_u^q, S_{0u}^q, and also, by E, we denote one of the sequence spaces $w_\infty(F,Q,p), w_1(F,Q,p), w_0(F,Q,p), S^q, S_0^q$. What conditions should satisfy $u=(u_k)$ in order that $E[u]=E$?

If one considers the sequnce spaces:

1) $w_Z(f,q,p,u)$ instead of $w_Z(F,Q,p,u)$,

2) $w_Z(f,Q,p,u)$ instead of $w_Z(F,Q,p,u)$,

3) $w_Z(F,q,p,u)$ instead of $w_Z(F,Q,p,u)$,

4) $w_Z(F,Q,p)$ instead of $w_Z(F,Q,p,u)$,

5) $w_Z(F,Q,u)$ instead of $w_Z(F,Q,p,u)$,

6) $w_Z\ (F,Q)$ instead of $w_Z(F,Q,p,u)$,

7) $w_Z\ (F,p,u)$ instead of $w_Z(F,Q,p,u)$,

8) $w_Z\ (Q,p,u)$ instead of $w_Z(F,Q,p,u)$,

9) $w_Z\ (p,u)$ instead of $w_Z(F,Q,p,u)$,

10) S^q and S_0^q instead of S_u^q and S_{0u}^q,

11) S_u and S_{0u} instead of S_u^q and S_{0u}^q,

one will get that most of the results proved in the previous sections will be true for these spaces too.

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