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USE OF REFINED FINITE ELEMENT MODELS FOR SOLVING THE CONTACT THERMOALASTICITY PROBLEM OF GAS TURBINE ROTORS

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A refined mathematical model of gas turbine engine rotors using three-dimensional finite elements of a curvilinear form is developed. All the calculations were performed for rotors, which are widely used in power machine building and shipbuilding. The fact is that such components have a constructive heterogeneity that can hardly be correctly explained using well-known finite elements and their shape functions. On the other hand, the mathematical model should be as simple as possible with a view to its wide use in the process of designing a rotor. Therefore, a new refined finite-element mathematical model was developed, consisting of three-dimensional curvilinear hexahedral finite elements. It was used to calculate the displacement field caused by the complex action of the heat flux and contact load at the junction of rotor elements. This approach makes it possible to describe the entire rotor as a superposition of the developed curvilinear finite element models and make the calculation process more correct and compact. To solve this problem, a system of matrix equations was compiled. It is based on the use of energy balance dependences in the mechanical contact interaction of rotor elements, as well as the heat balance under the influence of non-stationary heat flow. When creating a numerical algorithm for solving the problem, the direct decomposition of Cholesky was used. To make the solution more compact, the Sherman scheme was used. All the calculations of displacement and temperature fields were carried out for two widely used types of joints, which are used to create such rotors, namely: joints with clearance and interference.

Keywords: *three-dimensional finite elements, gas turbine rotors, displacement and temperature fields, contact thermoelasticity problem, clearance, interference.*

Introduction

The working process of gas turbine rotors that are used in modern turbines is constantly influenced by various high intensity mechanical and thermal effects. This causes changes in the stress-deformed state of the entire motor as well as its components, such as the disk, shaft, and blades due to their mechanical contact and the heat flow passing through their contacting surfaces. This correlation is especially important for gas turbine engine components due to their extremely complex working process.

It should be noticed that the main conditions of contact between rotor components are not always identical even when one-type parts contact. [1]. Firstly, the shaft and rotor are mounted before the start of the working process. This means that each pair of contacting surfaces has its own definite conjugation conditions. But during the working process the conjugation conditions can rapidly change. This fact causes the changes of the mechanical contact pressure. Therefore, changes of heat flow parameters can also be observed on the shaft and blade row contact surfaces [2]. Therefore, the mathematical model used for solving a gas turbine engine rotor thermoelasticity problem needs to take into consideration all these mechanical and temperature changes on the contacting surfaces of gas turbine engine rotor components.

Two main approaches are used for solving contact problems of deformable solid mechanics by the finite element method (FEM). The main idea of the first approach consists in using a contact layer of definite thermal conductivity, that is located among between the surfaces of contacting solids. The finite element model of the contact layer is based on the finite elements similar to those of solids. But the thermo conductivity features of the layer elements are different from those of solids. [1]. Such an approach is rather useful, but its application to thermoelasticity problems for studying real assembly constructions is inconvenient, because it is extremely difficult to calculate the layer deformation caused by the thermal gradient on contacting surfaces. The second main approach consists in using a definite function, that distinctly determines the dependences between the heat flux and displacements of the finite element nodes located on contacting surfaces [3–6]. The above-mentioned solution to problems could only be obtained in the case of fulfilling several conditions. The first one is the condition of the contacting components non-penetrating each other; the second one is the condition of equality of normal and tangential forces for each node pair of contacting finite elements [4–6].

The main aim of this article is to develop a more correct mathematical model based on three dimensional finite elements, that could be used for solving gas turbine engine rotor thermoelasticity problems.

Formulation of the problem

The gas turbine engine rotor under consideration is located in the right rectangular Cartesian coordinate system $x\,y\,z$ with the beginning at the shaft end centre O. z axis is normal to the shaft axis of rotation; x matches the shaft axis of rotation. The whole coordinate system is rotating with constant angular velocity together with the rotor.

$$\begin{gathered} \delta L = 0 \\ L = \Pi - W \end{gathered} \tag{1}$$

where L – the Lagrangian function; Π – potential deformation energy of a mechanical system; W – the work of external forces.

The potential deformation energy can be found as follows:

$$\Pi = \frac{1}{2}\int_V \varepsilon^T \sigma dV, \tag{2}$$

where V – the investigated model volume; $\boldsymbol{\varepsilon}$ – the elastic deformation vector; $\boldsymbol{\sigma}$ – the stress vector.

After the FEM approximation, to determine the potential deformation the relation (2) can be transformed as follows:

$$\Pi = \frac{1}{2}\left(\delta^T K \delta\right), \tag{3}$$

where $\mathbf{K}$ – the global stiffness matrix of a finite element model; $\boldsymbol{\delta}$ – the vector generalized displacements of the model nodes.

Consequently, the energy balance equation of a mechanical system (1), taking into consideration the dependency (3), can be transformed to

$$\mathbf{K}\boldsymbol{\delta} = \mathbf{F}, \tag{4}$$

where $\boldsymbol{F}$ – the vector of external forces [7].

The temperature state of a the solid under the stationary heat flow can be described by the dependency [5] as

$$\lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + Q = 0, \tag{5}$$

where T – the temperature of a solid under consideration, ^{0}K; λ – the thermal conductivity coefficient W/m*K; x, y, z – the Cartesian coordinates of the solid; Q – the internal sources of heat.

To solve the dependency (5), it is necessary to introduce the following boundary conditions

$$\lambda\left(\frac{\partial T}{\partial x} l_x + \frac{\partial T}{\partial y} l_y + \frac{\partial T}{\partial z} l_z\right) + \lambda\left(T - T_o\right) + q = 0, \tag{6}$$

where T_o – ambient temperature. ^{0}K; l_x, l_y, l_z – direction cosines of the normal vector to the surface; q – heat flow density, W/m.

The dependences (5) and (6) form the functional (7). Its minimization makes it possible to solve the thermal conductivity problem

$$\phi=\frac{1}{2}\int_V\left[\lambda\left(\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2-2QT\right)\right]dV+\int_S\left[qT+\frac{1}{2}h(T-T_o)^2\right]dS \qquad (7)$$

After the FEM approximation of (7), the equation for the gas turbine engine rotor heat balance can be obtained:

$$\mathbf{K_T T}=\mathbf{Q}\,, \qquad (8)$$

where $\mathbf{K_T}$ – the global thermal conductivity matrix of a finite element; $\mathbf{T}$ – the temperature vector located in the finite element nodes; $\mathbf{Q}$ – the vector of external heat loads.

Therefore, in order to solve a gas turbine engine rotor thermoelasticity problem, a system of matrix equations, obtained on the basis of the dependences (4) and (8), must be first solved.

Problem solution

For a refined solution of the given problem, a special three-dimensional hexahedral finite element was designed (Fig. 1). It has eight nodes with five degrees of freedom in each node. Such type of finite elements makes it possible to provide the FEM approximation of solids that form the turbine rotor since their form corresponds to those of the rotor components. That is why, using a mathematical model formed on the basis of such finite elements makes it possible to more correctly solve the thermoelasticity contact problem of gas turbine rotors [7].

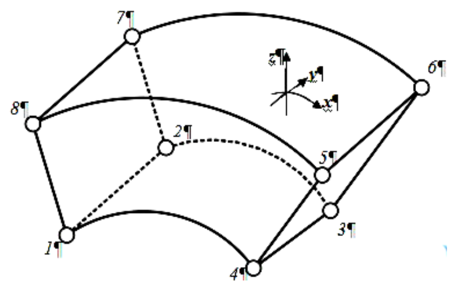

Fig. 1. Three-dimensional hexahedral finite element

Transition from the global Cartesian coordinate system *x y z* to the finite element local coordinate system ζ η ξ can be described by the following dependences

$$x=\{N_i\}^T\begin{Bmatrix}x_1\\x_2\\\vdots\\x_8\end{Bmatrix};\quad y=\{N_i\}^T\begin{Bmatrix}y_1\\y_2\\\vdots\\y_8\end{Bmatrix};\quad z=\{N_i\}^T\begin{Bmatrix}z_1\\z_2\\\vdots\\z_8\end{Bmatrix}\ ,\ (i=1,2,\ldots,8), \qquad (9)$$

where (x, y, z) – the global Cartesian coordinates of a finite element; $(x_1,x_2,\ldots,x_8;\ y_1,y_2,\ldots,y_8;\ z_1,z_2,\ldots,z_8)$ – the Cartesian coordinates of the finite element nodes; N_i – the finite element shape functions.

The shape functions of the finite element under consideration in the local curvilinear coordinate system ($-1\le\xi\le1$; $-1\le\zeta\le1$; $-1\le\eta\le1$) are presented by the dependences

$$\begin{aligned}
N_1&=\frac{1}{8}(1-\eta)(1-\xi)(1-\zeta); & N_2&=\frac{1}{8}(1-\eta)(1+\xi)(1-\zeta);\\
N_3&=\frac{1}{8}(1+\eta)(1+\xi)(1-\zeta); & N_4&=\frac{1}{8}(1+\eta)(1-\xi)(1-\zeta);\\
N_5&=\frac{1}{8}(1-\eta)(1-\xi)(1+\zeta); & N_6&=\frac{1}{8}(1-\eta)(1+\xi)(1+\zeta);\\
N_7&=\frac{1}{8}(1+\eta)(1+\xi)(1+\zeta); & N_8&=\frac{1}{8}(1+\eta)(1-\xi)(1+\zeta).
\end{aligned} \qquad (10)$$

The displacement of finite element nodes towards the ***x y z*** axes can be obtained by solution of the dependencies (9) and (10). Thus

$$\boldsymbol{\delta}^e = \begin{bmatrix} N_1 & 0 & 0 & \cdots & N_8 & 0 & 0 \\ 0 & N_1 & 0 & \cdots & 0 & N_8 & 0 \\ 0 & 0 & N_1 & \cdots & 0 & 0 & N_8 \end{bmatrix} \begin{Bmatrix} \delta_1^1 \\ \delta_1^2 \\ \delta_1^3 \\ \vdots \\ \delta_8^1 \\ \delta_8^2 \\ \delta_8^3 \end{Bmatrix}, (e = 1,2,\ldots,n), \qquad (11)$$

where $\boldsymbol{\delta}^{\mathbf{e}}$ – the vector of the *e*-st finite element generalized displacements; δ_1^1, δ_1^2, δ_1^3 – the generalized displacements of the finite element node *1* towards the *x y z* axes respectively; *n* – the quantity of the finite element nodes taken into consideration.

The temperature vector (T^e) for the *e*-st three-dimensional finite element can be obtained by means of the dependences

$$T^e = \sum_{i=1}^{n} N_i T_i, \; (i = 1,2,\ldots,8), \qquad (12)$$

where T_i – the temperatures in the finite element nodes.

It should also be noticed noted that for the solution to the systems of equations (8 – 11) the numerical method of Kholetsky direct expansion is used. To make global matrixes more compact, a procedure of matrix reordering is also used. To reduce the computer RAM, necessary for solving the set problem, the Sherman compact scheme is used.

Main results and their analysis

In order to evaluate the adequacy of the developed mathematical model and calculation algorithm the temperature and displacement fields of gas turbine engine rotor components are calculated. All the calculations are realized using a special program complex. In the assembly unit under consideration the shaft diameter *d*=120h7 mm; the shaft material is the 20Х3НМФА heat-resistant steel. The shaft rotation frequency is 1000 revolutions per minute. The thermal conductivity coefficient λ=500 W/m*K.

The frontal surfaces of the two components forming the assembly unit are axially fixed. Between the shaft and disk radial surfaces there is an interference, whose value is 0.01 mm. On the front surfaces of the two components the boundary conditions of the first kind used for the thermal problem are given. In the initial state, both parts of the assembly unit have the temperature of 100 C. Then the shaft front surface is heated up to 1000 C.

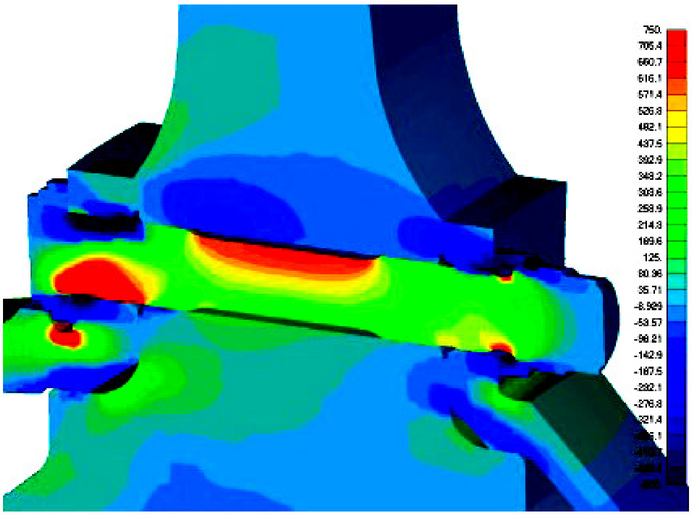

Fig. 2. Displacement fields of a gas turbine engine rotor

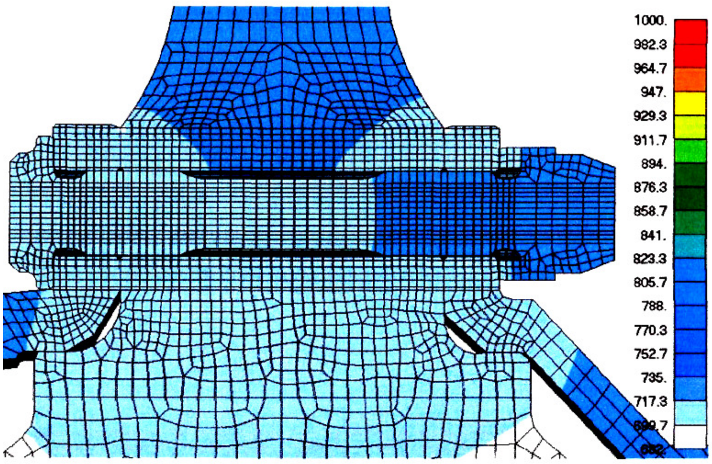

Fig. 3. Temperature fields of a gas turbine engine rotor

Figures 2 and 3 show the displacement and temperature fields in the area of the rotor shaft and disk.

According to figure 2, it should be taken into consideration that on the shaft and disk front surfaces the conjugation conditions were changed from clearance to interference. Such changes could be explained by the influence of the heat flow, that causes heat extension of the contacting surfaces. These processes are described by Fick's law [5].

The temperature field in the gas turbine shaft and disk rotor is shown in figure 3. It could be observed that it is practically homogeneous and has no sharp gradients. This fact could be explained by the change of conjugation conditions from the mounting clearance to the interference, caused by the influence of thermal deformation of contact surfaces. Thus, the absence of an air clearance between the shaft and disk front surfaces caused the absence of sharp temperature gradients in the places of their conjugation.

Conclusion

A refined mathematical model that could be used for solving thermoelasticity problems of gas turbine engine rotors has been created. This model is based on using three-dimensional hexagonal finite elements. The displacement and temperature fields of on the surfaces of contacting components have also been obtained. It has also been established that the conjugation conditions in such type of assembly units change from clearance to interference due to the heat extension of material. On the basis of this mathematical model the thermal stress-strained state of such rotors, widely used in marine propulsion engineering, could also be investigated.

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Використання уточнених скінченноелементних моделей для розв'язання задачі термопружності роторів турбомашин

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Розроблена уточнена математична модель роторів газотурбінних двигунів з використанням тривимірних скінченних елементів криволінійної форми. Всі розрахунки виконані для роторів, що дуже поширені в енергетичному машино-та суднобудуванні. Деталі такого типу мають конструктивну неоднорідність, яку навряд чи можна було б правильно пояснити, використовуючи добре відомі скінченні елементи та їхні функції форми. З іншого боку, математична модель повинна бути максимально простою з метою її широкого використання в процесі проектування ротора. Тому була розроблена нова уточнена скінченноелементна математична модель, що складається з тривимірних криволінійних скінченних елементів типу гексаедр. Вона використовувалась для розрахунку поля переміщень, викликаного комплексним впливом теплового потоку, і контактного навантаження в місцях з'єднання елементів ротора. Такий підхід дає можливість описати весь ротор як суперпозицію розроблених криволінійних

моделей скінченних елементів і зробити процес розрахунку більш правильним і компактним. Для вирішення поставленого завдання була складена система матричних рівнянь. Вона ґрунтується на використанні залежностей енергетичного балансу під час механічної контактної взаємодії елементів ротора, а також теплового балансу у разі впливу нестаціонарного теплового потоку. Під час створення чисельного алгоритму розв'язання поставленої задачі використовувалося пряме розкладання Холецького. Для додання розв'язку більшої компактності застосовувалася схема Шермана. Всі розрахунки полів переміщень і температур проведені для двох широко поширених типів з'єднань, які використовуються для створення таких роторів, а саме: з'єднань з зазором та натягом.

Ключові слова: *тривимірні скінченні елементи, ротори газових турбін, поля переміщень і температур, задача контактної термопружності, зазор, натяг.*

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STEPWISE OPTIMIZATION OF I-SECTION FLEXIBLE ELEMENTS UNDER A FUZZY APPROACH TO TAKING INTO ACCOUNT CORROSION AND PROTECTIVE PROPERTIES OF ANTICORROSIVE COATING

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This paper is a continuation of research in the field of optimal design of structures under a combined approach to the measurement of corrosion and anticorrosion protective properties of coatings. As noted earlier, such coatings are barrier layers that impede the penetration of aggressive media to the surface of a structure and delay the onset of the process of intense corrosion. In this case, it is important to take into account not only the corrosive effect on the structure, but also be able to estimate the period of time for which the anticorrosive coating loses its protective properties. Since structural elements with damaged protective coatings are able to continue to be subjected to current loads over a considerable period of time, their accelerated corrosion wear is to be taken into account in the damaged areas of coatings. Consequently, the work of the structures protected by coatings consists of two periods: with a protective coating (during which the coating loses protective properties and collapses) and with a damaged protective coating (when there is a severe corrosion wear of unprotected structural areas). The model proposed in the previous study (and implemented on the example of optimization of the flexible elements of a rectangular beam) allows taking into account the smooth transition of the work of structures with protective coatings and the time when the protective properties of anticorrosive coatings practically do not work. This paper considers a solution to a more complicated (due to its multi-extremity) problem of optimization (finding the optimal form) of I-section (double-T) flexible structural elements under a fuzzy approach to taking into account corrosion and anticorrosive coating protective properties.

Keywords: *corrosion, anticorrosive coatings, optimization.*