

# 2.5D SIMULATION OF PLASMA WAKEFIELD EXCITATION BY A NONRESONANT TRAIN OF RELATIVISTIC ELECTRON BUNCHES

*K.V. Lotov<sup>1</sup>, V.I. Maslov, I.N. Onishchenko, E.N. Svistun<sup>2</sup>*

*National Science Center “Kharkov Institute of Physics and Technology”, Kharkov, Ukraine;*

*<sup>1</sup>Budker Institute of Nuclear Physics, and Novosibirsk State University, Novosibirsk, Russia;*

*<sup>2</sup>Karazin Kharkov National University, Kharkov, Ukraine*

By using 2d3v code LCODE the investigations in detail of the reconstruction of a long nonresonant train of relativistic electron bunches into resonant one at plasma wakefield excitation are presented. Because of bunch repetition and plasma frequencies detuning the wakefield beatings are occurred. The bunches in the maximum of beating experience defocusing radial force and go out the interaction with wakefield. It leads to the shortening of the beating period and to the asymmetry between the energy loss of decelerated bunches of the beating front and energy gain of accelerated bunches of the beating rear. In result along with wakefield amplitude beating the monotonical growth takes place with the rate equivalent to the resonant train with smaller current.

PACS: 29.17.+w; 41.75.Lx;

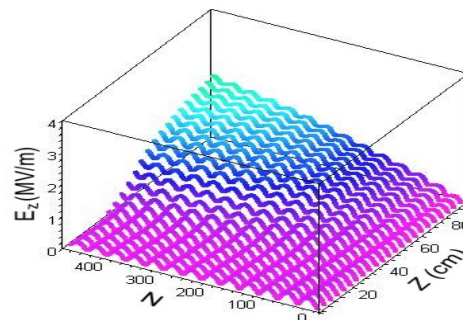
## 1. INTRODUCTION

Resonant plasma wakefield excitation by long train of relativistic electron bunches is difficult because realization of homogeneous and stationary plasma in experiments is difficult. Results of 2.5D numerical simulation by the code LCODE [1] of plasma wakefield excitation by a nonresonant train of relativistic electron bunches (repetition frequency  $\omega_m$  is not coincided with plasma frequency  $\omega_p$ ) with parameters close to experiment [2] have been presented in [3]. Train of short relativistic electron bunches of energy 2 MeV, charge 0.32 nC, rms length  $2\sigma_z=1.7$  cm, rms radius  $\sigma_r=0.5$  cm, rms angular spread  $\sigma_\theta=0.05$  mrad, repetition period 360 ps excites a wakefield. The plasma of density, smaller than resonant one  $10^{11}$  cm<sup>-3</sup>, is simulated, so the frequency of the excited wave is smaller than the frequency of bunches repetition  $\omega_p < \omega_m$ . The mechanism of the wakefield excitation in this case is the self-consistent self-cleaning of electron bunches. As a result of interaction with wakefield the train becomes more resonant.

In this paper we consider the mechanism of this excitation more details, taking into account bunch defocusing in radial field and changing of the coupling of bunches with wakefield.

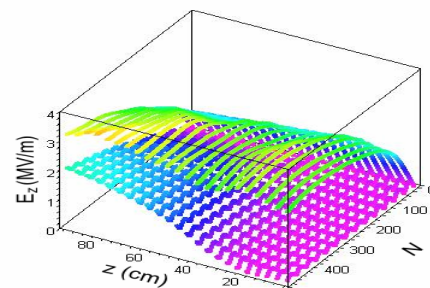
## 2. RESULTS OF SIMULATION

It is difficult to support plasma strongly resonant ( $\omega_m$  is coincided with  $\omega_p$ ) in experiments. Therefore, the wakefield excitation by nonresonant train of electron bunches in plasma is of great interest. We simulate dynamics, using cylindrical coordinate system ( $r, z$ ). At first we consider 500 electron bunches. Fig.1 shows the amplitude of the on-axis electric field as a function of the coordinate along the plasma and the number of bunches for  $\omega_p \approx 0.97\omega_m$ . At small coordinate along the plasma, we observe beatings of the field, as it is excited by a periodic force of a different frequency. Number of bunches in beating equals  $N=1/(1-\omega_p/\omega_m) \approx 39$ . Number of beatings along train equals  $N_0=500(1-\omega_p/\omega_m) \approx 13$ .



*Fig.1. The amplitude of the on-axis longitudinal electric field as a function of the coordinate along the plasma and the number of bunches for  $\omega_p \approx 0.97\omega_m$*

Some distance downstream the wakefield grows, because the train becomes resonant due to self-cleaning at radial defocusing of nonresonant bunch electrons [3]. The effect of reconstruction of resonant train due to radial defocusing of nonresonant bunch electrons is useful, because during evolution the nonlinear wave frequency  $\omega^{(NL)}$  change takes place. Really, from Fig.1 one can estimate this nonlinear frequency change  $(\omega^{(NL)}/\omega_p - 1) \approx 0.1\%$  using increase of beating number at amplitude growth.



*Fig.2. The amplitude of the on-axis electric field as a function of the coordinate along the plasma and the number of bunches for  $\omega_p \approx 1.0025\omega_m$  and  $\omega_p \approx 0.97\omega_m$*

We simulate and compare the case, close to optimal  $\omega_p \approx 1.0025\omega_m$  (initial compensation of nonlinear  $\omega_p$  decrease), and the nonresonant case  $\omega_p \approx 0.97\omega_m$ . For optimized difference  $\omega_m - \omega_p$  the wakefield amplitude reaches value 4.5 MV/m, corresponding to 15% of the wave breaking limit. From Fig.2 one can see that near

exit of the system after 500 bunches the wakefield in the case  $\omega_p \approx 0.97\omega_m$  is smaller only in 1.5 times in comparison with the case, close to optimal  $\omega_p \approx 1.0025\omega_m$ .

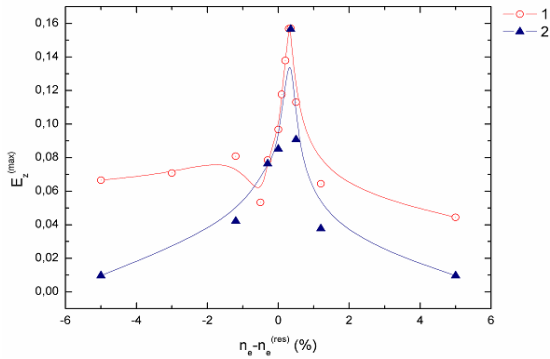


Fig.3. Dependence of the wakefield amplitude on plasma density difference from resonant one, determined by bunch repetition frequency. 1 -  $\gamma_b=5$ , 2 -  $\gamma_b=1000$ .  $E_z^{(max)}$  is normalised on  $m_e\omega_p c/e$

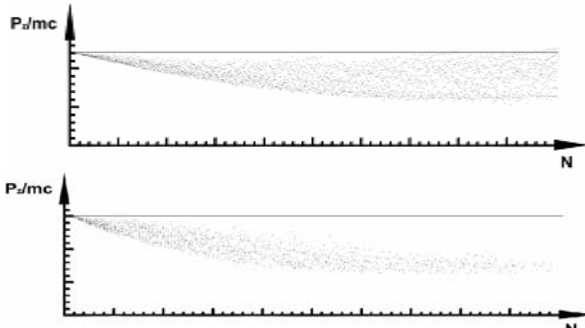


Fig.4. Longitudinal momentum of train of 318 bunches in the middle ( $z=50$  cm) and on the exit of the system ( $z=100$  cm) for  $\omega_p \approx 0.999\omega_m$

We simulate dependence of the wakefield amplitude on plasma density difference from resonant one, determined by  $\omega_p = \omega_m$ . Maximal wakefield is achieved at  $\omega_p \approx 1.0018\omega_m$ . It is determined by nonlinear wave frequency  $\omega^{(NL)}$  change. Maximal wakefield is excited by nonresonant train of bunches (see curve 2 in Fig.3, ob-

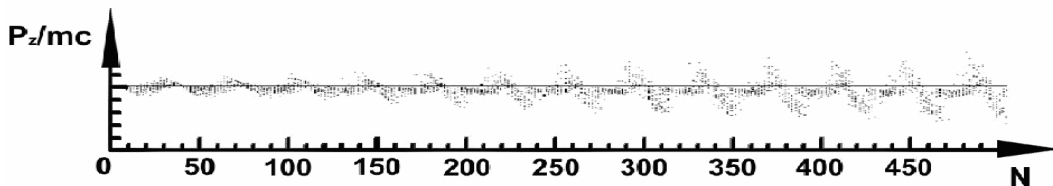


Fig.5. Longitudinal momentum of 500 bunches on the end of the system ( $z=100$  cm) for  $\omega_p \approx 0.97\omega_m$

### 3. SELF-CLEANING MECHANISM

Let us consider self-cleaning mechanism of the train of 32 bunches in the case  $\omega_p \approx 0.92\omega_m$ . From Fig.6 one can see that the radial distribution of bunch density is asymmetrical along beating. In front of beating the bunch electrons are located closer to the axis and in the rear of beating they are located far from the axis.

From Figs.6-8 one can see that the middle radii of bunches are larger (Fig.6) and the mean field  $E_0 = \int E_z n_b dr / \int n_b dr$  is smaller (Fig.7) in rear of beating (phases of electron acceleration) in comparison with front of beating (phases of electron deceleration).

tained for ultra relativistic electrons, for which radial motion is negligible). Due to the self-cleaning and reconstruction of the nonresonant train to resonant one (see curve 1 in Fig.3 for slightly relativistic case with essential radial dynamics) the achieved wakefield is especially larger for large detuning.

One can see self-cleaning due to defocusing in Fig.4. Near exit of the plasma the bunch electrons, which get in large radial defocusing fields and consequently in small longitudinal electrical field  $E_z$ , are defocused.

For the train of 318 bunches (Fig.4), we observe that approximately 100 bunches lose their energy linearly. Further bunches lose their energy slower. Let us consider energy exchange of bunches with wakefield for small difference  $\omega_m$  and  $\omega_p$ . Energy losses of N-th bunch on wakefield excitation on  $\lambda$  equals

$$\varepsilon_N = 2\pi e n_b c E_{Nc} / \omega_p, E_{Nc} = E_N + (\beta - 1)\delta E_N, \beta \approx 1/2.$$

Then wave energy  $W$  changes on

$$W_N - W_{N-1} = \eta \varepsilon_N, \eta \approx 1.7 \cdot 10^{-3}, \ell_b / \lambda \approx 1/6,$$

$\eta$  is the part of volume, occupied by bunches in comparison with volume, occupied by wakefield

$$\delta E_N = E_N - E_{N-1}, E_N \approx N \delta E_N, \delta E_N = E_1 = e n_b c \eta (2\pi)^2 / \omega_p.$$

Energy losses of train of bunches  $\sum \varepsilon_i$  equal

$$\sum \varepsilon_i = \pi \eta (e c n_b N 2\pi / \omega_p)^2.$$

Energy losses of train of  $N$  bunches are proportional to  $N^2$ . On 1st asymptotic the wakefield amplitude grows linearly with time

$$\partial_t E_N = 2\pi e n_b c \eta = \text{const.}$$

When bunches completely lose the energy

$$\sum \varepsilon_i = (N - K) m c^2 (\gamma_b - 1) n_b + \pi \eta c (e c n_b K 2\pi / \omega_p)^2.$$

When each bunch loses a significant part of the energy, the wakefield amplitude grows with time as  $\sqrt{t}$ .

From Fig.5 one can see that there is electron group which is decelerated and there is no another similar electron group which is accelerated.

Because the wakefield is excited by nonresonant train the wakefield represents beatings. Near injection boundary the symmetry between phases of decelerated electrons in wakefield in front of beating and phases of accelerated electrons in wakefield in rear of beating is realized. As a result of bunch radial defocusing in the middle of beating the wavelength becomes larger far from the injection boundary. This leads to phase symmetry braking, i.e. accelerated bunches in rear of beating get in the smaller (larger) longitudinal (radial) fields comparatively to decelerated bunches in front of beating. Therefore middle radii of bunches are larger in rear of beating, than in front of beating. It decreases coupling of bunches with wakefield in rear of beating in

comparison with front of beating. The train continues to excite the wakefield.

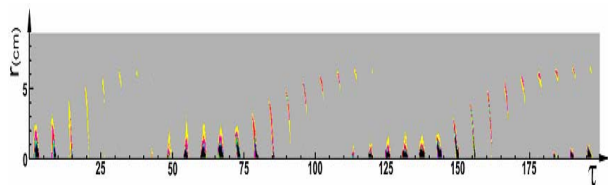


Fig. 6. Evolution of bunch density on the plasma end ( $z=100$  cm) for  $\omega_p \approx 0.92 \omega_m$

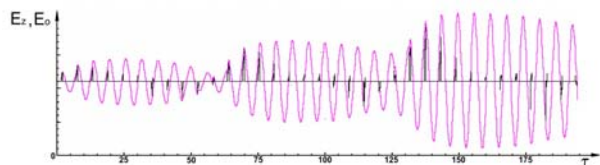


Fig. 7. The on-axis wakefield excitation  $E_z$  by train of 32 bunches ( $z=100$  cm) in the nonresonant case  $\omega_p \approx 0.92 \omega_m$ . The mean field  $E_0$  is shown to be black

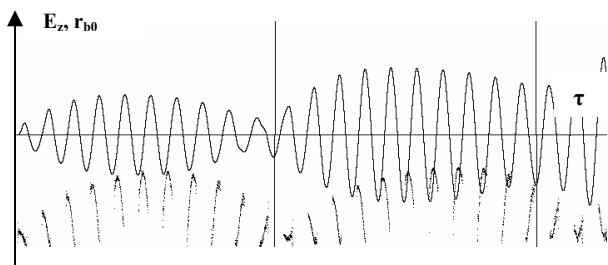


Fig. 8. The on-axis wakefield excitation  $E_z$  and middle radius of bunches  $r_{b0}$  in the nonresonant case  $\omega_p \approx 0.92 \omega_m$

## CONCLUSIONS

As a result of bunch radial defocusing in the middle of beating, excited by nonresonant train of electron bunches, the wavelength becomes larger. This leads to bunch phase shift, in rear of beating accelerated bunches get in the smaller (larger) longitudinal (radial) fields comparatively to decelerated bunches in front of beating. The train continues to excite the wakefield.

## ACKNOWLEDGEMENTS

This work is supported by Russian Science Support Foundation, Russian President grants MD-4704.2007.2 and NSh-6046.2008.2, RFBR grant 06-02-16757, and Russian Ministry of Education grant RNP.2.2.1.1.3653.

## REFERENCES

1. K.V. Lotov, V.I. Maslov, I.N. Onishchenko, E.N. Svistun. Simulation of plasma wakefield excitation by a sequence of relativistic electron bunches // *Problems of Atomic Science and Technology. Series: "Plasma Physics"* (14), 2008, N6, p.114-116.
2. A.K. Berezin, Ya.B. Fainberg, V.A. Kiselev, et al. Wakefield excitation in plasma by relativistic electron beam, consisting regular train of short bunches // *Fizika Plasmy*. 1994, v.20, N7-8, p.663-670.
3. K.V. Lotov, V.I. Maslov, I.N. Onishchenko, E. Svistun, Resonant excitation of plasma wakefields by a nonresonant train of short electron bunches // *Laser and Plasma Accelerators Workshop*. Greece. 2009, p.69.

Статья поступила в редакцию 29.11.2009 г.

## 2,5-МЕРНОЕ МОДЕЛИРОВАНИЕ ВОЗБУЖДЕНИЯ КИЛЬВАТЕРНОЙ ВОЛНЫ В ПЛАЗМЕ НЕРЕЗОНАНСНОЙ ЦЕПОЧКОЙ РЕЛЯТИВИСТСКИХ ЭЛЕКТРОННЫХ СГУСТКОВ

К.В. Лотов, В.И. Маслов, И.Н. Онищенко, Е.Н. Свистун

Представлены детальные исследования с использованием 2d3v кода LCODE превращения длинной нерезонансной цепочки релятивистских электронных сгустков в резонансную при возбуждении кильватерного поля в плазме. Вследствие отличия частоты следования сгустков и плазменной частоты происходят биения кильватерного поля. На сгустки в середине биения действует радиальная дефокусирующая сила, что приводит к уменьшению их взаимодействия с кильватерным полем. Это приводит к укорочению биений и к асимметрии между величинами теряемой энергии тормозящимися на фронте биения сгустками и приобретаемой энергии ускоряемыми в конце биения сгустками. В результате наряду с биениями амплитуда кильватерного поля монотонно нарастает с инкрементом, эквивалентным резонансной цепочке с меньшим током.

## 2,5-ВИМІРНЕ МОДЕЛЮВАННЯ ЗБУДЖЕННЯ КИЛЬВАТЕРНОЇ ХВИЛІ В ПЛАЗМІ НЕРЕЗОНАНСНИМ ЛАНЦЮЖКОМ РЕЛЯТИВІСТСЬКИХ ЕЛЕКТРОННИХ ЗГУСТКІВ

К.В. Лотов, В.І. Маслов, І.М. Онищенко, О.М. Свистун

Надано детальні дослідження з використанням 2d3v-коду LCODE перетворення довгого нерезонансного ланцюжка релятивістських електронних згустків у резонансний при збудженні кильватерного поля у плазмі. Завдяки відміні частоти входження згустків та плазмової частоти спостерігається биття кильватерного поля. На згустки в середині биття діє радіальна дефокусуюча сила, що призводить до зменшення їх взаємодії з кильватерним полем. Це веде до скорочення биття та до асиметрії між величинами енергій, що втрачається згустками, які гальмуються на фронті биття, та що набувається згустками, які прискорюються на краю биття. В результаті поряд з биттям амплітуда кильватерного поля монотонно зростає з інкрементом, еквівалентним резонансному ланцюжку з меншим струмом.