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SUBSTANTIATION OF BOUNDARY ACCELERATIONS OF ROLLER FORMING UNIT OPTIMAL REVERSAL MODE ACCORDING TO FOURTH-ORDER ACCELERATION

[1] **Viacheslav S. Loveykin**
lovvs@ukr.net,
ORCID: 0000-0003-4259-3900

[2] **Konstantyn I. Pochka**
shanovniy@ukr.net,
ORCID: 0000-0002-0355-002X

[2] **Nikolay A. Pristaylo**
pristaylo_na@ukr.net,
ORCID: 0000-0003-3151-4680

[2] **Olga B. Pochka**
zasedkoolga@ukr.net,
ORCID: 0000-0001-5701-978X

[1] National University of Life and Environmental Sciences of Ukraine,
15, Heroiv Oborony St., Kyiv, 03041, Ukraine

[2] Kyiv National University of Construction and Architecture
31, Povitroflotsky Ave., Kyiv, 03037, Ukraine

In order to increase the reliability and durability of a roller forming unit, we calculated a combined mode of the reciprocating movement of a forming trolley with the reversal according to the fourth-order acceleration with the optimal values of boundary accelerations. In determining the combined mode of the reciprocating movement of the forming trolley with the reversal according to the fourth-order acceleration with the optimal values of boundary accelerations, the criterion of the movement was the criteria action, which is a time integral with the integrand function expressing the "energy" of the unit fourth-order accelerations. We calculated the functions of changing the kinematic characteristics of the forming trolley moving from one extreme position to another, with the functions corresponding to the combined mode of the reciprocating movement of the forming trolley with the reversal according to the fourth-order acceleration with the optimal values of boundary accelerations. It is proposed that the design use a drive in the form of a cam mechanism, for which we constructed a cam profile to provide for the combined mode of the reciprocating movement of the forming trolley with the reversal according to the fourth-order acceleration with the optimal values of boundary accelerations. It is also proposed that the roller forming unit design use a drive from a high-torque stepper motor embedded into the compaction rollers of the forming trolley. The use of the specified drive mechanism in the unit leads to a reduction in the dynamic loads in the drive mechanism elements and, accordingly, to an increase in the reliability and durability of the unit as a whole. The results of the work may further be useful for refining and improving the existing engineering methods for calculating the drive mechanisms of roller forming machines both at the design stages and in real operation modes. Also, the results of the work can be used in designing or improving mechanisms with the reciprocating movement of their actuating elements.

Keywords: *installation, forming trolley, movement mode, drive, acceleration, cam, stepper motor.*

Introduction

In the existing units for the surface compaction of reinforced concrete products, the reciprocating movement of the forming trolley with compaction rollers is provided either by a crank-slider drive or hydraulic drive [1–7]. During constant accelerating and decelerating movement modes, significant dynamic loads occur in the elements of both the drive mechanism and forming trolley, which can lead to the unit premature failure.

Analysis of Recent Research and Publications

In the existing theoretical and experimental studies of the units for the roller forming of reinforced concrete products, their design parameters and productivity are substantiated [1–4, 7–13]. However, there is a lack of attention to the study of the existing dynamic loads and movement modes [5, 6, 14], which greatly affects both the unit operation and quality of the finished products.

In [15], the optimization of the dynamic reversal mode of a roller forming unit was carried out. However, in this mode, the acceleration and the second-order acceleration (jerk) of the forming trolley are of great importance in its extreme positions. In optimizing the jerk reversal mode of the roller forming unit [16], the acceleration of the trolley in the extreme positions occurs smoothly, but the jerk changes abruptly and is quite significant. The optimization of the reversal mode of the roller forming unit according to the third-order acceleration [17, 18] results in the situation in which in the extreme positions of the unit, both the acceleration and jerk occur smoothly, but the third-order acceleration is quite large and changes abruptly from zero to its maximum value. Therefore, the problem of improving the drive mechanism of a roller forming unit is important to ensure such a mode of movement of the forming trolley, which would reduce the dynamic loads in the unit elements and increase the unit durability.

Objective

The objective of this work is to improve the design of the drive mechanism of a roller forming unit to increase its reliability and durability.

Setting the Research Task

In order for a roller forming unit to compact a concrete mixture, the forming trolley should have a constant velocity reciprocating movement on the entire area, which would positively affect the quality of the finished product. However, in practice, this mode of movement cannot be implemented, because it has no acceleration and deceleration phases, without which no cyclical movement can occur. Therefore, it is proposed to implement such a mode of the forming trolley movement from one extreme position to another, in which there are areas for reversing with minimal dynamic loads and moving with constant velocity.

For a smooth process of the forming trolley reversal, it was proposed to implement it in agreement with the optimal movement mode according to the fourth-order acceleration.

The criteria for the mode of movement of mechanisms and machines can be the coefficients of non-uniform motion and dynamism [19–21]. In this paper, as a criterion of the mode of movement, a criteria action is used, which is a time integral with an integrand function that expresses the value characterizing the movement or system operation. For the optimal reversal mode according to the fourth-order acceleration, the optimal movement criterion is as follows:

$$I_Q = \int_0^{t_\mathrm{p}} Q\,dt \to \min , \qquad (1)$$

where t is the time; t_p is the reversal duration; Q is the "energy" of the fourth- order accelerations

$$Q = \frac{1}{2} \cdot m \cdot \overset{V}{x}{}^2 , \qquad (2)$$

where m is the mass of the forming trolley; $\overset{V}{x}$ is the fourth-order acceleration.

In this paper, the "energy" of the fourth-order accelerations is designated by Q. Different designations of energies are used to determine a complex optimality criterion of the mode of movement of the forming trolley. In a number of works, "energy" has the following designations: V for the "energy" of accelerations [15], W for the "energy" of jerks (second-order accelerations) [16], Z for the "energy" of the third-order accelerations [17, 18], Q for the "energy" of the fourth-order accelerations [19].

The minimum condition for criterion (1) is the Poisson equation:

$$\frac{\partial Q}{\partial x} - \frac{d}{dt}\frac{\partial Q}{\partial \dot{x}} + \frac{d^2}{dt^2}\frac{\partial Q}{\partial \ddot{x}} - \frac{d^3}{dt^3}\frac{\partial Q}{\partial \dddot{x}} + \frac{d^4}{dt^4}\frac{\partial Q}{\partial \overset{IV}{x}} - \frac{d^5}{dt^5}\frac{\partial Q}{\partial \overset{V}{x}} = 0 , \qquad (3)$$

where x, $\dot{x}$, $\ddot{x}$, $\dddot{x}$, $\overset{IV}{x}$ are the forming trolley displacement coordinate, velocity, acceleration, acceleration of the second and third orders, respectively.

From expression (3), taking into account dependence (2), we can write

$$\frac{\partial Q}{\partial x} = \frac{\partial Q}{\partial \dot{x}} = \frac{\partial Q}{\partial \ddot{x}} = \frac{\partial Q}{\partial \dddot{x}} = \frac{\partial Q}{\partial \overset{IV}{x}} = 0 ; \quad \frac{\partial Q}{\partial \overset{V}{x}} = m \cdot \overset{V}{x} ; \quad \frac{d^5}{dt^5}\frac{\partial Q}{\partial \overset{V}{x}} = m \cdot \overset{X}{x} = 0 . \qquad (4)$$

The first five terms in equations (4) are taken equal to zero $\frac{\partial Q}{\partial x}=\frac{\partial Q}{\partial \dot{x}}=\frac{\partial Q}{\partial \ddot{x}}=\frac{\partial Q}{\partial \dddot{x}}=\frac{\partial Q}{\partial \overset{IV}{x}}=0$ on the basis of dependence (2). Since in expression (2) $\overset{V}{x}$ is the fourth-order acceleration, then the forming trolley displacement coordinate x, velocity $\dot{x}$, acceleration $\ddot{x}$, acceleration of the second order $\dddot{x}$ and third order $\overset{IV}{x}$ have no influence on equation (2). Therefore, the partial derivatives from the "energy" of the fourth-order accelerations for the indicated parameters are zero. After integrating the last relation of equations (4), these conditions are also satisfied.

From the last equation in (4) we obtain the differential equation and its solutions

$$\overset{X}{x}=0;\quad \overset{IX}{x}=C_1;\quad \overset{VIII}{x}=C_1\cdot t+C_2;\quad \overset{VII}{x}=\frac{1}{2}\cdot C_1\cdot t^2+C_2\cdot t+C_3;$$

$$\overset{VI}{x}=\frac{1}{6}\cdot C_1\cdot t^3+\frac{1}{2}\cdot C_2\cdot t^2+C_3\cdot t+C_4;\quad \overset{V}{x}=\frac{1}{24}\cdot C_1\cdot t^4+\frac{1}{6}\cdot C_2\cdot t^3+\frac{1}{2}\cdot C_3\cdot t^2+C_4\cdot t+C_5;$$

$$\overset{IV}{x}=\frac{1}{120}\cdot C_1\cdot t^5+\frac{1}{24}\cdot C_2\cdot t^4+\frac{1}{6}\cdot C_3\cdot t^3+\frac{1}{2}\cdot C_4\cdot t^2+C_5\cdot t+C_6;$$

$$\dddot{x}=\frac{1}{720}\cdot C_1\cdot t^6+\frac{1}{120}\cdot C_2\cdot t^5+\frac{1}{24}\cdot C_3\cdot t^4+\frac{1}{6}\cdot C_4\cdot t^3+\frac{1}{2}\cdot C_5\cdot t^2+C_6\cdot t+C_7;$$

$$\ddot{x}=\frac{1}{5040}\cdot C_1\cdot t^7+\frac{1}{720}\cdot C_2\cdot t^6+\frac{1}{120}\cdot C_3\cdot t^5+\frac{1}{24}\cdot C_4\cdot t^4+\frac{1}{6}\cdot C_5\cdot t^3+\frac{1}{2}\cdot C_6\cdot t^2+C_7\cdot t+C_8; \qquad (5)$$

$$\dot{x}=\frac{1}{40320}\cdot C_1\cdot t^8+\frac{1}{5040}\cdot C_2\cdot t^7+\frac{1}{720}\cdot C_3\cdot t^6+\frac{1}{120}\cdot C_4\cdot t^5+\frac{1}{24}\cdot C_5\cdot t^4+\frac{1}{6}\cdot C_6\cdot t^3+$$
$$+\frac{1}{2}\cdot C_7\cdot t^2+C_8\cdot t+C_9;$$

$$x=\frac{1}{362880}\cdot C_1\cdot t^9+\frac{1}{40320}\cdot C_2\cdot t^8+\frac{1}{5040}\cdot C_3\cdot t^7+\frac{1}{720}\cdot C_4\cdot t^6+\frac{1}{120}\cdot C_5\cdot t^5+\frac{1}{24}\cdot C_6\cdot t^4+$$
$$+\frac{1}{6}\cdot C_7\cdot t^3+\frac{1}{2}\cdot C_8\cdot t^2+C_9\cdot t+C_{10},$$

where C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10} are the integration constants, which are determined from the initial and final conditions at each of the phases of the forming trolley movement.

Divide the reversal process into two phases: deceleration and acceleration.

At the deceleration phase, the initial conditions are the following: $t=0$: $x=-x_1$; $\dot{x}=\dot{x}_{\text{у}}$; $\ddot{x}=0$; $\dddot{x}=0$; $\overset{IV}{x}=0$. The final conditions at the acceleration phase are the following: $t=t_{\text{т}}$: $x=0$; $\dot{x}=0$; $\ddot{x}=a$; $\dddot{x}=0$; $\overset{IV}{x}=0$. Here, x_1 is the coordinate of the beginning of the deceleration process; $\dot{x}_{\text{у}}$ is the trolley movement velocity in the steady-state mode before the start of the deceleration phase; a is the trolley acceleration at the end of the deceleration phase.

At the acceleration phase, the initial conditions are the following: $t=0$: $x=0$; $\dot{x}=0$; $\ddot{x}=a$; $\dddot{x}=0$; $\overset{IV}{x}=0$. The final conditions at this phase are the following: $t=t_{\text{п}}$: $x=-x_1$; $\dot{x}=-\dot{x}_{\text{у}}$; $\ddot{x}=0$; $\dddot{x}=0$; $\overset{IV}{x}=0$.

Consider the deceleration process. Substituting the initial and final conditions into equations (5), we obtain

$$t=0 \quad \Rightarrow \quad C_{10}=-x_1;\quad C_9=\dot{x}_{\text{у}};\quad C_8=0;\quad C_7=0;\quad C_6=0; \qquad (6)$$

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$$t = t_{\text{т}} \;\Rightarrow\; \begin{cases} \dfrac{1}{362880}\cdot C_1\cdot t_{\text{т}}^9+\dfrac{1}{40320}\cdot C_2\cdot t_{\text{т}}^8+\dfrac{1}{5040}\cdot C_3\cdot t_{\text{т}}^7+\dfrac{1}{720}\cdot C_4\cdot t_{\text{т}}^6+\dfrac{1}{120}\cdot C_5\cdot t_{\text{т}}^5+ \\ +\dot{x}_{\text{у}}\cdot t_{\text{т}}-x_1=0; \\ \dfrac{1}{40320}\cdot C_1\cdot t_{\text{т}}^8+\dfrac{1}{5040}\cdot C_2\cdot t_{\text{т}}^7+\dfrac{1}{720}\cdot C_3\cdot t_{\text{т}}^6+\dfrac{1}{120}\cdot C_4\cdot t_{\text{т}}^5+\dfrac{1}{24}\cdot C_5\cdot t_{\text{т}}^4+\dot{x}_{\text{у}}=0; \\ \dfrac{1}{5040}\cdot C_1\cdot t_{\text{т}}^7+\dfrac{1}{720}\cdot C_2\cdot t_{\text{т}}^6+\dfrac{1}{120}\cdot C_3\cdot t_{\text{т}}^5+\dfrac{1}{24}\cdot C_4\cdot t_{\text{т}}^4+\dfrac{1}{6}\cdot C_5\cdot t_{\text{т}}^3=a; \\ \dfrac{1}{720}\cdot C_1\cdot t_{\text{т}}^6+\dfrac{1}{120}\cdot C_2\cdot t_{\text{т}}^5+\dfrac{1}{24}\cdot C_3\cdot t_{\text{т}}^4+\dfrac{1}{6}\cdot C_4\cdot t_{\text{т}}^3+\dfrac{1}{2}\cdot C_5\cdot t_{\text{т}}^2=0; \\ \dfrac{1}{120}\cdot C_1\cdot t_{\text{т}}^5+\dfrac{1}{24}\cdot C_2\cdot t_{\text{т}}^4+\dfrac{1}{6}\cdot C_3\cdot t_{\text{т}}^3+\dfrac{1}{2}\cdot C_4\cdot t_{\text{т}}^2+C_5\cdot t_{\text{т}}=0. \end{cases} \tag{7}$$

Having solved the system of equations (7), we have integration constants C_1, C_2, C_3, C_4 and C_5

$$\begin{gathered} C_1=907200\cdot\left(3\cdot\frac{a}{t_{\text{т}}^7}-14\cdot\frac{\dot{x}_{\text{у}}}{t_{\text{т}}^8}+28\cdot\frac{x_1}{t_{\text{т}}^9}\right);\quad C_2=100800\cdot\left(-13\cdot\frac{a}{t_{\text{т}}^6}+64\cdot\frac{\dot{x}_{\text{у}}}{t_{\text{т}}^7}-126\cdot\frac{x_1}{t_{\text{т}}^8}\right); \\ C_3=5040\cdot\left(53\cdot\frac{a}{t_{\text{т}}^5}-280\cdot\frac{\dot{x}_{\text{у}}}{t_{\text{т}}^6}+540\cdot\frac{x_1}{t_{\text{т}}^7}\right);\quad C_4=2520\cdot\left(-11\cdot\frac{a}{t_{\text{т}}^4}+64\cdot\frac{\dot{x}_{\text{у}}}{t_{\text{т}}^5}-120\cdot\frac{x_1}{t_{\text{т}}^6}\right); \\ C_5=420\cdot\left(3\cdot\frac{a}{t_{\text{т}}^3}-20\cdot\frac{\dot{x}_{\text{у}}}{t_{\text{т}}^4}+36\cdot\frac{x_1}{t_{\text{т}}^5}\right). \end{gathered} \tag{8}$$

After substituting certain integration constants (6) and (8) into the system of equations (5), we obtain the function of changing the fourth-order acceleration of the forming trolley during the process of deceleration from the steady velocity $\dot{x}_{\text{у}}$ to a full stop

$$\overset{V}{x}=\frac{420}{t_{\text{т}}^3}\cdot\left[\begin{gathered} 90\cdot\left(3\cdot a-14\cdot\frac{\dot{x}_{\text{у}}}{t_{\text{т}}}+28\cdot\frac{x_1}{t_{\text{т}}^2}\right)\cdot\frac{t^4}{t_{\text{т}}^4}+40\cdot\left(-13\cdot a+64\cdot\frac{\dot{x}_{\text{у}}}{t_{\text{т}}}-126\cdot\frac{x_1}{t_{\text{т}}^2}\right)\cdot\frac{t^3}{t_{\text{т}}^3}+ \\ +6\cdot\left(53\cdot a-280\cdot\frac{\dot{x}_{\text{у}}}{t_{\text{т}}}+540\cdot\frac{x_1}{t_{\text{т}}^2}\right)\cdot\frac{t^2}{t_{\text{т}}^5}+6\cdot\left(-11\cdot a+64\cdot\frac{\dot{x}_{\text{у}}}{t_{\text{т}}}-120\cdot\frac{x_1}{t_{\text{т}}^2}\right)\cdot\frac{t}{t_{\text{т}}}+ \\ +\left(3\cdot a-20\cdot\frac{\dot{x}_{\text{у}}}{t_{\text{т}}}+36\cdot\frac{x_1}{t_{\text{т}}^2}\right) \end{gathered}\right]. \tag{9}$$

After this, the criterion of movement optimality in the deceleration process, with expressions (2) and (9) taken into account, will have the form

$$I_{Q\text{т}}=\frac{m}{2}\int_0^{t_{\text{т}}}\overset{V}{x}{}^2\,dt=\frac{88200\cdot m}{t_{\text{т}}^5}\cdot\left[\frac{9}{5}\cdot a^2-\frac{104}{7}\cdot\frac{\dot{x}_{\text{у}}}{t_{\text{т}}}\cdot a+\frac{216}{7}\cdot\frac{x_1}{t_{\text{т}}^2}\cdot a+\frac{256}{7}\cdot\frac{\dot{x}_{\text{у}}^2}{t_{\text{т}}^2}-144\cdot\frac{x_1}{t_{\text{т}}^3}\cdot\dot{x}_{\text{у}}+144\cdot\frac{x_1^2}{t_{\text{т}}^4}\right]. \tag{10}$$

Consider the acceleration process. Substituting the initial and final acceleration conditions into equations (5), we obtain

$$t=0 \;\Rightarrow\; C_6=0;\;\; C_7=0;\;\; C_8=a;\;\; C_9=0;\;\; C_{10}=0; \tag{11}$$

$$t = t_{\text{п}} \Rightarrow \begin{cases} \frac{1}{362880} \cdot C_1 \cdot t_{\text{п}}^9 + \frac{1}{40320} \cdot C_2 \cdot t_{\text{п}}^8 + \frac{1}{5040} \cdot C_3 \cdot t_{\text{п}}^7 + \frac{1}{720} \cdot C_4 \cdot t_{\text{п}}^6 + \frac{1}{120} \cdot C_5 \cdot t_{\text{п}}^5 + \\ + \frac{1}{2} \cdot a \cdot t_{\text{п}}^2 = -x_1; \\ \frac{1}{40320} \cdot C_1 \cdot t_{\text{п}}^8 + \frac{1}{5040} \cdot C_2 \cdot t_{\text{п}}^7 + \frac{1}{720} \cdot C_3 \cdot t_{\text{п}}^6 + \frac{1}{120} \cdot C_4 \cdot t_{\text{п}}^5 + \frac{1}{24} \cdot C_5 \cdot t_{\text{п}}^4 + a \cdot t_{\text{п}} = -\dot{x}_{\text{у}}; \\ \frac{1}{5040} \cdot C_1 \cdot t_{\text{п}}^7 + \frac{1}{720} \cdot C_2 \cdot t_{\text{п}}^6 + \frac{1}{120} \cdot C_3 \cdot t_{\text{п}}^5 + \frac{1}{24} \cdot C_4 \cdot t_{\text{п}}^4 + \frac{1}{6} \cdot C_5 \cdot t_{\text{п}}^3 + a = 0; \\ \frac{1}{720} \cdot C_1 \cdot t_{\text{п}}^6 + \frac{1}{120} \cdot C_2 \cdot t_{\text{п}}^5 + \frac{1}{24} \cdot C_3 \cdot t_{\text{п}}^4 + \frac{1}{6} \cdot C_4 \cdot t_{\text{п}}^3 + \frac{1}{2} \cdot C_5 \cdot t_{\text{п}}^2 = 0; \\ \frac{1}{120} \cdot C_1 \cdot t_{\text{п}}^5 + \frac{1}{24} \cdot C_2 \cdot t_{\text{п}}^4 + \frac{1}{6} \cdot C_3 \cdot t_{\text{п}}^3 + \frac{1}{2} \cdot C_4 \cdot t_{\text{п}}^2 + C_5 \cdot t_{\text{п}} = 0. \end{cases} \tag{12}$$

Having solved the system of equations (12), we have integration constants C_1, C_2, C_3, C_4 and C_5

$$C_1 = 907200 \cdot \left(-3 \cdot \frac{a}{t_{\text{п}}^7} + 14 \cdot \frac{\dot{x}_{\text{у}}}{t_{\text{п}}^8} - 28 \cdot \frac{x_1}{t_{\text{п}}^9} \right); \; C_2 = 201600 \cdot \left(7 \cdot \frac{a}{t_{\text{п}}^6} - 31 \cdot \frac{\dot{x}_{\text{у}}}{t_{\text{п}}^7} + 63 \cdot \frac{x_1}{t_{\text{п}}^8} \right);$$

$$C_3 = 5040 \cdot \left(-63 \cdot \frac{a}{t_{\text{п}}^5} + 260 \cdot \frac{\dot{x}_{\text{у}}}{t_{\text{п}}^6} - 540 \cdot \frac{x_1}{t_{\text{п}}^7} \right); \; C_4 = 2520 \cdot \left(15 \cdot \frac{a}{t_{\text{п}}^4} - 56 \cdot \frac{\dot{x}_{\text{у}}}{t_{\text{п}}^5} + 120 \cdot \frac{x_1}{t_{\text{п}}^6} \right); \tag{13}$$

$$C_5 = 420 \cdot \left(-5 \cdot \frac{a}{t_{\text{п}}^3} + 16 \cdot \frac{\dot{x}_{\text{у}}}{t_{\text{п}}^4} - 36 \cdot \frac{x_1}{t_{\text{п}}^5} \right).$$

After substituting certain integration constants (11) and (13) into the system of equations (5), we obtain the function of changing the fourth-order acceleration of the forming trolley during the acceleration process from the rest state to the steady-state movement mode

$$\overset{V}{x} = \frac{420}{t_{\text{п}}^3} \cdot \left[\begin{array}{l} 90 \cdot \left(-3 \cdot a + 14 \cdot \frac{\dot{x}_{\text{у}}}{t_{\text{п}}} - 28 \cdot \frac{x_1}{t_{\text{п}}^2} \right) \cdot \frac{t^4}{t_{\text{п}}^4} + 80 \cdot \left(7 \cdot a - 31 \cdot \frac{\dot{x}_{\text{у}}}{t_{\text{п}}} + 63 \cdot \frac{x_1}{t_{\text{п}}^2} \right) \cdot \frac{t^3}{t_{\text{п}}^3} + \\ + 6 \cdot \left(-63 \cdot a + 260 \cdot \frac{\dot{x}_{\text{у}}}{t_{\text{п}}} - 540 \cdot \frac{x_1}{t_{\text{п}}^2} \right) \cdot \frac{t^2}{t_{\text{п}}^2} + 6 \cdot \left(15 \cdot a - 56 \cdot \frac{\dot{x}_{\text{у}}}{t_{\text{п}}} + 120 \cdot \frac{x_1}{t_{\text{п}}^2} \right) \cdot \frac{t}{t_{\text{п}}} + \\ + \left(-5 \cdot a + 16 \cdot \frac{\dot{x}_{\text{у}}}{t_{\text{п}}} - 36 \cdot \frac{x_1}{t_{\text{п}}^2} \right) \end{array} \right]. \tag{14}$$

After this, the criterion of movement optimality in the acceleration process, with expressions (2) and (14) taken into account, will have the form

$$I_{Q\text{п}} = \frac{m}{2} \int_0^{t_{\text{п}}} \overset{V}{x}{}^2 \, dt = \frac{88200 \cdot m}{t_{\text{п}}^5} \cdot \left[\frac{9}{5} \cdot a^2 - \frac{104}{7} \cdot \frac{\dot{x}_{\text{у}}}{t_{\text{п}}} \cdot a + \frac{216}{7} \cdot \frac{x_1}{t_{\text{п}}^2} \cdot a + \frac{256}{7} \cdot \frac{\dot{x}_{\text{у}}^2}{t_{\text{п}}^2} - 144 \cdot \frac{x_1}{t_{\text{п}}^3} \cdot \dot{x}_{\text{у}} + 144 \cdot \frac{x_1^2}{t_{\text{п}}^4} \right]. \tag{15}$$

Having taken the equality of the duration of the trolley deceleration and acceleration processes $t_{\text{т}} = t_{\text{п}} = t_1$, the general criterion for the movement optimality during the reversal process, with expressions (10) and (15) taken into account, will be determined by the following expression:

$$I_Q = \frac{176400 \cdot m}{t_1^5} \cdot \left[\frac{9}{5} \cdot a^2 - \frac{104}{7} \cdot \frac{\dot{x}_{\text{у}}}{t_1} \cdot a + \frac{216}{7} \cdot \frac{x_1}{t_1^2} \cdot a + \frac{256}{7} \cdot \frac{\dot{x}_{\text{у}}^2}{t_1^2} - 144 \cdot \frac{x_1}{t_1^3} \cdot \dot{x}_{\text{у}} + 144 \cdot \frac{x_1^2}{t_1^4} \right]. \tag{16}$$

To ensure the fulfillment of inequality (1), it is necessary, on the basis of expression (16), to fulfill the conditions:

$$\begin{cases} \dfrac{\partial I_Q}{\partial x_1} = \dfrac{176400 \cdot m}{t_1^5} \cdot \left[\dfrac{216}{7} \cdot \dfrac{a}{t_1^2} - 144 \cdot \dfrac{\dot{x}_y}{t_1^3} + 288 \cdot \dfrac{x_1}{t_1^4} \right] = \dfrac{12700800 \cdot m}{t_1^7} \cdot \left[\dfrac{3}{7} \cdot a - 2 \cdot \dfrac{\dot{x}_y}{t_1} + 4 \cdot \dfrac{x_1}{t_1^2} \right] = 0; \\ \dfrac{\partial I_Q}{\partial a} = \dfrac{176400 \cdot m}{t_1^5} \cdot \left[\dfrac{18}{5} \cdot a - \dfrac{104}{7} \cdot \dfrac{\dot{x}_y}{t_1} + \dfrac{216}{7} \cdot \dfrac{x_1}{t_1^2} \right] = \dfrac{352800 \cdot m}{t_1^5} \cdot \left[\dfrac{9}{5} \cdot a - \dfrac{52}{7} \cdot \dfrac{\dot{x}_y}{t_1} + \dfrac{108}{7} \cdot \dfrac{x_1}{t_1^2} \right] = 0. \end{cases} \quad (17)$$

From expressions (17) we can get

$$\begin{cases} \left[\dfrac{3}{7} \cdot a - 2 \cdot \dfrac{\dot{x}_y}{t_1} + 4 \cdot \dfrac{x_1}{t_1^2} \right] = 0 \\ \left[\dfrac{9}{5} \cdot a - \dfrac{52}{7} \cdot \dfrac{\dot{x}_y}{t_1} + \dfrac{108}{7} \cdot \dfrac{x_1}{t_1^2} \right] = 0 \end{cases} \Rightarrow \quad x_1 = \frac{17}{24} \cdot \dot{x}_y \cdot t_1; \quad a = -\frac{35}{18} \cdot \frac{\dot{x}_y}{t_1}. \quad (18)$$

Substituting the last two expressions of (18) into equality (6) and (8), we have the integration constants for the forming trolley during the deceleration phase

$$C_1 = 0; \; C_2 = 2800 \cdot \frac{\dot{x}_y}{t_1^7}; \; C_3 = -2800 \cdot \frac{\dot{x}_y}{t_1^6}; \; C_4 = 980 \cdot \frac{\dot{x}_y}{t_1^5}; \; C_5 = -140 \cdot \frac{\dot{x}_y}{t_1^4};$$
$$C_6 = 0; \; C_7 = 0; \; C_8 = 0; \; C_9 = \dot{x}_y; \; C_{10} = -\frac{17}{24} \cdot \dot{x}_y \cdot t_1. \quad (19)$$

After that, with integration constants (19) taken into account, the functions of changing the forming trolley displacement, velocity, acceleration, accelerations of the second, third and fourth orders during the deceleration phase are obtained:

$$x = \frac{1}{3} \cdot \dot{x}_y \cdot \left(\frac{5}{24} \cdot \frac{t^8}{t_1^7} - \frac{5}{3} \cdot \frac{t^7}{t_1^6} + \frac{49}{12} \cdot \frac{t^6}{t_1^5} - \frac{7}{2} \cdot \frac{t^5}{t_1^4} + 3 \cdot t - \frac{17}{8} \cdot t_1 \right);$$
$$\dot{x} = \frac{1}{3} \cdot \dot{x}_y \cdot \left(\frac{5}{3} \cdot \frac{t^7}{t_1^7} - \frac{35}{3} \cdot \frac{t^6}{t_1^6} + \frac{49}{2} \cdot \frac{t^5}{t_1^5} - \frac{35}{2} \cdot \frac{t^4}{t_1^4} + 3 \right);$$
$$\ddot{x} = \frac{35}{3} \cdot \dot{x}_y \cdot \left(\frac{1}{3} \cdot \frac{t^3}{t_1^3} - 2 \cdot \frac{t^2}{t_1^2} + \frac{7}{2} \cdot \frac{t}{t_1} - 2 \right) \cdot \frac{t^3}{t_1^4}; \qquad \dddot{x} = \frac{70}{3} \cdot \dot{x}_y \cdot \left(\frac{t^3}{t_1^3} - 5 \cdot \frac{t^2}{t_1^2} + 7 \cdot \frac{t}{t_1} - 3 \right) \cdot \frac{t^2}{t_1^4}; \quad (20)$$
$$\overset{IV}{x} = 70 \cdot \dot{x}_y \cdot \left(\frac{5}{3} \cdot \frac{t^3}{t_1^3} - \frac{20}{3} \cdot \frac{t^2}{t_1^2} + 7 \cdot \frac{t}{t_1} - 2 \right) \cdot \frac{t}{t_1^4}; \qquad \overset{V}{x} = 140 \cdot \dot{x}_y \cdot \left(\frac{10}{3} \cdot \frac{t^3}{t_1^3} - 10 \cdot \frac{t^2}{t_1^2} + 7 \cdot \frac{t}{t_1} - 1 \right) \cdot \frac{1}{t_1^4}.$$

Substituting the last two expressions from (18) into equalities (11) and (13), we obtain integration constants for the forming trolley at the deceleration phase

$$C_1 = 0; \; C_2 = 2800 \cdot \frac{\dot{x}_y}{t_1^7}; \; C_3 = 0; \; C_4 = -420 \cdot \frac{\dot{x}_y}{t_1^5}; \; C_5 = \frac{280}{3} \cdot \frac{\dot{x}_y}{t_1^4};$$
$$C_6 = 0; \; C_7 = 0; \; C_8 = -\frac{35}{18} \cdot \frac{\dot{x}_y}{t_1}; \; C_9 = 0; \; C_{10} = 0. \quad (21)$$

After that, with integration constants (21) taken into account, the functions of changing the forming trolley displacement, velocity, acceleration, accelerations of the second, third and fourth orders at the acceleration phase can be obtained

$$x=\frac{1}{3}\cdot\dot{x}_{\text{y}}\cdot\left(\frac{5}{24}\cdot\frac{t^6}{t_1^6}-\frac{7}{4}\cdot\frac{t^4}{t_1^4}+\frac{7}{3}\cdot\frac{t^3}{t_1^3}-\frac{35}{12}\right)\cdot\frac{t^2}{t_1};\qquad \dot{x}=\frac{5}{9}\cdot\dot{x}_{\text{y}}\cdot\left(\frac{t^6}{t_1^6}-\frac{63}{10}\cdot\frac{t^4}{t_1^4}+7\cdot\frac{t^3}{t_1^3}-\frac{7}{2}\right)\cdot\frac{t}{t_1};$$

$$\ddot{x}=\frac{35}{9}\cdot\dot{x}_{\text{y}}\cdot\left(\frac{t^6}{t_1^6}-\frac{9}{2}\cdot\frac{t^4}{t_1^4}+4\cdot\frac{t^3}{t_1^3}-\frac{1}{2}\right)\cdot\frac{1}{t_1};\qquad \dddot{x}=\frac{70}{3}\cdot\dot{x}_{\text{y}}\cdot\left(\frac{t^3}{t_1^3}-3\cdot\frac{t}{t_1}+2\right)\cdot\frac{t^2}{t_1^4};\tag{22}$$

$$\overset{IV}{x}=\frac{70}{3}\cdot\dot{x}_{\text{y}}\cdot\left(5\cdot\frac{t^3}{t_1^3}-9\cdot\frac{t}{t_1}+4\right)\cdot\frac{t}{t_1^4};\qquad \overset{V}{x}=\frac{140}{3}\cdot\dot{x}_{\text{y}}\cdot\left(10\cdot\frac{t^3}{t_1^3}-9\cdot\frac{t}{t_1}+2\right)\cdot\frac{1}{t_1^4}.$$

At the steady-state mode of the movement of the forming trolley, the coordinate of displacement, velocity, acceleration, accelerations of the second, third and fourth orders of its center of mass are described by equations [19]

$$x=x_{0\text{y}}+\frac{(x_{1\text{y}}-x_{0\text{y}})\cdot t}{t_{\text{y}}};\quad \dot{x}=\frac{(x_{1\text{y}}-x_{0\text{y}})}{t_{\text{y}}}=\text{const};\quad \ddot{x}=0;\quad \dddot{x}=0;\quad \overset{IV}{x}=0;\quad \overset{V}{x}=0,\tag{23}$$

where $x_{0\text{y}}$ and $x_{1\text{y}}$ are the coordinates of the initial and final positions of the center of mass of the trolley at the steady-state movement; t_{y} is the duration of the steady-state movement.

In expressions (23), the coordinate of the initial position of the center of mass of the trolley at the steady-state movement $x_{0\text{y}}$ is taken equal to x_1. Then, with the amplitude of the trolley movement from one extreme position to another taken as Δx, the final coordinate of the position of the center of mass of the trolley at the steady-state movement can be determined as $x_{1\text{y}}=\Delta x-x_1$.

Substituting the obtained coordinates $x_{0\text{y}}$ and $x_{1\text{y}}$ into the second expression in (23), we obtain the dependence for determining the trolley velocity at the steady-state movement $\dot{x}_{\text{y}}$

$$\dot{x}_{\text{y}}=\frac{\Delta x-2\cdot x_1}{t_{\text{y}}}=\frac{\Delta x-\frac{17}{12}\cdot\dot{x}_{\text{y}}\cdot t_1}{t_{\text{y}}}\quad\Rightarrow\quad \dot{x}_{\text{y}}=\frac{\Delta x}{t_{\text{y}}+\frac{17}{12}\cdot t_1}.\tag{24}$$

With the total time of the forming trolley movement from one extreme position to the other taken as t_{o}, it can be divided into three phases: acceleration phase time – $t_{\text{п}}$; steady-state movement phase time – t_{y}; deceleration phase time – $t_{\text{т}}$. For the concrete mixture compaction to be ensured by the forming trolley moving with constant working stroke velocity, take the time of the steady-state movement equal to, say, $t_{\text{y}}=\frac{2}{3}\cdot t_{\text{o}}$ [4]. Then, with the condition of equality of the acceleration and deceleration phase times given, they can be determined by the corresponding expressions: $t_{\text{п}}=t_{\text{т}}=t_1=1/6\cdot t_{\text{o}}$.

The functions of the trolley steady-state movement velocity and the coordinate x_1, with dependencies (24) taken into account, will have the form

$$\dot{x}_{\text{y}}=\frac{72\cdot\Delta x}{65\cdot t_{\text{o}}};\qquad x_1=\frac{17}{130}\cdot\Delta x.\tag{25}$$

Considering the movement of the forming trolley from one extreme position to another and substituting expressions (25) into equations (20), (23) and (22), we obtain the functions of changing the forming trolley displacement, velocity, acceleration, acceleration of the second, third and fourth orders:

– at the acceleration phase

$$
\begin{aligned}
&x=-\frac{144}{65}\cdot\Delta x\cdot\left(9720\cdot\frac{t^6}{t_o^6}-2268\cdot\frac{t^4}{t_o^4}+504\cdot\frac{t^3}{t_o^3}-\frac{35}{12}\right)\cdot\frac{t^2}{t_o^2};\\
&\dot{x}=-\frac{48}{13}\cdot\Delta x\cdot\left(46656\cdot\frac{t^6}{t_o^6}-\frac{81648}{10}\cdot\frac{t^4}{t_o^4}+1512\cdot\frac{t^3}{t_o^3}-\frac{7}{2}\right)\cdot\frac{t}{t_o^2};\\
&\ddot{x}=-\frac{336}{13}\cdot\Delta x\cdot\left(46656\cdot\frac{t^6}{t_o^6}-5832\cdot\frac{t^4}{t_o^4}+864\cdot\frac{t^3}{t_o^3}-\frac{1}{2}\right)\cdot\frac{1}{t_o^2};\\
&\dddot{x}=\frac{870912}{13}\cdot\Delta x\cdot\left(108\cdot\frac{t^3}{t_o^3}-9\cdot\frac{t}{t_o}+1\right)\cdot\frac{t^2}{t_o^5};\\
&\overset{IV}{x}=\frac{870912}{13}\cdot\Delta x\cdot\left(540\cdot\frac{t^3}{t_o^3}-27\cdot\frac{t}{t_o}+2\right)\cdot\frac{t}{t_o^5};\\
&\overset{V}{x}=\frac{1741824}{13}\cdot\Delta x\cdot\left(1080\cdot\frac{t^3}{t_o^3}-27\cdot\frac{t}{t_o}+1\right)\cdot\frac{1}{t_o^5};
\end{aligned}
\qquad (26)
$$

– at the steady-state movement phase

$$
x=\frac{\Delta x}{130}\cdot\left(17+144\cdot\frac{t}{t_{\hat{\imath}}}\right);\quad \dot{x}=\frac{72\cdot\Delta x}{65\cdot t_{\hat{\imath}}}=const;\quad \ddot{x}=0;\quad \dddot{x}=0;\quad \overset{{}^2V}{x}=0;\quad \overset{V}{x}=0; \qquad (27)
$$

– at the deceleration phase

$$
\begin{aligned}
&x=\Delta x+\frac{24}{65}\cdot\Delta x\cdot\left(58320\cdot\frac{t^8}{t_o^8}-77760\cdot\frac{t^7}{t_o^7}+31752\cdot\frac{t^6}{t_o^6}-4536\cdot\frac{t^5}{t_o^5}+3\cdot\frac{t}{t_o}-\frac{17}{48}\right);\\
&\dot{x}=\frac{72}{65}\cdot\Delta x\cdot\left(155520\cdot\frac{t^7}{t_o^7}-181440\cdot\frac{t^6}{t_o^6}+63504\cdot\frac{t^5}{t_o^5}-7560\cdot\frac{t^4}{t_o^4}+1\right)\cdot\frac{1}{t_o};\\
&\ddot{x}=\frac{217728}{13}\cdot\Delta x\cdot\left(72\cdot\frac{t^3}{t_o^3}-72\cdot\frac{t^2}{t_o^2}+21\cdot\frac{t}{t_o}-2\right)\cdot\frac{t^3}{t_o^5};\\
&\dddot{x}=\frac{1306368}{13}\cdot\Delta x\cdot\left(72\cdot\frac{t^3}{t_o^3}-60\cdot\frac{t^2}{t_o^2}+14\cdot\frac{t}{t_o}-1\right)\cdot\frac{t^2}{t_o^5};\\
&\overset{IV}{x}=\frac{2612736}{13}\cdot\Delta x\cdot\left(180\cdot\frac{t^3}{t_o^3}-120\cdot\frac{t^2}{t_o^2}+21\cdot\frac{t}{t_o}-1\right)\cdot\frac{t}{t_o^5};\\
&\overset{V}{x}=\frac{2612736}{13}\cdot\Delta x\cdot\left(720\cdot\frac{t^3}{t_o^3}-360\cdot\frac{t^2}{t_o^2}+42\cdot\frac{t}{t_o}-1\right)\cdot\frac{1}{t_o^5}.
\end{aligned}
\qquad (28)
$$

With the amplitude of the forming trolley taken as $\Delta x = 0.4$ m and the duration of its movement from one extreme position to another taken as $t_o = 3$ sec [4], according to equations (26)–(28), kinematic characteristics were calculated, and graphs (Fig. 1) were built for the forming trolley moving from one extreme position to another and in the reverse direction with the reversal mode according to the fourth-order acceleration with the optimal values of boundary accelerations, Fig. 1, a showing the change of displacement; Fig. 1, b, velocities; Fig. 1, c, accelerations; Fig. 1, d, second-order accelerations (jerk), Fig. 1, e, third-order accelerations; and Fig. 1, f, fourth-order accelerations.

The analysis of the graphs in Fig. 1 shows that the functions of the forming trolley velocity, acceleration, second-order acceleration (jerk) and third-order acceleration change smoothly, without creating significant dynamic loads in the roller forming unit, which in turn has a positive effect on its durability. At the same time, the value of the velocity of the forming trolley at the steady-state movement phase is

$\dot{x} = \dfrac{72 \cdot \Delta x}{65 \cdot t_o} = 0.148$ m/sec, which is 5% less than the mode of movement according to the third-order acceleration [18]; the maximum value of the acceleration at the acceleration and deceleration phases is $\ddot{x} = 0{,}574$ m/sec^2, which is 7% less than the mode of movement according to the third-order acceleration [18]; the maximum value of the second-order acceleration (jerk) at the acceleration and deceleration phases is $\dddot{x} = 2{,}163$ m/sec^3, which is 2.33 times less than the mode of movement according to the third-order acceleration [18]. Besides, the graphs in Fig. 1 show that the change in the second-order acceleration function (jerk) at the acceleration and deceleration phases occurs in one quadrant, that is, during the acceleration and deceleration processes, the second-order acceleration (jerk) changes from zero to the extreme position and returns to zero, the curve not intersecting the x-axis, compared with the mode of movement according to the third-order acceleration [18]. Therefore, with this mode of movement of the forming trolley, the number of changes in the sign of the second-order acceleration (jerk) decreases, which leads to a decrease in the alternating forces in the roller forming unit, and, consequently, reduces the stress level in the trolley.

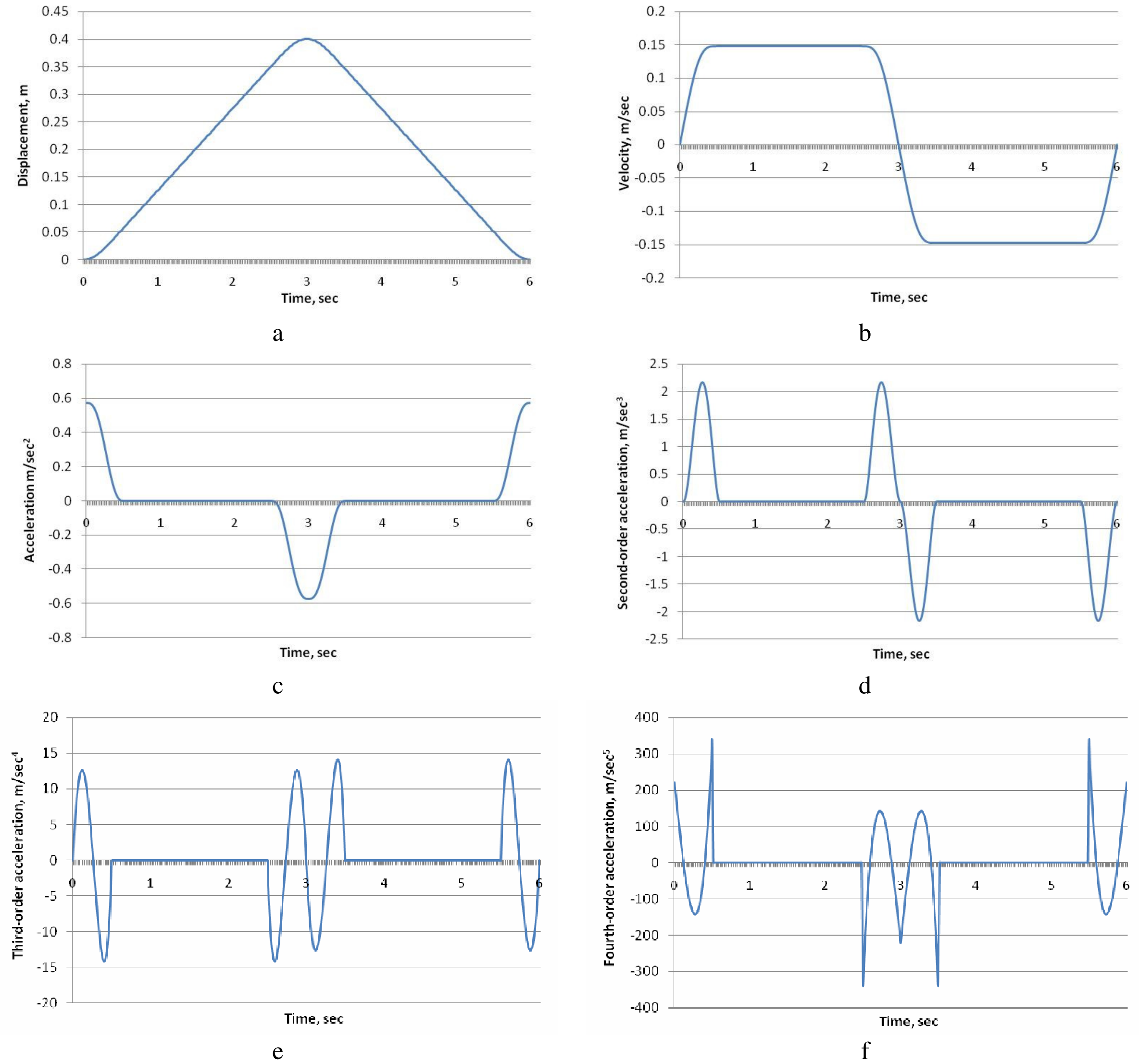

Fig. 1. Graphs of changes in the kinematic characteristics of the forming trolley:
a – displacement; b – velocity; c – acceleration; d – second-order acceleration;
e – third-order acceleration; f – fourth-order acceleration

Research Results

The law of movement of the forming trolley, described by equations (26)–(28), can be fulfilled by the trolley cam drive (Fig. 2) providing the reciprocating movement of the trolley. In this case, the movement of the trolley in one direction is carried out by rotating cam 1 by half a turn (i.e. $\varphi = \pi$) and in the reverse direction, by another half a turn, the full cycle of the trolley movement being one revolution of the cam. In order to implement the described law of the trolley movement, it is necessary that the increment of the cam radius correspond to the increment of the trolley movement.

From expressions (26)–(28) we exclude the time t, since $t = \varphi/\omega$, and $t_1 = \pi/\omega$. Here, φ is the angular coordinate of the cam rotation, and ω is the angular velocity of the cam. Since the acceleration phase time of the forming trolley is taken equal to

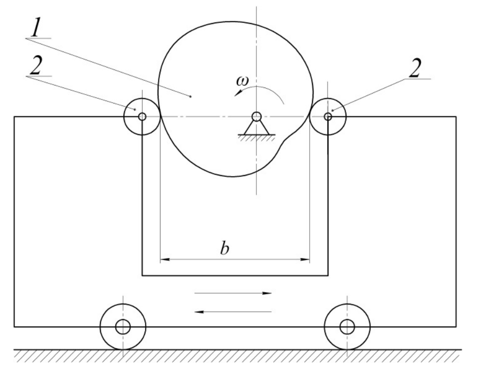

Fig. 2. Scheme of a cam drive mechanism to provide for trolley reciprocating movement:
1 – cam; 2 – pusher;
ω – cam angular velocity

$t_п = \frac{1}{6} \cdot t_о$, then the deceleration process will be provided out by rotating the cam to an angle ranging from $\varphi = 0$ to $\varphi = \pi/6$. Since the time of the steady-state movement is taken equal to $t_у = \frac{2}{3} \cdot t_о$ then the steady-state movement of the forming trolley will be provided by rotating the cam to an angle ranging from $\varphi = \pi/6$ to $\varphi = 5\pi/6$. Since the deceleration phase time is taken equal to $t_т = \frac{1}{6} \cdot t_о$, then the deceleration process will be carried out by rotating the cam to an angle in the range from $\varphi = 5\pi/6$ to $\varphi = \pi$. According to the above, with the first equations of expressions (26) - (28) transformed for the case where the origin of coordinates is measured from the middle position of the forming trolley, the cam radius, which describes its profile, is associated with the angular coordinate by the following expressions:

$$\rho = \frac{b}{2} - \frac{144}{65} \cdot \Delta x \cdot \left(9720 \cdot \frac{\varphi^6}{\pi^6} - 2268 \cdot \frac{\varphi^4}{\pi^4} + 504 \cdot \frac{\varphi^3}{\pi^3} - \frac{35}{12} \right) \cdot \frac{\varphi^2}{\pi^2} - \frac{\Delta x}{2}, \qquad 0 \le \varphi \le \frac{\pi}{6}; \tag{29}$$

$$\rho = \frac{b}{2} + \frac{\Delta x}{130} \cdot \left[17 + 144 \cdot \left(\varphi - \frac{\pi}{6} \right) \cdot \frac{1}{\pi} \right] - \frac{\Delta x}{2}, \qquad \frac{\pi}{6} < \varphi < \frac{5\pi}{6}; \tag{30}$$

$$\rho = \frac{b}{2} + \frac{\Delta x}{2} + \frac{24}{65} \cdot \Delta x \cdot \left[\begin{array}{l} 58320 \cdot \left(\varphi - \frac{5\pi}{6} \right)^8 \cdot \frac{1}{\pi^8} - 77760 \cdot \left(\varphi - \frac{5\pi}{6} \right)^7 \cdot \frac{1}{\pi^7} + \\ + 31752 \cdot \left(\varphi - \frac{5\pi}{6} \right)^6 \cdot \frac{1}{\pi^6} - 4536 \cdot \left(\varphi - \frac{5\pi}{6} \right)^5 \cdot \frac{1}{\pi^5} + \\ + 3 \cdot \left(\varphi - \frac{5\pi}{6} \right) \cdot \frac{1}{\pi} - \frac{17}{48} \end{array} \right], \qquad \frac{5\pi}{6} < \varphi \le \pi, \tag{31}$$

where b is the distance between the pushers (Fig. 2).

The cam profile can be determined in a similar way in the area of its rotation from π to 2π, which is described by a radius that varies according to the dependencies

$$\rho = \frac{b}{2} + \frac{144}{65} \cdot \Delta x \cdot \left[9720 \cdot \frac{(\varphi - \pi)^6}{\pi^6} - 2268 \cdot \frac{(\varphi - \pi)^4}{\pi^4} + 504 \cdot \frac{(\varphi - \pi)^3}{\pi^3} - \frac{35}{12} \right] \cdot \frac{(\varphi - \pi)^2}{\pi^2} + \frac{\Delta x}{2}, \ \pi \le \varphi \le \frac{7\pi}{6}; \tag{32}$$

$$\rho = \frac{b}{2} - \frac{\Delta x}{130} \cdot \left[17 + 144 \cdot \left(\varphi - \frac{7\pi}{6} \right) \cdot \frac{1}{\pi} \right] + \frac{\Delta x}{2}, \qquad \frac{7\pi}{6} < \varphi < \frac{11\pi}{6}; \tag{33}$$

$$\rho = \frac{b}{2} - \frac{\Delta x}{2} - \frac{24}{65} \cdot \Delta x \cdot \left[\begin{array}{l} 58320 \cdot \left(\varphi - \frac{11\pi}{6} \right)^8 \cdot \frac{1}{\pi^8} - 77760 \cdot \left(\varphi - \frac{11\pi}{6} \right)^7 \cdot \frac{1}{\pi^7} + \\ + 31752 \cdot \left(\varphi - \frac{11\pi}{6} \right)^6 \cdot \frac{1}{\pi^6} - 4536 \cdot \left(\varphi - \frac{11\pi}{6} \right)^5 \cdot \frac{1}{\pi^5} + \\ + 3 \cdot \left(\varphi - \frac{11\pi}{6} \right) \cdot \frac{1}{\pi} - \frac{17}{48} \end{array} \right], \qquad \frac{11\pi}{6} < \varphi \le 2\pi. \tag{34}$$

In order to prevent cam impacts against the pushers when the trolley changes its direction of movement, the cam profile described by equations (29)–(34) (Fig. 3) has the form where, in any position, its diameter d is constant and equal to the distance between the pushers b ($d = b$).

In order to reduce the dynamic loads in the elements of the roller forming unit and increase its reliability, it was proposed that the unit design use a drive mechanism providing the reciprocating movement of the forming trolley with the reversal mode according to the fourth-order acceleration with optimal boundary accelerations (Fig. 4). The drive mechanism is made in the form of cam mechanisms that are hinged on the portal and come into contact with the pushers rigidly attached to the forming trolley. The presence of two pushers on each side of the forming trolley allows creating a rigid kinematical connection at its direct and reverse movement. The cams are rotated by engines.

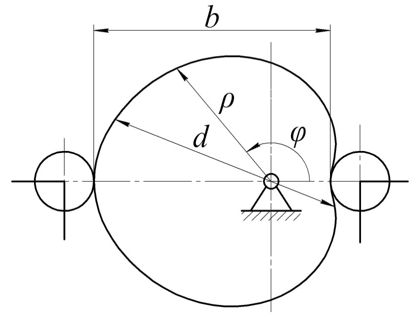

Fig. 3. Cam profile that implements the trolley reversing mode according to the fourth-order acceleration:
b is the distance between the pushers;
d, ρ are the diameter and radius of the cam;
φ is the angular coordinate of the cam rotation

When a cam drive mechanism is used in the roller forming unit, on each side of the forming trolley, the possibility of its axial offset is prevented, the dynamic loads in the drive elements are reduced, and, accordingly, the durability of the unit as a whole increases.

The law of movement of the forming trolley, described by equations (26)–(28), can be fulfilled by the drive from a high-torque stepper motor that is embedded into the forming trolley compaction rollers. Taking the forming trolley acceleration phase time $t_п = 1/6 \cdot t_о$, the time of steady-state movement $t_у = 2/3 \cdot t_о$ and the deceleration phase time $t_т = 1/6 \cdot t_о$ [4], from expressions (26)–(28), we obtain the law of changing the drive stepper motor angular velocity for the forming trolley moving from one extreme position to another

$$\dot{\varphi} = -\frac{48 \cdot \Delta x}{13 \cdot R} \cdot \left(46656 \cdot \frac{t^6}{t_о^6} - \frac{81648}{10} \cdot \frac{t^4}{t_о^4} + 1512 \cdot \frac{t^3}{t_о^3} - \frac{7}{2} \right) \cdot \frac{t}{t_о^2}, \qquad 0 \le t \le \frac{t_о}{6};$$

$$\dot{\varphi} = \frac{72 \cdot \Delta x}{65 \cdot t_о \cdot R}, \qquad \frac{t_о}{6} < t < \frac{5 \cdot t_о}{6};$$

$$\dot{\varphi} = \frac{72 \cdot \Delta x}{65 \cdot R} \cdot \left[\begin{array}{l} 155520 \cdot \left(t - \frac{5 \cdot t_о}{6} \right)^7 \cdot \frac{1}{t_о^7} - 181440 \cdot \left(t - \frac{5 \cdot t_о}{6} \right)^6 \cdot \frac{1}{t_о^6} + \\ + 63504 \cdot \left(t - \frac{5 \cdot t_о}{6} \right)^5 \cdot \frac{1}{t_о^5} - 7560 \cdot \left(t - \frac{5 \cdot t_о}{6} \right)^4 \cdot \frac{1}{t_о^4} + 1 \end{array} \right] \cdot \frac{1}{t_о}, \qquad \frac{5 \cdot t_о}{6} < t \le t_о,$$

where R is the radius of the compaction rollers.

The law of change in the angular velocity of the drive stepper motor for the forming trolley moving in the reverse direction can be determined similarly:

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$$\dot{\varphi} = \frac{48\cdot\Delta x}{13\cdot R}\cdot\left[46656\cdot\frac{(t-t_o)^6}{t_o^6} - \frac{81648}{10}\cdot\frac{(t-t_o)^4}{t_o^4} + 1512\cdot\frac{(t-t_o)^3}{t_o^3} - \frac{7}{2}\right]\cdot\frac{(t-t_o)}{t_o^2},\quad t_o \le t \le \frac{7\cdot t_o}{6};$$

$$\dot{\varphi} = -\frac{72\cdot\Delta x}{65\cdot t_o\cdot R},\qquad \frac{7\cdot t_o}{6} < t < \frac{11\cdot t_o}{6};$$

$$\dot{\varphi} = -\frac{72\cdot\Delta x}{65\cdot R}\cdot\left[\begin{matrix}155520\cdot\left(t-\frac{11\cdot t_o}{6}\right)^7\cdot\frac{1}{t_o^7} - 181440\cdot\left(t-\frac{11\cdot t_o}{6}\right)^6\cdot\frac{1}{t_o^6} + \\ +63504\cdot\left(t-\frac{11\cdot t_o}{6}\right)^5\cdot\frac{1}{t_o^5} - 7560\cdot\left(t-\frac{11\cdot t_o}{6}\right)^4\cdot\frac{1}{t_o^4} + 1\end{matrix}\right]\cdot\frac{1}{t_o},\qquad \frac{11\cdot t_o}{6} < t \le 2\cdot t_o.$$

It is also proposed that the roller forming unit design use a drive from a high-torque stepper motor providing for the reciprocating movement of the forming trolley with the reversal mode according to the fourth-order acceleration with optimal boundary accelerations (Fig. 5). The trolley is set in reciprocating movement by a high-torque stepper motor embedded into the compaction rollers, the roller axis playing the role of a stator, and the roller itself, of a rotor [22].

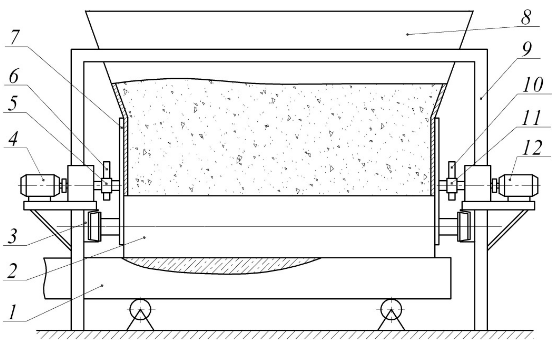

Fig. 4. Roller forming unit with a cam drive mechanism

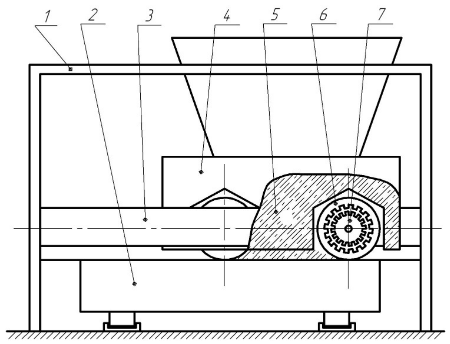

Fig. 5. Roller forming unit with a drive from a stepper motor

When the roller forming unit uses a drive from a high-torque stepper motor embedded into the compaction rollers, with the motor angular velocity change law described by the above equations, the dynamic loads in the drive elements are reduced and, accordingly, the durability of the roller forming unit as a whole increases.

Conclusions

1. As a result of the conducted research, in order to increase the reliability and durability of the roller forming unit, we calculated a combined mode of the reciprocating movement of the forming trolley with the reversal according to the fourth-order acceleration with the optimal values of boundary accelerations.

2. Kinematic characteristics of the forming trolley during the reversal mode according to the fourth-order acceleration with the optimal values of boundary accelerations were obtained.

3. It was proposed that the roller forming unit design use a cam-type drive, and a cam profile be built to provide for a combined mode of the reciprocating movement of the forming trolley with the reversal according to the fourth order acceleration with the optimal values of boundary accelerations.

4. Consideration was given to the roller forming unit design using a drive from a high-torque stepper motor embedded into the forming trolley compaction rollers.

5. The results of the work may further be useful for refining and improving the existing engineering methods for calculating the drive mechanisms of roller forming machines both at the design stages and in real operation modes. Also, the results of the work can be used in designing or improving mechanisms with the reciprocating motion of their actuating elements.

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Обґрунтування граничних прискорень оптимального режиму реверсування роликової формувальної установки за прискоренням четвертого порядку

[1] Ловейкін В. С., [2] Почка К. І., [2] Пристайло М. О., [2] Почка О. Б.

[1] Національний університет біоресурсів і природокористування України
03041, Україна, м. Київ, вул.. Героїв оборони, 15

[2] Київський національний університет будівництва і архітектури
03037, Україна, м. Київ, пр. Повітрофлотський, 31

З метою підвищення надійності та довговічності роликової формувальної установки розраховано комбінований режим зворотно-поступального руху формувального візка з реверсуванням за прискоренням четвертого порядку з оптимальними значеннями крайових прискорень. Під час визначення комбінованого режиму зворотно-поступального руху формувального візка з реверсуванням за прискоренням четвертого порядку з оптимальними значеннями крайових прискорень як критерій режиму руху використано критеріальну дію, яка являє собою інтеграл за часом з підінтегральною функцією, що виражає «енергію» прискорень четвертого порядку установки. Отримано функції зміни кінематичних характеристик формувального візка за його руху від одного крайнього положення в інше, що відповідають комбінованому режиму зворотно-поступального руху формувального візка з реверсуванням за прискоренням четвертого порядку з оптимальними значеннями крайових прискорень. Запропоновано використовувати в конструкції установки привід у вигляді кулачкового механізму. Побудовано профіль кулачка для забезпечення комбінованого режиму зворотно-поступального руху формувального візка з реверсуванням за прискоренням четвертого порядку з оптимальними значеннями крайових прискорень. Розглянуто застосування в конструкції роликової формувальної установки приводу від високомоментного крокового двигуна, що вмонтований в укочувальні ролики формувального візка установки. Використання в установці вказаного привідного механізму приводить до зменшення динамічних навантажень в його елементах і, відповідно, до підвищення надійності та довговічності установки в цілому. Результати роботи можуть в подальшому бути корисними для уточнення та удосконалення існуючих інженерних методів розрахунку привідних механізмів машин роликового формування як на стадіях проектування/конструювання, так і в режимах реальної експлуатації, а також використовуватися під час проектування або удосконалення механізмів із зворотно-поступальним рухом виконавчих елементів.

Ключові слова: *установка, формувальний візок, режим руху, привод, прискорення, кулачок, кроковий двигун.*

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