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TOPOLOGICAL ANALYSIS OF ANGULAR MOMENTUM RANGE VALUES OF THE GYRO MOMENT CLUSTERS BASED ON COLLINEAR GYRODINES PAIRS

Introduction. To ensure the high dynamic characteristics of Earth remote sensing satellites in their orientation systems, the gyro moment clusters (GMCs) based on excessive number (more than three) two-gimbals control moment gyrodines (GDs) can be used as actuators. The attitude control by GD actuators task is the most difficult among the tasks of spacecraft (SC) reorientation control. The central issue in solving this task is the synthesis of the control laws for precession angles of individual GDs when there are excessive. Success in solving the control problem is substantially determined by the choice of the GMC structure, it means the number of GDs used and their mutual positions of the precession axes. From this choice depends on the possibility of forming by GMC the necessary control momentum, the existence and number of special GMC states, the complexity of the control laws for the precession angles of the individual GDs included in the GMC. This is because in order to maintain the desired SC orientation for a long time and to perform its turns with the required angular rate, the GMC must have a sufficient margin of angular momentum. The allowable values of the total angular momentum created by the GDs form a certain area that is bounded by a closed surface of complex shape in a coordinate system rigidly attached to main SC body. Inside this area there are particular surfaces on which the control of the GDs is complicated or unfeasible. These surfaces are called singular. In this regard, for SC attitude control in addition to control the precession rate of individual GDs it is also necessary to control the mutual orientation of the angular momenta of the GDs in GMC. That is why one of the most important problems of the control laws synthesis with the use of GMC is the identifying singular surfaces (topological analysis) in the area allowable angular momentum of the GMC.

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The purpose of the article is to develop a technique for detecting singular states in GMC based on three collinear pairs.

Results. The analysis was carried out and the singular states of the GMC with three collinear pairs were revealed.

Conclusion. An original technique of a topological analysis of GMC based on collinear GD's pairs is proposed. This technique may be useful to developers of SC attitude control systems.

Keywords: spacecraft, gyrodrone, singular vector, singular surface.

INTRODUCTION

The task of a SC attitude control is the control by angular motion around CS mass center. Nowadays this task is very actual due to the ever-increasing requirements for the dynamic characteristics of CS angular maneuvers. The turns must run from any current position to any given position. Wherein the attitude accuracy after turning should be angular minutes, and the angular rates can attain the values 2–3 degrees per second. For example, the French SC Spot 7 for shooting the Earth's surface with high-resolution launched on June 30, 2014, provides the following dynamic characteristics of spatial maneuvers:

- angular orientation accuracy — 1,7 arcmin;
- the maximum angular rate — 2,1 deg/s.

The assurance of such high dynamic characteristics is complicated by the fact that the mass increase trend is observed for high resolution Earth remote sensing satellites. If the mass of the earlier satellites Ikonos, OrbView-3 was 720 and 304 kg, respectively, then the mass of the subsequent satellites QuickBird-2, WorldView-1, Geoeye-1, WorldView-2 exceeds 2000 kg. It is known that for SC with a considerable mass the most effective actuators of attitude control systems are the GMCs based on excessive number (more than three) two-gimbals control moment GDs [1, 2]. The main advantage of GMC is that they have the best “created control moment /own mass” ratio among other actuator types and at the same time allow for complex rotational movements of the SC necessary for solving many practically important attitude control tasks. The tasks of controlling angular motion with the help of GMC are one of the most difficult tasks among the SC attitude control tasks. The general approaches to the solution of these tasks and some fundamental results were first presented in the domestic publications by papers of Ye. N. Tokar in the 1970s – 1980s. A significant contribution to the development of this subject matter was also made by the results presented in [3, 4, 5, 6]. The modern approaches to the development SC attitude control algorithms using GDs are considered in [7, 8].

In order to maintain the SC given orientation for a long time and to turn with the required angular rate the GMC must have a sufficient angular momentum store. Possible summary angular momentum generated by the GDs form a certain area S_H in SC body fixed frame. This area is bounded by a closed surface of complex shape. Inside area S_H there are particular surfaces on which the control of the GDs is complicated or unfeasible. These surfaces are called singular. There are two types of singular surfaces: passable and impassable. Passable are called surfaces that can be passed by changing the mutual configuration of GDs angular momenta without changing the summary angular momentum of GMC. If this is not possible, then the surface is called impassable. In the class of GDs systems, the most rational are the GMC on the base of collinear pairs. The

aggregating of GDs into so-called collinear groups, in which the precession axes are arranged in parallel, gives one very important advantage: if there are six or more GDs, all the singular surfaces of such schemes are strictly passable [9]. Quite a lot of work has been devoted to the problem of analyzing singular surfaces. As an example, one can cite the works [10, 11], in which singular passable and impassable surfaces were classified based on analytical criteria for GMC arbitrary schemes, and a technique for the type surface determining was proposed. The works [12–16] deserve attention, where the attitude control problems using GMC are studied in detail. Despite the fact that a lot of work has been devoted to the problem of GMC topological analysis, interest to this problem continues unabated today.

The main goal of this paper is to develop an analytical technique for identifying singular surfaces for GMC based on collinear GD's pairs and to carry out a topological analysis of such GMC built on six GDs.

FORMULATION OF THE PROBLEM

The SC will be considered as a rigid body containing arbitrarily mounted collinear pairs of GDs. Let us introduce the right orthogonal coordinate system B rigidly attached to main SC body or Body Frame (BF) and the right orthogonal coordinate system G_i rigidly attached to mounting plains of the GDs in the i^{th} pair. Let us determine the position of the basis G_i relative to the basis B by the transition matrix C_{BG_i} . We assume that all the GDs have the same proper angular momenta $h_0 = \text{const}$.

For the GDs angular momenta vectors h_{2i-1} , h_{2i} , $i = 1, 2, \dots, N$ in the i^{th} collinear pair, the following relationships are valid:

$$\begin{aligned} h_{2i-1B} &= C_{BG_i} h_{2i-1G_i}, \\ h_{2iB} &= C_{BG_i} h_{2iG_i}. \end{aligned} \tag{1}$$

Hereinafter, the subscript in the vector designation indicates what basis it is projected.

Introduce the vector $\alpha = (\alpha_1, \dots, \alpha_{2N})^T$ and Jacobi gradient matrix

$$L(\alpha) = \begin{pmatrix} \frac{\partial h_{1B}}{\partial \alpha_1} & \dots & \frac{\partial h_{2NB}}{\partial \alpha_{2N}} \end{pmatrix}. \tag{2}$$

To ensure SC angular motion complete controllability it is necessary that the rank of the Jacobi gradient matrix $L(\alpha)$, which is determined by the dimension of its column space, is three. Consider the following problem: to find restrictions on the vector of the GDs precession angles α in the scalar equations form (equations of special surfaces)

$$f_k(\alpha) = 0, \quad k = 1, 2, \dots, K,$$

where K is the number of singular surfaces in GMC at which the rank of the Jacobi gradient matrix $L(\alpha)$ is less than three

$$\text{rang } L(\alpha) < 3 \tag{3}$$

The solution of the problem. If the condition (3) is satisfied, then the columns of the matrix $L(\alpha)$ lie in the same plane. In this case GMC cannot generate control torque components along unit vector λ_B of the normal to this plane. Such an unit vector $\lambda_B = (\lambda_{1B}, \lambda_{2B}, \lambda_{3B})^T$ is called the GMC singular vector and is determined from the condition

$$\lambda_B^T L(\alpha) = 0 \tag{4}$$

This task can be reformulated as follows: to find all the restrictions on the GDs precession angles vector, for which the homogeneous overdetermined system of equations (4) has nonzero solutions. Equation (4) taking into account expressions (1) and (2) can be represented as follows:

$$\begin{pmatrix} \frac{\partial}{\partial \alpha_1} h_{1G_1}^T C_{G_1B} \\ \frac{\partial}{\partial \alpha_2} h_{2G_1}^T C_{G_1B} \\ \vdots \\ \frac{\partial}{\partial \alpha_{2N-1}} h_{2N-1G_{N-1}}^T C_{G_{N-1}B} \\ \frac{\partial}{\partial \alpha_{2N}} h_{2NG_{2N}}^T C_{G_{2N}B} \end{pmatrix} \lambda_B = 0 \tag{5}$$

Denote by $g_i = \frac{h_i}{h_0}$ the angular momenta of the GDs in relative units. Then equation (5) can be represented as:

$$\begin{pmatrix} \frac{\partial}{\partial \alpha_1} g_{1G_1}^T C_{G_1B} \\ \frac{\partial}{\partial \alpha_2} g_{2G_1}^T C_{G_1B} \\ \vdots \\ \frac{\partial}{\partial \alpha_{2N-1}} g_{2N-1G_{N-1}}^T C_{G_{N-1}B} \\ \frac{\partial}{\partial \alpha_{2N}} g_{2NG_{2N}}^T C_{G_{2N}B} \end{pmatrix} \lambda_B = 0 \tag{6}$$

Suppose R_i is the coordinate system associated with i^{th} GD rotor. The axis x_R of this system coincides with the precession axis of the i^{th} GD, the axis y_R coincides with the rotor rotation axis of i^{th} GD and the z_R axis complements the system to a right-hand one. Further assume that the mutual orientation of the bases G_i and R_i corresponds to Fig. 1.

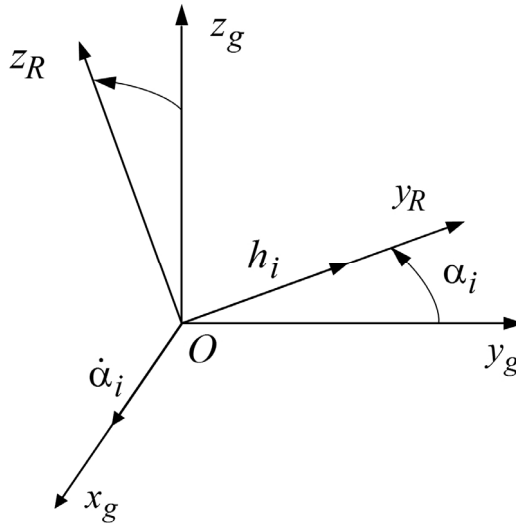


Fig. 1. Mutual orientation of the bases G_i and R_i

In this case, for the i^{th} collinear pair GDs angular momenta in the relative units, the following expressions are valid:

$$g_{G_j} = \begin{pmatrix} 0 \\ \cos \alpha_j \\ \sin \alpha_j \end{pmatrix} = \begin{pmatrix} 0 \\ y_j \\ z_j \end{pmatrix}, \quad j = 1, 2, \dots, 2N. \tag{7}$$

It follows from (7) that

$$\frac{\partial g_{G_j}}{\partial \alpha_j} = \begin{pmatrix} 0 \\ -\sin \alpha_j \\ \cos \alpha_j \end{pmatrix} = \begin{pmatrix} 0 \\ -z_j \\ y_j \end{pmatrix}. \tag{8}$$

By substituting (8) into (6), we obtain N subsystems

$$\begin{pmatrix} 0 & -z_{2i-1} & y_{2i-1} \\ 0 & -z_{2i} & y_{2i} \end{pmatrix} \lambda_{G_i} = 0, \quad i = 1, 2, \dots, N, \tag{9}$$

where

$$\lambda_{G_i} = C_{G_i B} \lambda_B. \tag{10}$$

Consider the matrices

$$A_{2i-1, 2i} = \begin{pmatrix} -z_{2i-1} & y_{2i-1} \\ -z_{2i} & y_{2i} \end{pmatrix}. \tag{11}$$

Denote by $D_{2i-1, 2i}$ the determinant of the matrix $A_{2i-1, 2i}$. Finding this determinant, we obtain

$$D_{2i-1,2i} = z_{2i}y_{2i-1} - z_{2i-1}y_{2i} = \sin \alpha_{2i} \cos \alpha_{2i-1} - \cos \alpha_{2i} \sin \alpha_{2i-1} = \sin(\alpha_{2i} - \alpha_{2i-1}) \quad (12)$$

The solution of the systems (9) depends on the values of the determinants and is determined by the expression:

$$\lambda_B = C_{BG_i} \lambda_{G_i} = \begin{cases} \begin{pmatrix} \pm 1 \\ 0 \\ 0 \end{pmatrix}, & D_{2i-1,2i} \neq 0; \\ \begin{pmatrix} \sin \varepsilon_i \\ y_{2i} \cos \varepsilon_i \\ z_{2i} \cos \varepsilon_i \end{pmatrix} = \begin{pmatrix} \sin \varepsilon_i \\ \pm y_{2i-1} \cos \varepsilon_i \\ \pm z_{2i-1} \cos \varepsilon_i \end{pmatrix}, & D_{2i-1,2i} = 0; \end{cases} \quad (13)$$

where ε_i is an arbitrary angle. The geometric interpretation of the solution (13) is as follows. According to equation (9), the vector λ_{G_i} is a singular vector of the i^{th} collinear pair. If the angular momenta of the included in the collinear pair GDs do not lie on one straight line, then the singular vector λ_{G_i} is perpendicular to the plane passing through the vectors $g_{G_{2i-1}}$, $g_{G_{2i}}$ and parallel to the GDs of precession axes, which, according to Fig. 1, is the axis x_i . If the angular momenta of the included into collinear pair GDs lie on one straight line, then they form a straight line to which an infinite number of perpendiculars can be drawn, which position in the basis G_i is determined by the angle ε_i . Since, if the condition $D_{2i-1,2i} = 0$ is satisfied, the angular momenta in the i^{th} collinear pair are related by the relationship $g_{G_{2i-1}} = \pm g_{G_{2i}}$, hereafter only vectors $g_{G_{2i}}$ are considered. Then, taking into account expressions (7) and (10), for a singular vector λ_B it is true the relationship

$$\lambda_B = C_{BG_i} \lambda_{G_i} = \begin{cases} \pm c_1^{BG_i}, & D_{2i-1,2i} \neq 0; \\ c_1^{BG_i} \sin \varepsilon_i + c_2^{BG_i} y_{2i} \cos \varepsilon_i + c_3^{BG_i} z_{2i} \cos \varepsilon_i, & D_{2i-1,2i} = 0; \end{cases} \quad (14)$$

where $c_k^{BG_i}, k=1, 2, 3$ are the columns of matrix C_{BG_i} . N relationships (14) define the same vector λ_B , therefore, if λ_B is not a zero solution to system (4), then for any $i \neq j, i, j = 1, 2, \dots, N$, the equalities should be satisfied

$$C_{BG_i} \lambda_{G_i} = C_{BG_j} \lambda_{G_j} \quad (15)$$

Thus, if there are the GDs precession angles vector α and angles ε_i for which all $N(N-1)/2$ equalities (15) hold, then there exist nonzero solutions of system (4) and there is a singular state. If there are no such angles, then $\lambda_B = 0$ and there is no singular state.

TOPOLOGICAL ANALYSIS OF THE 3-SPE SCHEME

As an example of the application of the proposed technique, consider the GMC containing three collinear pairs. In the original work of J.W.Crenshaw [17], the excessive multiple scheme based on three collinear GDs pairs was named as 3-Scissored Pair Ensemble (3-SPE).

We will assume that the precession axes of the GDs of the first group coincide with the axis z_B , the precession axes of the second group coincide with the axis y_B and the precession axes of the third group coincide with the axis x_B (Fig. 2). For such a GMC scheme the transition matrixes C_{BG_i} , $i = 1, 2, 3$ have the forms:

$$C_{BG_1} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad C_{BG_2} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad C_{BG_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

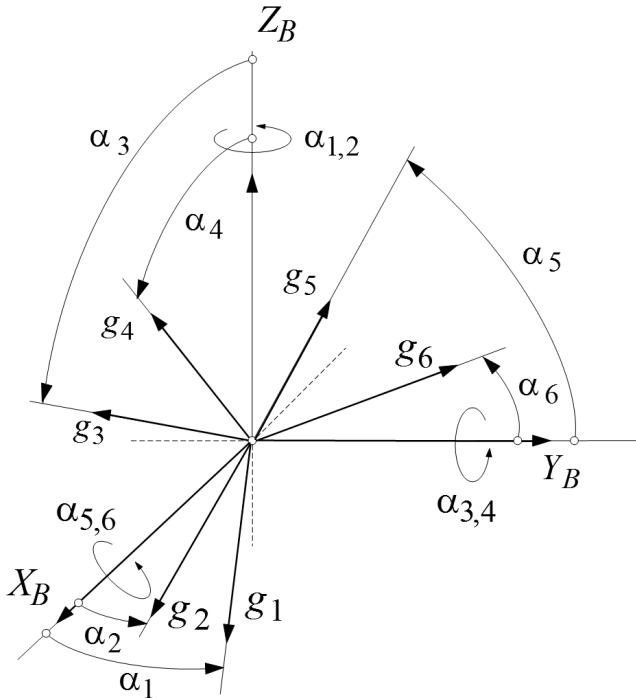


Fig. 2. Precession axes arrangement of GDs in 3-SPE scheme

In this case, for the projections of the relative angular momenta along the axes of B basis, the following expressions are valid:

$$\begin{aligned}
 \mathbf{g}_{1B} &= \begin{pmatrix} -\sin \bar{\sigma}_1 \\ \cos \bar{\sigma}_1 \\ 0 \end{pmatrix} = \begin{pmatrix} -z_1 \\ y_1 \\ 0 \end{pmatrix} = \begin{pmatrix} x_{1B} \\ y_{1B} \\ 0 \end{pmatrix}, & \mathbf{g}_{2B} &= \begin{pmatrix} -\sin \bar{\sigma}_2 \\ \cos \bar{\sigma}_2 \\ 0 \end{pmatrix} = \begin{pmatrix} -z_2 \\ y_2 \\ 0 \end{pmatrix} = \begin{pmatrix} x_{2B} \\ y_{2B} \\ 0 \end{pmatrix}, \\
 \mathbf{g}_{3B} &= \begin{pmatrix} -\cos \bar{\sigma}_3 \\ 0 \\ \sin \alpha_3 \end{pmatrix} = \begin{pmatrix} -y_3 \\ 0 \\ z_3 \end{pmatrix} = \begin{pmatrix} x_{3B} \\ 0 \\ z_{3B} \end{pmatrix}, & \mathbf{g}_{4B} &= \begin{pmatrix} -\cos \bar{\sigma}_4 \\ 0 \\ \sin \alpha_4 \end{pmatrix} = \begin{pmatrix} -y_4 \\ 0 \\ z_4 \end{pmatrix} = \begin{pmatrix} x_{4B} \\ 0 \\ z_{4B} \end{pmatrix}, & (16) \\
 \mathbf{g}_{5B} &= \begin{pmatrix} 0 \\ \cos \alpha_5 \\ \sin \alpha_5 \end{pmatrix} = \begin{pmatrix} 0 \\ y_5 \\ z_5 \end{pmatrix} = \begin{pmatrix} 0 \\ y_{5B} \\ z_{5B} \end{pmatrix}, & \mathbf{g}_{6B} &= \begin{pmatrix} 0 \\ \cos \alpha_6 \\ \sin \alpha_6 \end{pmatrix} = \begin{pmatrix} 0 \\ y_6 \\ z_6 \end{pmatrix} = \begin{pmatrix} 0 \\ y_{6B} \\ z_{6B} \end{pmatrix}.
 \end{aligned}$$

To determine the conditions under which the homogeneous system (4) has a nonzero solution, we analyze the dependence of the solution of this system on the values of the determinants $D_{1,2}$, $D_{3,4}$, $D_{5,6}$. According to (12) these determinants are defined by the formulae

$$D_{1,2} = \sin(\alpha_2 - \alpha_1), \quad D_{3,4} = \sin(\alpha_4 - \alpha_3), \quad D_{5,6} = \sin(\alpha_6 - \alpha_5).$$

Depending on the numerical values of the determinants $D_{1,2}$, $D_{3,4}$, $D_{5,6}$, the following different options are possible:

1) $D_{1,2} \neq 0$, $D_{3,4} \neq 0$, $D_{5,6} \neq 0$.

In this case, according to (15), the condition for the existence of a nonzero solution of system (4) is $\begin{pmatrix} 0 \\ 0 \\ \pm 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \pm 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \pm 1 \\ 0 \\ 0 \end{pmatrix}$.

But obviously equalities are impossible. Therefore, $\lambda_B = 0$ and no singular state.

2) $D_{1,2} = 0$, $D_{3,4} \neq 0$, $D_{5,6} \neq 0$.

From relationships (14) and (15) it follows that for the existence of a nonzero solution the equalities

$$\begin{pmatrix} -z_2 \cos \varepsilon_1 \\ y_2 \cos \varepsilon_1 \\ \sin \varepsilon_1 \end{pmatrix} = \begin{pmatrix} 0 \\ \pm 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \pm 1 \\ 0 \\ 0 \end{pmatrix}$$

must be satisfied, which is impossible and there is no singular state.

3) $D_{1,2} \neq 0$, $D_{3,4} = 0$, $D_{5,6} \neq 0$.

In this case, for the existence of a nonzero solution the equalities

$$\begin{pmatrix} 0 \\ 0 \\ \pm 1 \end{pmatrix} = \begin{pmatrix} -y_4 \cos \varepsilon_2 \\ \sin \varepsilon_2 \\ z_4 \cos \varepsilon_2 \end{pmatrix} = \begin{pmatrix} \pm 1 \\ 0 \\ 0 \end{pmatrix}.$$

must be satisfied, which is impossible and there is no singular state.

4) $D_{1,2} \neq 0, D_{3,4} \neq 0, D_{5,6} = 0.$

In this case, for the existence of a nonzero solution the equalities

$$\begin{pmatrix} 0 \\ 0 \\ \pm 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \pm 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \sin \varepsilon_3 \\ y_6 \cos \varepsilon_3 \\ z_6 \cos \varepsilon_3 \end{pmatrix}$$

must be satisfied. These equalities are not satisfied, $\lambda_B = 0$, therefore there is no single state.

5) $D_{1,2} \neq 0, D_{3,4} = 0, D_{5,6} = 0.$

A necessary condition for the existence of a nonzero solution for this case is the validity following equalities:

$$\begin{pmatrix} 0 \\ 0 \\ \pm 1 \end{pmatrix} = \begin{pmatrix} -y_4 \cos \varepsilon_2 \\ \sin \varepsilon_2 \\ z_4 \cos \varepsilon_2 \end{pmatrix} = \begin{pmatrix} \sin \varepsilon_3 \\ y_6 \cos \varepsilon_3 \\ z_6 \cos \varepsilon_3 \end{pmatrix}.$$

These equalities are possible when $\varepsilon_2 = 0, \pi; \varepsilon_3 = 0, \pi; y_4 = 0, y_6 = 0, z_4 = \pm 1, z_6 = \pm 1$. Therefore $\lambda_B \neq 0$ and there is a singular state. Let us find the equation of the surface in the basis B on which this state arises. From the equalities of the determinants $D_{3,4}$ and $D_{5,6}$ to zero and expressions (16) it follows that $y_3 = \pm y_4 = -x_{3B} = \mp x_{4B} = 0, y_5 = \pm y_6 = y_{5B} = \pm y_{6B} = 0$. Then

$$x_{3B}^2 + x_{4B}^2 + y_{5B}^2 + y_{6B}^2 = 0. \tag{17}$$

The constraint (17) defines a surface in the basis B in the form of a unit circle lying in a plane $OX_B Y_B$, to which the singular vector $\lambda_B = (0, 0, \pm 1)^T$ corresponds.

6) $D_{1,2} = 0, D_{3,4} \neq 0, D_{5,6} = 0.$

In this case, the condition for the zero solution existence is

$$\begin{pmatrix} -z_2 \cos \varepsilon_1 \\ y_2 \cos \varepsilon_1 \\ \sin \varepsilon_1 \end{pmatrix} = \begin{pmatrix} 0 \\ \pm 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \sin \varepsilon_3 \\ y_6 \cos \varepsilon_3 \\ z_6 \cos \varepsilon_3 \end{pmatrix}.$$

It is satisfied if $\varepsilon_1 = 0, \pi; \varepsilon_3 = 0, \pi; y_2 = \pm 1, y_6 = \pm 1, z_2 = 0, z_6 = 0$. From the conditions $D_{1,2} = 0$ and $D_{5,6} = 0$, the equalities follow: $y_1 = \pm y_2 = -x_{1B} = \mp x_{2B} = 0, z_5 = \pm z_6 = z_{5B} = \pm z_{6B} = 0$. A nonzero solution will occur if there is a constraint

$$x_{1B}^2 + x_{2B}^2 + z_{5B}^2 + z_{6B}^2 = 0. \tag{18}$$

To the constraint (18) corresponds a circle lying in a plane OX_BZ_B of the basis B and a singular vector $\lambda_B = (0, \pm 1, 0)^T$.

7) $D_{1,2} = 0, D_{3,4} = 0, D_{5,6} \neq 0$.

Making the transformations similar to those performed in cases 6) and 7), we obtain the following equation for the singular surface:

$$y_{1B}^2 + y_{2B}^2 + z_{3B}^2 + z_{4B}^2 = 0. \tag{19}$$

The constraint (19) defines a circle lying in a plane OY_BZ_B of the basis B and a singular vector $\lambda_B = (\pm 1, 0, 0)^T$.

The spatial positions of singular planes for cases 5) – 7) are shown in Fig. 3, [14].

8) $D_{12} = 0, D_{13} = 0, D_{23} = 0$.

The condition for the existence of a nonzero solution in this case is

$$\lambda_B = \begin{pmatrix} -z_2 \cos \varepsilon_1 \\ y_2 \cos \varepsilon_1 \\ \sin \varepsilon_1 \end{pmatrix} = \begin{pmatrix} -y_4 \cos \varepsilon_2 \\ \sin \varepsilon_2 \\ z_4 \cos \varepsilon_2 \end{pmatrix} = \begin{pmatrix} \sin \varepsilon_3 \\ y_6 \cos \varepsilon_3 \\ z_6 \cos \varepsilon_3 \end{pmatrix}. \tag{20}$$

Considering that the sines of angles $\varepsilon_k, k=1,2,3$ are the coordinates of the vector λ_B :

$$\lambda_{1B} = \sin \varepsilon_3, \quad \lambda_{2B} = \sin \varepsilon_2, \quad \lambda_{3B} = \sin \varepsilon_1 \tag{21}$$

and λ_B is the normalized vector, then from the equality of its norm to one follows the condition for the existence of a nonzero solution

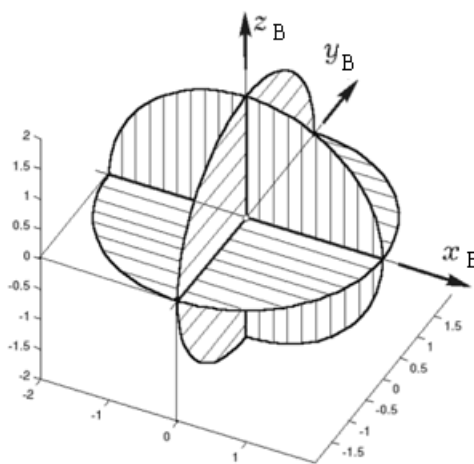


Fig. 3. The set of special GMC surfaces when two pairs GDs are in singular state

$$\sin^2 \varepsilon_1 + \sin^2 \varepsilon_2 + \sin^2 \varepsilon_3 = 1. \quad (22)$$

To satisfy equalities (20), it is necessary that the following relationships hold for the coordinates of vectors g_{2i} :

$$\begin{aligned} y_2 &= \frac{\sin \varepsilon_2}{\cos \varepsilon_1}, & z_2 &= -\frac{\sin \varepsilon_3}{\cos \varepsilon_1}; \\ y_4 &= -\frac{\sin \varepsilon_3}{\cos \varepsilon_2}, & z_4 &= \frac{\sin \varepsilon_1}{\cos \varepsilon_2}; \\ y_6 &= -\frac{\sin \varepsilon_2}{\cos \varepsilon_3}, & z_6 &= \frac{\sin \varepsilon_1}{\cos \varepsilon_3}. \end{aligned} \quad (23)$$

Consider a few special cases:

- a) $\varepsilon_1 = 0, \varepsilon_2 \neq 0, \varepsilon_3 \neq 0$;
- b) $\varepsilon_1 \neq 0, \varepsilon_2 = 0, \varepsilon_3 \neq 0$;
- c) $\varepsilon_1 \neq 0, \varepsilon_2 \neq 0, \varepsilon_3 = 0$;
- d) $\varepsilon_1 = 0, \varepsilon_2 = 0, \varepsilon_3 = \pm \frac{\pi}{2}$.

In the case a) from conditions (22–23) we have the following relationships:

$$\begin{aligned} \sin^2 \varepsilon_1 + \sin^2 \varepsilon_2 + \sin^2 \varepsilon_3 &= \sin^2 \varepsilon_2 + \sin^2 \varepsilon_3 = 1, \\ \sin^2 \varepsilon_2 &= 1 - \sin^2 \varepsilon_3 = \cos^2 \varepsilon_3, \quad \sin \varepsilon_2 = \pm \cos \varepsilon_3, \quad \cos \varepsilon_2 = \pm \sin \varepsilon_3, \end{aligned}$$

$$y_2 = \frac{\sin \varepsilon_2}{\cos \varepsilon_1} = \pm \cos \varepsilon_3, \quad z_2 = -\frac{\sin \varepsilon_3}{\cos \varepsilon_1} = -\sin \varepsilon_3,$$

$$y_4 = -\frac{\sin \varepsilon_3}{\cos \varepsilon_2} = \pm 1, \quad z_4 = \frac{\sin \varepsilon_1}{\cos \varepsilon_2} = 0,$$

$$y_6 = -\frac{\sin \varepsilon_2}{\cos \varepsilon_3} = \pm 1, \quad z_6 = \frac{\sin \varepsilon_1}{\cos \varepsilon_3} = 0,$$

$$y_1 = \pm y_2, \quad z_1 = \pm z_2,$$

$$y_3 = \pm y_4, \quad z_3 = \pm z_4,$$

$$y_5 = \pm y_6, \quad z_5 = \pm z_6.$$

By substituting these relationships into (16) we find

$$g_{1B} = \begin{pmatrix} x_{1B} \\ y_{1B} \\ 0 \end{pmatrix} = \begin{pmatrix} \pm z_2 \\ \pm y_2 \\ 0 \end{pmatrix} = \begin{pmatrix} \mp \sin \varepsilon_3 \\ \pm \cos \varepsilon_3 \\ 0 \end{pmatrix}, \quad g_{2B} = \begin{pmatrix} x_{2B} \\ y_{2B} \\ 0 \end{pmatrix} = \begin{pmatrix} -z_2 \\ y_2 \\ 0 \end{pmatrix} = \begin{pmatrix} \sin \varepsilon_3 \\ \pm \cos \varepsilon_3 \\ 0 \end{pmatrix},$$

$$g_{3B} = \begin{pmatrix} x_{3B} \\ 0 \\ z_{3B} \end{pmatrix} = \begin{pmatrix} -y_3 \\ 0 \\ z_3 \end{pmatrix} = \begin{pmatrix} \pm 1 \\ 0 \\ 0 \end{pmatrix}, \quad g_{4B} = \begin{pmatrix} x_{4B} \\ 0 \\ z_{4B} \end{pmatrix} = \begin{pmatrix} -y_4 \\ 0 \\ z_4 \end{pmatrix} = \begin{pmatrix} \pm 1 \\ 0 \\ 0 \end{pmatrix},$$

$$g_{5B} = \begin{pmatrix} 0 \\ y_{5B} \\ z_{5B} \end{pmatrix} = \begin{pmatrix} 0 \\ y_5 \\ z_5 \end{pmatrix} = \begin{pmatrix} 0 \\ \pm 1 \\ 0 \end{pmatrix}, \quad g_{6B} = \begin{pmatrix} 0 \\ y_{6B} \\ z_{6B} \end{pmatrix} = \begin{pmatrix} 0 \\ y_6 \\ z_6 \end{pmatrix} = \begin{pmatrix} 0 \\ \pm 1 \\ 0 \end{pmatrix}.$$

In this case, the singular surface is a unit circle lying in a plane $OX_B Y_B$.

Having done the similar calculations for cases b) and c), we obtain that in case b) the singular surface is a unit circle in the plane $OX_B Z_B$, and in case c) it is a unit circle lying in the plane $OY_B Z_B$.

For the case d) a nonzero solution will exist if the following equalities hold

$$n_B = \begin{pmatrix} -z_2 \\ y_2 \\ 0 \end{pmatrix} = \begin{pmatrix} -y_4 \\ 0 \\ z_4 \end{pmatrix} = \begin{pmatrix} \pm 1 \\ 0 \\ 0 \end{pmatrix},$$

from which it follows that $y_2 = y_{2B} = 0$, $z_2 = -x_{2B} = \pm 1$, $y_4 = -x_{4B} = \pm 1$, $z_4 = z_{4B} = 0$ and the singular surface is a straight line coinciding with the axis OX_B . Similarly, it can be shown that when $\varepsilon_1 = \pm \frac{\pi}{2}$, $\varepsilon_2 = 0$, $\varepsilon_3 = 0$ the singular surface is transformed into a straight line that coincides with the axis OZ_B , and as $\varepsilon_1 = 0$, $\varepsilon_2 = \pm \frac{\pi}{2}$, $\varepsilon_3 = 0$ the singular surface is transformed into a straight line that coincides with the axis OY_B .

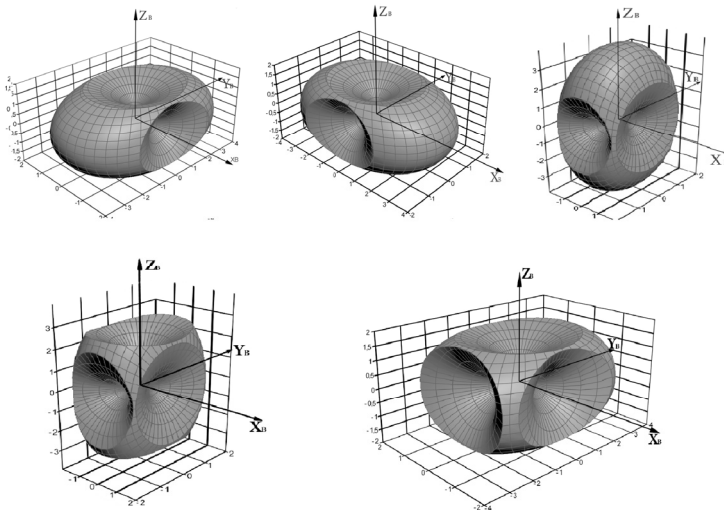


Fig. 4. The set of special GMC surfaces when three pairs GDs are in singular state

Thus, when one of the angles ε_k , $k=1, 2, 3$ tends to zero or to π , the singular surface is transformed into a circle lying in one of the coordinate planes, and when one of the angles ε_k tends to $\frac{\pi}{2}$ or to $\frac{3\pi}{2}$ the singular surface is transformed into a straight line coinciding with one of the BF axes. In the general case, when $D_{12} = 0$, $D_{13} = 0$, $D_{23} = 0$, the singular surface has a quite complex shape. The boundary of this shape has six cavity (“craters”), the axes of which coincide with the BF axes. The set of possible native singular states of the 3-SPE scheme is shown in Fig. 4.

CONCLUSION

An original technique of a topological analysis of GMC based on collinear GD’s pairs is proposed. Using this technique, the analysis was carried out and the singular states of the GMC containing three collinear pairs were revealed. The proposed technique may be useful to developers of SC attitude control systems.

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ТОПОЛОГІЧНИЙ АНАЛІЗ ОБЛАСТІ ДОПУСТИМИХ ЗНАЧЕНЬ КІНЕТИЧНОГО МОМЕНТУ СИЛОВИХ ГІРОСКОПІЧНИХ КОМПЛЕКСІВ КРАТНИХ СХЕМ

Вступ. Для забезпечення високих динамічних характеристик супутників дистанційного зондування Землі в їхніх системах орієнтації в якості виконавчих органів можуть використовуватися силові гіроскопічні комплекси (СГК), які є надлишковою (більше 3) системою двоступеневих силових гіроскопів (гіродінів). Завдання гіросилового керування кутів рухом є одними з найскладніших серед завдань керування переорієнтацією КА. Центральним питанням під час вирішення цих завдань є питання синтезу законів керування кутами прецесії окремих гіродінів за їх надлишковості. Успіх у розв'язанні завдання керування багато в чому визначається вибором структури СГК, під якою розуміється кількість використуваних гіродінів і взаємне розташування їхніх осей прецесії. Від такого вибору залежить можливість формування СГК необхідного керуючого моменту, наявність і кількість особливих станів СГК, складність законів керування кутами прецесії окремих гіродінів входять в СГК. Це зумовлено тим, що для тривалого підтримання постійної орієнтації апарата і виконання ним розворотів з необхідною кутною швидкістю СГК повинен мати достатній запас кінетичного моменту (КМ). Допустимі значення сумарного КМ, створюваного гіродінами, утворюють в системі координат, жорстко пов'язаної з КА, деяку область, яка обмежена замкнутою поверхнею складної форми. Усередині цієї області розташовуються особливі поверхні, на яких керування гіродінами ускладнено або взагалі неможливе. Ці поверхні прийнято називати сингулярними. У зв'язку з цим, у разі керуванні орієнтацією КА за допомоги СГК, крім керування швидкістю прецесії окремих гіродінів, необхідно керувати і взаємною орієнтацією кінетичних моментів гіродінів, що входять в СГК. Водночас одним з найважливіших завдань синтезу законів керування з використанням СГК є завдання виявлення сингулярних поверхонь (топологічного аналізу) області допустимого кінетичного моменту СГК.

Мета роботи — розроблення методики виявлення сингулярних поверхонь в СГК кратних схем.

Результат — проведено аналіз та виявлено сингулярні стани схеми СГК, що містить три Колінеарні пари.

Висновки. Запропоновано оригінальну методику проведення топологічного аналізу СГК кратних схеми. Методика може бути корисна розробникам систем орієнтації КА.

Ключові слова: космічний апарат, гіродін, сингулярний вектор, сингулярна поверхня.

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ТОПОЛОГИЧЕСКИЙ АНАЛИЗ ОБЛАСТИ ДОПУСТИМЫХ ЗНАЧЕНИЙ КИНЕТИЧЕСКОГО МОМЕНТА СИЛОВЫХ ГИРОСКОПИЧЕСКИХ КОМПЛЕКСОВ КРАТНЫХ СХЕМ

Введение. Для обеспечения высоких динамических характеристик спутников дистанционного зондирования Земли в их системах ориентации в качестве исполнительных органов могут использоваться силовые гироскопические комплексы (СГК), представляющие собой избыточную (более 3) систему двухступенных силовых гироскопов (гиродинов). Задачи гиросилового управления угловым движением являются одними из наиболее сложных среди задач управления переориентацией КА. Центральным вопросом при решении этих задач является вопрос синтеза законов управления углами прецессии отдельных гиродинов при их избыточности. Успех в решении задачи управления во многом определяется выбором структуры СГК, под которой понимается количество используемых гиродинов и взаимное расположение их осей прецессии. От такого выбора зависит возможность формирования СГК требуемого управляющего момента, наличие и количество особых состояний СГК, сложность законов управления углами прецессии отдельных гиродинов, входящих в СГК. Обусловлено это тем, что для продолжительного поддержания заданной ориентации аппарата и выполнения им разворотов с требуемой угловой скоростью, СГК должен обладать достаточным запасом кинетического момента (КМ). Допустимые значения суммарного КМ, создаваемого гиродинами, образуют в системе координат, жестко связанной с КА, некоторую область, которая ограничена замкнутой поверхностью сложной формы. Внутри этой области располагаются особые поверхности, на которых управление гиродинами усложнено или вообще неосуществимо. Эти поверхности принято называть сингулярными. В связи с этим, при управлении ориентацией КА с помощью СГК, кроме управления скоростью прецессии отдельных гиродинов, необходимо управлять и взаимной ориентацией кинетических моментов гиродинов, входящих в СГК. При этом одной из важнейших задач синтеза законов управления с использованием СГК является задача выявления сингулярных поверхностей (топологического анализа) области допустимого кинетического момента СГК.

Цель работы — разработка методики выявления сингулярных поверхностей в СГК кратных схем.

Результат — проведен анализ и выявлены сингулярные состояния схемы СГК, содержащей три коллинеарные пары.

Выводы. Предложена оригинальная методика проведения топологического анализа СГК кратных схем. Методика может быть полезна разработчикам систем ориентации КА.

Ключевые слова: космический аппарат, гиродин, сингулярный вектор, сингулярная поверхность.