Sergei Nikolaevich Chernikov. Memoirs

B. Plotkin

Dedicated to the 100th anniversary of the birth of Sergei Nikolaevich Chernikov

I met Sergei Nikolaevich Chernikov in 1945, when I matriculated to the Ural University, straight after the war and after being released from the hospital. Sergei Nikolaevich was 33 years old at the time; he was a remarkable person — the head of the Department of Mathematics, a scientist of worldwide reputation and a great specialist in group theory.

Despite the difference in age, we interacted often. I especially remember our stay in the settlement of Sarana, near Krasnoufimsk, in 1946 or 1947. The university's recreation site, where students and professors spent their summer holidays, was located there. Sergei Nikolaevich was among the professors there that year. He was a lean young man and frequently swam in the river Ufimka. His passion for swimming remained unchanged all his life. It so happened that at the time of the Chernobyl disaster he was swimming in the Dnieper River...

Sergei Nikolaevich came to the Ural University from the Polytechnic Institute. His results and knowledge attracted people, and even in the Polytechnic he had many students. For example, Jury Nikolaevich Nefed'ev was one of them. During his work at the University, Sergei Nikolaevich was a Ph.D. advisor to Viktor Mikhailovich Glushkov, who at that time taught at the Ural Forest Engineering Institute. Glushkov later became an Academician and the Director of the Institute of Cybernetics in Kiev. Another student of S.N. was Professor Victor Silvestrovich Charin. Many other prominent mathematicians were also his students and followers.

In general, his ability to encourage young, talented people to make research in his area was very characteristic for S.N. Chernikov.

I entered the University as a physics major. I entered together with my friend Lenya Kobelev, who was also injured during the World War II.

He later became a Professor of Theoretical Physics. At the time, at the end of the 40s, there was a joint Department of Physics and Mathematics at the Ural University, which formally remains the present situation. But I should mention that the atmosphere at the University was quite different. Students used to go to lectures of professors from various departments to listen to interesting talks and used to interact closely despite pursuing different majors. Lectures on physics were given to us by Anton Panteleimonovich Komar, and I did not like those lectures. Note that in 1950, Academician A.P. Komar became the Director of the LPTI replacing the recently fired Academician A.F. Ioffe.

On the other hand, algebra was taught by Professor Petr Grigoryevich Kontorovich, who later became my supervisor. He spoke with little enunciation, often swallowing letters and whole words, but in spite of this manner of speaking, a huge expanse for thoughts and creativity quickly opened to the audience.

Kontorovich organized the Department of Algebra of the Ural University and was also its permanent head. He, just like Sergei Nikolaevich, was an outstanding specialist in group theory, one of those who created the theory of abstract infinite groups.

S.N. Chernikov delivered lectures on classical analysis, talked very brightly and very unconventionally, and I very much enjoyed analysis in his exposition. I visited these lectures, but still preferred algebra, and then group theory, participating in P.G.'s and S.N.'s seminars.

Once I went to see Kontorovich and complained about Komar's physics lectures. He answered that if I don't like them, I should learn physics from books and peacefully study algebra. So I switched to the Department of Mathematics. By the way, not long before that, in 1944, the first edition of Kurosh's book "Group Theory" appeared. This book immediately became for many algebraists of my generation a bible.

Both Petr Grigoryevich Kontorovich and Sergei Nikolaevich were wonderful algebraists. In the period around the war they introduced the concepts that became the topic of numerous studies in the following decades.

It must be mentioned that the situation was not that easy for P.G. and S.N. It happens quite often when two outstanding scientists who work in the same area find themselves in the walls of the same educational institution. Their relations were influenced by scientific competition and by a struggle to attract good students as well as to teach the main courses. For instance, it is clear that S.N. Chernikov would have been glad to teach an algebra course, which was traditionally, well, and logically, delivered by the Kontorovich.

Once S.N. described to me a problem about divisible torsion-free nilpotent groups. I constructed an example, which Chernikov found to be

interesting, because it involved the divisible groups that he was actively studying. At the same time, in my example, roots were unique, i.e., the constructed group was an R-group. Such groups were an important subject of study for Kontorovich. Overall, I worked directly with Kontorovich, and maintained contact with S.N.

Time passed and in 1967 in Riga, I organized the All-union Colloquium. Many well-known algebraists were there, including S.N. Chernikov and P.G. Kontorovich. Everyone was pleased to see how well they interacted. They both were dear to me, both were teachers in my student years.

It comes to mind that S.N. had an even temperament and a firm character. In Sverdlovsk there was an incident when S.B. Stechkin, the first director of the Sverdlovsk Branch of the Institute of Mathematics and a quite harsh man, during one of the meetings was rude to S.N.: "And you be quiet, son of a priest!" Sergei Nikolaevich stood up and answered calmly and with dignity: "My priest-father is not in the least worse than your father-academician." The incident happened in the time when accusations of poor social character were not just a sign of boorishness, but could still be a direct threat and one could draw corresponding conclusions.

Back during my studentship, I met Nina Vasilyevna Baeva, the future wife of Sergei Nikolaevich. She was also a university student, but one year senior to me. We were friends. Later, when the Chernikovs already lived in Ukraine, I visited them each time I was in Kiev. I remember how sad S.N. was when he saw me and my wife off — this was after Chernobyl, and it was the last time we met.

Sergei Nikolaevich Chernikov started his work in algebra and group theory back in the end of the 30s, and immediately became known by his new, unconventional approaches. He built the theory in which group properties were considered in connection with the properties of their finitely generated subgroups. He defined locally nilpotent and locally solvable groups and proved that these infinite groups are never simple, that is, they have nontrivial normal subgroups.

Moreover, S.N.'s papers introduced a general perspective on the idea of solvability and nilpotency in infinite groups.

The work of S.N. gave reason for A.I. Malcev to indicate a general method of obtaining local theorems in groups, and not only in groups. Malcev used logic for this, and it was the first application of logic and model theory to purely algebraic problems.

In many respects, the ideas of S.N. Chernikov also influenced my scientific interests. Referring to this, I would like to mention how I came to the Engel problem.

In 1947, in the UMN (Russian Mathematical Surveys), an important review of Kurosh and Chernikov "Solvable and Nilpotent Groups" was released. This review impressed many algebraists including me.

Among other things, there were a lot of open problems in the review. For myself, I singled out the problem about groups satisfying the normalizer condition — groups, in which every proper subgroup does not coincide with its normalizer. All nilpotent groups have this property. The question was whether the converse is also true. This problem, dating back to O.Yu. Schmidt, was highlighted in the review of Kurosh-Chernikov.

I noticed that if G is a group obeying the normalizer condition and g is its element, then for each element a in G there is an integer $n = n(a, g)$ so that

$$[a, g, g, ..., g] = 1,$$

where the repeated commutator is taken $n = n(a, g)$ times. I called an element g with such a property a nilelement, and the group G which consists of nilelements, a nilgroup. Each locally nilpotent group is a nilgroup, but the converse is not true (1964, Golod-Shafarevich). It is true, however, if G satisfies the normalizer condition. Thus, all such groups are locally nilpotent.

H. Heineken and I. Mohamed proved that the groups that obey the normalizer condition and have trivial center exist, so they are not nilpotent.

On the other hand, Sergei Nikolaevich proposed to check more generally whether groups with the normalizer condition can have a nontrivial abelian normal subgroup. If there is no abelian normal subgroup, then the group with such a property is not solvable. I am not sure what is known about this problem of Sergei Nikolaevich today.

Let us consider an infinite nilgroup G in which n is independent of a and g. That is, we are talking about the identity:

$$[x, y, y, ..., y] = 1,$$

where the repeated commutator is taken n times. Groups with such an identity are called n-Engel groups. The Engel problem is to prove that n-Engel groups are not necessarily locally nilpotent. In other words, it means that a free n-Engel group of finite rank is not nilpotent for all sufficiently large n.

This problem remained unsolved for exactly 60 years. The difficulty was to develop the methods of combinatorial and geometric group theory necessary to solve such sorts of problems. This year, I listened to a course of lectures given by Ilya Rips, from which the solution of the problem follows. Now E. Rips and A. Juhász are preparing a text.

In group theory, three avenues of investigation have gradually emerged. First the structural properties of individual groups. Second, the combinatorial group theory. Finally groups in connection with other theories: geometry, logic, group presentations. The Kurosh-Chernikov review was mainly connected with structural theory as was S.N.'s further activity while he worked in Kiev. Lately interest in the combinatorial theory has increased dramatically. The combinatorial approach, along with geometric ideas often leads to solving difficult old problems. In particular, the Burnside problem and the Engel problem were solved by combinatorial geometric methods, although they came from structural group theory.

In any case, it took 60 years of hard work to solve the Engel problem, which was inspired by Kurosh-Chernikov's review.

We mention two more problems solved by combinatorial methods which grew out of structural group theory. We talk about Artinian and Noetherian groups. The following questions arose: Is every Artinian group a Chernikov group and every Noetherian group a finite extension of a polycyclic group? Recall that a group is a Chernikov group if it is a finite extension of a direct product of finitely many quasicyclic groups.

In both cases A.Yu. Olshansky obtained the general answer "no" in his studies on the Burnside problem. At the same time, we can answer in the positive if we assume that the group satisfies some additional conditions (R. Baer, V.P. Shunkov, N.S. Chernikov).

S.N. Chernikov devoted much attention to groups with different conditions of finiteness: to groups in which all abelian subgroups are finite, to Artinian or Noetherian groups. This research was surveyed in Sergei Nikolaevich's second wonderful review "Finiteness Conditions in the General Theory of Groups". It was published in 1959.

One of the favorites of S.N. were divisible groups. A group is divisible if every equation $x^n = a$ is solvable. It is known (A.I. Malcev) that every torsion-free locally nilpotent group can be embedded in a divisible group with the same properties. The identities of a group are preserved in this process, while quasi-identities are not.

S.N. Chernikov constructed the theory of divisible groups and treated conditions which provide (not only for torsion-free groups) a "good" completion to a divisible group.

It is well known that besides abstract group theory S.N. was also interested in applications of algebra. This resulted in the book "Linear inequalities", which was published in 1968, and a review with the same title released two years earlier in "Itogi Nauki".

In the work of S.N. Chernikov, especially in his Kiev period, a group is considered as a complicated organism with very different interacting properties. Summarizing, S.N. Chernikov made many important contributions to group theory, and his name ranks the same as the names of great

creators of this theory: A.I. Malcev, A.G. Kurosh, P. Hall, G. Higman, R. Baer, B. Neumann.

Many students of S.N. successfully continued his work and developed his ideas. This was the case in the Ural as well as in the Ukraine. In conclusion, I would like to name several Ukrainian scientists who worked in group theory.

It is known that Sergei Nikolaevich learned algebra by himself using the books of a great mathematician and teacher D.A. Grave, who spent a large part of his scientific tenure in Kiev. One of the students of Grave was O.Yu. Schmidt, whose book "Abstract theory of groups" played an important role in Sergei Nikolaevich's study of group theory.

The academician V.M. Glushkov, a direct student of S.N., became an outstanding scientist in the area of cybernetics. In his activities, he used algebraic methods, in particular group theory. Also works in group theory by R.I. Grigorchuk are known worldwide. In fact, R. Grigorchuk created a new direction in group theory.

I would like to mention the direct students of S.N. Chernikov in the Ukrainian period of his educational and scientific activity. These are D.I. Zaitsev, Ya.P. Sysak, O.D. Artemovich, A.V. Tushev and others.

Note that L.A. Kaluzhnin worked in Kiev for a long time. His students are: V.I. Sushchansky, V.A. Vyshensky, V.O. Ustimenko, A.G. Ganyushkin, M.H. Klin, M. Muzychuk, F. Lazebnik and others. Their works, like the works of Kaluzhnin himself, are connected with applications of group theory and with permutation groups.

Sergei Nikolaevich's son, N.S. Chernikov, actively and fruitfully works in the area of infinite groups. All mentioned mathematicians and, of course, S.N. Chernikov, contributed significantly to group theory.

Besides investigations in group theory, the achievements of Ukrainian mathematicians in representation theory are internationally acknowledged. This reputation is associated with the names of Yu. Drozd, V. Kirichenko, L. Nazarova, A. Roiter, V. Sergeichuk and many others.

Coming back to Sergei Nikolaevich Chernikov, I would like to emphasize once again that his role in the development of group theory all over the world is absolutely overwhelming. The same is true with respect to the development of Ukrainian school of algebra.

Contact information

B. Plotkin Department of Mathematics, Hebrew University
91904 Jerusalem, Israel
E-Mail: plotkin@macs.biu.ac.il