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METHOD OF INVERSE OPERATOR FOR THE RECOVER INPUT SIGNAL

There have been considered the methods of construction of the inverse operator for the restoration of signals in conditions of weak nonlinear dynamic and nonlinear static distortions in the registration devices and transmission of continuous signals are considered. The simplest structure of the implementation block of the inverse operator is based on the adder, whose second inverse communication competitor includes the simulation model of the direct operator. A linear inertia-free unit with a set transmission factor is used as regularizer.

The inverse operator, based on the output signal of an object and its mathematical model, is able to restore the input signal of an object.

Depending on the method of construction of the inverse feedback, there are considered various methods of the inverse operator construction.

It is necessary to specify that the amplitudes of input and output signals should be agreed. Failure to meet this condition leads to the inverse operator accuracy loss. The regularization parameter should differ from the unit, and the accurate functioning of the structure may go beyond the stability limits. Physical dimension of variables differ, but the ranges of their changes in numbers should coincide.

When using a regularizer with a value above than one, the computational process may vary however there is a small amount of stability thanks to digitalization being used as a regularization parameter.

The algorithms for the implementation of the inverse operator may be used while improving the efficiency of the operation of energy-saturated equipment, improving the resolution of surveillance systems, improving the information exchange rate in communication lines, improving the information capacity of the information registration means, etc.

The effectiveness of the offered approaches is studied on model problems implemented in the Matlab/Simulink system. There have been made the computational experiments that demonstrated the efficiency of the method of inverse operators for the signals restoration in the presence of weak nonlinear dynamic and nonlinear static distortions in the registration devices and transmission of continuous signals in real-time mode.

Key words: *restoration of signals, nonlinear dynamic distortions, stability.*

Introduction. At present the application of computers in various technical devices and control systems has become the habitual phenomenon. The special interest is represented with cases where the computing way of improvement of their physical and economical properties has no alternative. Application of analogue or digital filters for correction of dynamic characteristics of the element in system allows to lower the technical requirements to this element. It frequently allows not only to reduce the cost of the whole system, but also to improve its basic technical parameters essentially. It is possible to note as examples the inertial measuring converters or executive elements, communication line with large attenuation etc [1–3].

There is the perfect computer equipment for solving such problems. However, the known computer algorithms do not always provide the effective using of this equipment. This problem is not only the question how to solve a computing problem, but mainly how to formulate it.

The majority of computing problems arising in this case may be divided into two classes: the direct problems and the inverse ones. In technical systems the solution of an inverse problem usually names as recovering of signal $x = By$. The initial data are the output signal y and mathematical model of investigated object A. The block which solves this problem is named as the block of realisation of the inverse operator $B = 1 / A$, and the block, which solves a direct problem of simulation of object $y = Ax$, is named as the block of realisation of direct operator A.

One of the effective methods of solving the signal recovering problem is the inverse operator's method. The essential feature of this method is the explicit using of the direct operator for obtaining the inverse one. As a rule, block A is an element of one of feedback circuits in the structure of block B [2, 3].

The concept of direct and inverse operators is especially often used in the theory of ill posed problems solving. The solution of these problems is characterised by instability or high sensitivity to errors of the initial data. These problems are also called as inverse because their sense consists in recovering the input signal of the inertial measuring device from its registered output signal. On the contrary, the direct problem of simulation of such measuring device (or realisation of the direct operator) is correct. This problem is characterised by insensibility toinitial data errors, that is the rather rough measurements are allowed.

The most important advantage of the inverse operator's method is the technology of construction of stable computing process which realises unstable (complex) inverse operator on the base of explicit application of stable (simple) direct operator. Moreover, the inverse operators method allows to organise two independent loops for adaptation the block B to the processes of ageing of the object A model and changing the errors quality [3].

1. Examples of application in technical systems. In control systems the problem of maintaining the stability is solved rather good by known classical methods with the selection of regulator parameters at realisation of a principle of a deviation control. However, the necessity of use of the inverse operator arises at realisation of more simple principle of specifying influence control. It is necessary not for increasing the stability of a control system, but contrary for decreasing its roughness. Introduction of the inverse operator allows to solve automatically one more problem which is paid not enough attention in the theory of automatic control. It is a problem of dimensions. And not only in sense of discrepancy in dimensions of physical values on the input and output of controlled object, but also in the sense of discrepancy in number of inputs and outputs of this object.

The usual combined control system from proportional and specifying action can be represented in two variants: Fig. 1 and Fig. 2. Here *X* is input, and *Y* is output signal of controlled object *B*, *RB* is computing model of the inverse operator of this object, *I* is inverter, *S* is adder, *R* is regulator.

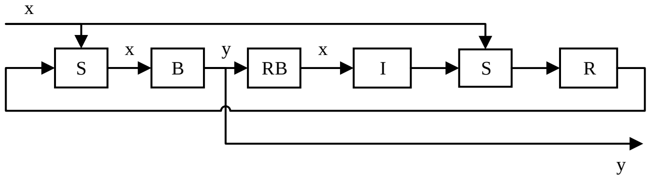

Fig. 1. *Control by input*

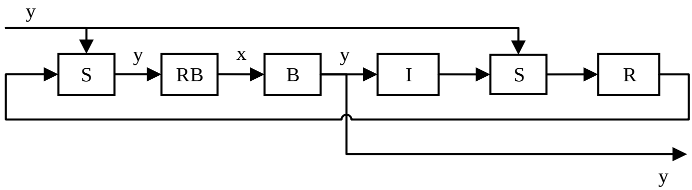

Fig. 2. *Control by output*

In the case when block *RB* is instantaneous element with unit gain (*RB* = 1) the both structures Fig. 1 and Fig. 2 coincide completely.

In case when *R* = 0 the specifying action control is realised. In case of switching-off of the first adder input, the second input of which is connected to output of regulator, the proportional action control is realised. The structures Fig. 1 and Fig. 2 differ only by a place of connection of block *RB*. Block *RB* carried out some of standard functions from the block *R*, for example, differentiation or integration. A case when *R* = 1 is quite possible, when block *RB* carried out all functions according to robustness and stability balance of control.

The organisation from output is especially convenient at control of multiconnected objects. So, for example, usual carburettor engine has at least two outputs: tachometer (crankshaft rotation speed) and flowmeter (fuel consumption). Inputs are the structure of a combustible mix (ratio petrol/air) and the ignition timing. The classical control circuit consists in control from the first input (at *RB* = 1) with organisation of a feedback of the first output on the second input. However, it is possible to organise the control from the first, or on the second output. In the first case the dynamics of automobile control is improved, and in the second case the fuel consumption decreases.

The algorithms of realisation of the inverse operator are important not only for increasing the efficiency of power-intensive equipment operation. Represent some other examples of their use. These are: increasing the resolution of observation systems; increasing the speed of information interchange in communication lines (compression of spectrum at digital transmitting of continuous signals, multitone coding); increasing the capacity of telephone exchange by reduction of signal recognition time in tone dialling; increasing the information capacity of information recording devices (magnetic and thermoplastic record); for images recognition in technical diagnostics by transients; for synthesis of band-pass filters [3].

The separate attention can be paid to full-scale `semivirtual reality’-type simulators, where simplified (base) object plus computer plays the role of complex object. On Fig. 3 the block diagram of such simulator is presented, where *U* are control signals, which the person generates acting on controls, block *RB* realises the inverse operator the base object mathematical model *B*, *M* is computing model of simulated object, *Z* are output signals of object or model.

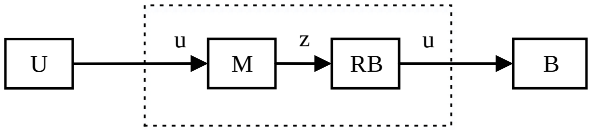

Fig. 3. *Simulation model*

The inverse operator’s method also useful in construction of numerical algorithms for solving the various applied problems.

2. Simple numerical experiment. As simple example we can consider the calculation of square root as inverse operation to square. This numerical experiment demonstrates an opportunity of application of the inverse operator’s method not only to solving the problems for linear dynamic systems or systems with weak nonlinearity usual in applications mentioned above, but also for essentially nonlinear problems. Certainly, the known classical algorithms for of a square root calculation are better in many senses, except one: they cannot be applied to calculation of other inverse functions. There is one more parameter, on which there may be superiority in comparison with traditional algorithms. It is balance of accuracy and speed.

The analogue prototype of our digital filter is the known circuit with connection of the direct operator in a feedback of operational amplifier Fig. 4, where *B* is the object, described by direct operator, *M* is mathematical (electronic) model of this object, *OA* is operational amplifier, *OR* are operational resistors, *I* is inverter. The dotted line leads round blocks, which form block *RB* which carries out the inverse function relative to object. In other words, according to the object signal and its mathematical model this block restores the input signal of object. But this structural diagram cannot be converted into discreet form explicitly. That is why the similar analogue structures received the name ‘nonalgorithmic’ [2, 3].

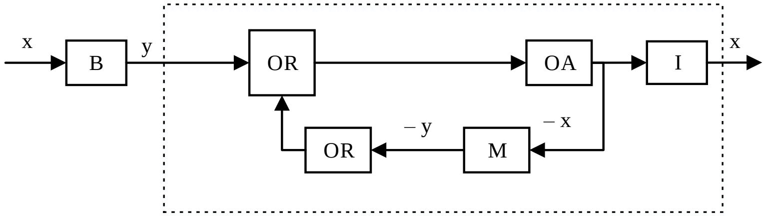

Fig. 4. Structural diagram of the inverse operator

On Fig. 5 the other variant of this computing structure transformed to algorithmic form is represented. Here *S* is adder, I is inverter, *R* is regularizator (linear instantaneous element with gain *r*). In fact, the second adder is accumulator (there is the feedback on its second input). This circuit is realized in digital computing elements, but already it cannot be realized in analogue ones.

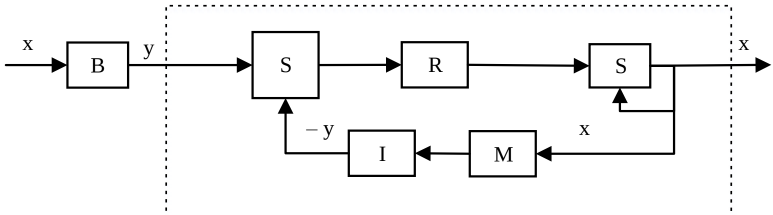

Fig. 5. Structural diagram of the inverse operator (short loop)

In other cases, e.g. for dynamic objects (differential or integral equations) the block *RB* has another structure (Fig. 6) and allows both types of realization (analogue and digital or nonalgorithmic and algorithmic). Here the regularizator not only limits the magnitude of residual between the output signals of object and its model, but also limits the amplification in a loop of positive feedback by restored signal. The modification of these structures with several regularizators also represent the practical interest.

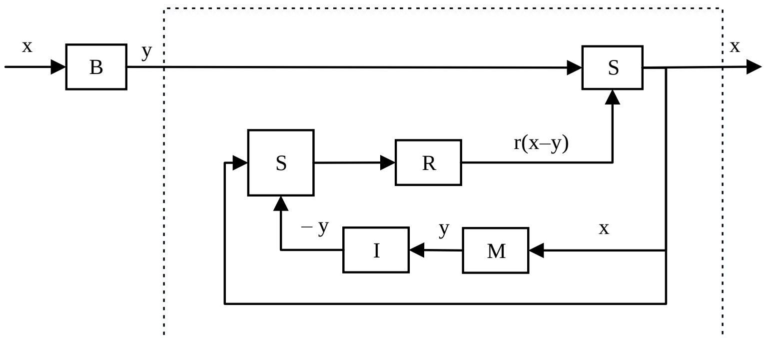

Fig. 6. Structural scheme of the inverse operator (long loop)

The structural diagrams Figs. 5 and 6 can be easily converted into corresponding recurrent formula

$$x(i) = x(i-1) + r\left(y(i) - M\,(x(i-1))\right), \tag{1}$$

$$x(i) = y(i) + r\left(x(i) - M\,(x(i-1))\right), \tag{2}$$

where y is output signal and x is input signal of computing process, M realizes the mathematical model of direct operator, and r is regularizing parameter. In the case of square root calculation $M(x) = x^2$, and we have two additions and two multiplications at the iteration step. Because of the double recursion by calculated output signal the term birecursive digital filter can be used to characterize such algorithms realized the inverse operator's method in application to signal recovering problems. In mathematical literature (1) corresponds to Friedman iterative regularization, and (2) corresponds to Lavrentyev algorithm respectively.

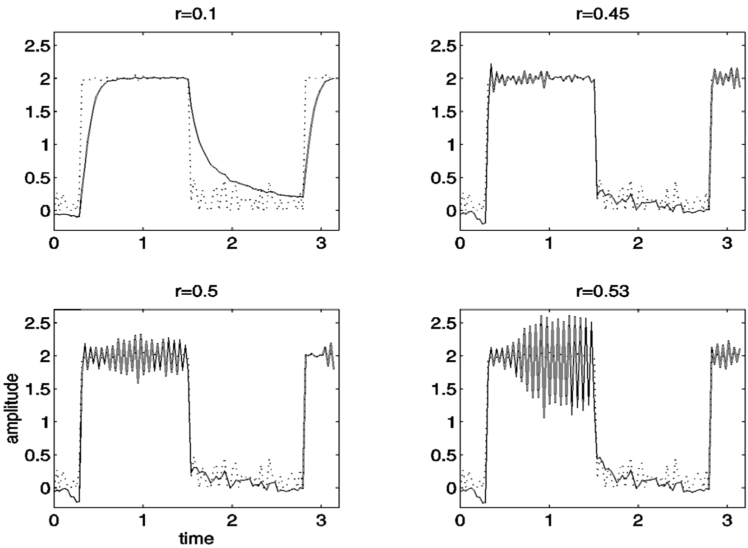

Fig. 7. *The initial signal sinusoidental type*

The graphs on Figs. 7 and 8 represent the results of restoring the squared signal with one iteration at every time step. The initial time interval was divided into 100 discretes. On Fig. 7 the initial signal $x(t)$ was rectangular meander with addition of normal white noise having the amplitude 0.1 of initial signal. Fig. 8 corresponds to the unnoised sinusoide. In both cases magnitude of initial signal was 4. Dotted lines correspond to standard function *sqrt*(*x*).

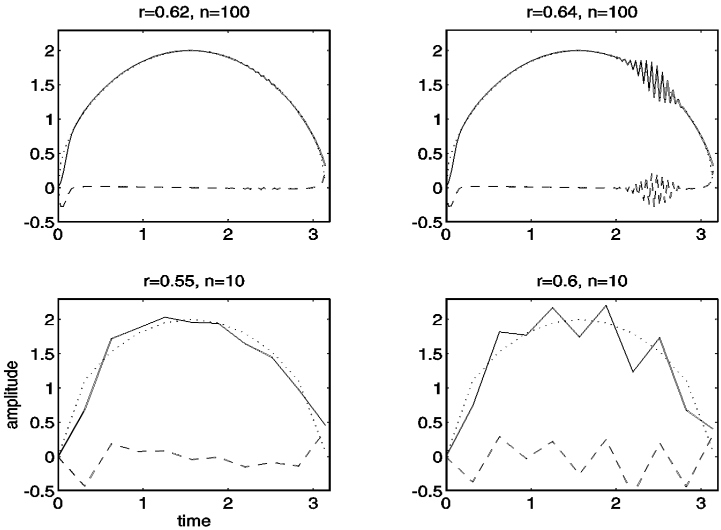

Fig. 8. *The initial signal rectangular meander with addition normal white noise*

From graphs Figs. 7 and 8 we can see that parameter *r* has the influence on the stability of computing process. At variations of magnitude and type of signal or characteristics of noise it is possible to choose such *r*, which provides the best balance of stability and accuracy of computing process. The instability is especially appearing in places of fast amplitude variations or sign of initial signal derivative. At change of digitalisation step the balance of accuracy and stability of calculations on BRDF algorithm will change, the traditional algorithm has not such property. With increasing the quantity of digitisation units, the accuracy, naturally, grows, and for providing the stability it is necessary to change the regularizing parameter *r*.

Conclusion. Algorithm can be applied to solving the systems of linear and nonlinear equations, including differential and integral ones. The most positive moment is an achievement of high speed, which is very important for real time signal processing.

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МЕТОД ОБЕРНЕНОГО ОПЕРАТОРА ДЛЯ ВІДНОВЛЕННЯ ВХІДНОГО СИГНАЛУ

Розглянуто методи побудови оберненого оператора для відновлення сигналів при умовах слабких нелінійних динамічних і нелінійних статичних спотворень у пристроях реєстрації та передачі неперервних сигналів. Найпростіша структура блока реалізації оберненого оператора будується на основі накопичуваного суматора, у другий контур оберненого зв'язку якого включена імітаційна модель прямого оператора. В якості регуляризатора використано лінійний безінерційний блок із визначеним коефіцієнтом передачі.

Обернений оператор по вихідному сигналу об'єкта і його математичної моделі відновлює вхідний сигнал об'єкта.

В залежності від підходу до побудови оберненого зв'язку розглядаються різні методи побудови оберненого оператора.

Важливо відмітити, що амплітуди вхідного і вихідного сигналів повинні бути узгодженні. Якщо ця умова не виконується точність оберненого оператора знизиться. Параметр регуляризації необхідно буде вибирати відмінним від одиниці, а точне функціонування структури може бути за межами стійкості. Фізичні розмірності величин можуть бути різними, але діапазони їх змін в числах повинні співпадати.

При використанні регуляризатора, значення якого більше одиниці, обчислювальний процес може розходитись, хоча залишається невеликий запас стійкості за рахунок кроку дискретизації, який використовується як регуляризуючий параметр.

Алгоритми реалізації оберненого оператора можуть бути використані при підвищенні ефективності експлуатації енергонасиченого обладнання, підвищення роздільної здатності систем спостереження, підвищення швидкості обміну інформації в лініях зв'язку, підвищення інформаційної ємності засобів реєстрації інформації тощо.

Ефективність запропонованих підходів досліджено на модельних задачах, які реалізовано в системі Matlab/Simulink. Проведені обчислювальні експерименти показали ефективність методу обернених операторів для відновлення сигналів при наявності слабких нелінійних динамічних і нелінійних статичних спотворень у пристроях реєстрації та передачі неперервних сигналів в умовах реального часу.

Ключові слова: *відновлення сигналів, нелінійні динамічні спотворення, стійкість.*

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