Algebra and Discrete Mathematics
Number 2. **(2004).** pp. 114 – 116
© Journal "Algebra and Discrete Mathematics"
A SHORT NOTE

A note to my paper "Multi-algebras from the viewpoint of algebraic logic"

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Communicated by L. Márki

30.09.2003

Abstract. The definition of a resolvent, introduced in the paper mentioned in the title, is simplified, and some misprints in that paper are corrected.

1. This note is, actually, a supplement to the recent paper [1]. We assume that the paper is available to the reader, and freely use the notation and terminology introduced there.

It was noted on p. 25 of [1] that

(*) a $\boldsymbol{T}$-shaped multi-algebra is completely determined by the restriction of its resolvent to the subset $M \subset X \times T$ consisting of those pairs (x, t) in which $x \notin \triangle t$.

An obvious consequence of this fact is that this restriction completely determines also the resolvent itself. As every finitary $\boldsymbol{T}$-resolvent (see Definition 3 in [1]) is a resolvent of some multi-algebra, the latter observation holds for these abstract resolvents as well. Our aim in this note is to characterise those functions of type $M \to \mathcal{P}(U^X)$ which are restrictions of $\boldsymbol{T}$-resolvents.

We also correct a number of misprints in [1]. On p. 26 of [1], the displayed formulas (7) and (8) presented two particular cases of the axiom (R2). The evidently wrong explanations to these formulas should read

2000 Mathematics Subject Classification: *08A99; 03G15, 08A62.*
Key words and phrases: *multi-algebra, resolvent.*

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as "with $x \notin \triangle s \cup \triangle r$" and, respectively, "with $t \in L_x$ and $x \neq y$, $x, y \notin \triangle r$, $y \notin \triangle t$". Misprints have occurred also in the third line of the last displayed formula on p. 30: there, μ must be changed to φ in all three occurrences. Finally, on p. 29, line 4, the phrase "(11) implies that $\psi \in R(z,s)$" should be replaced by the more appropriate "(R0) implies that $\psi \in R(z,z)$".

2. Let U be a non-empty set. For lack of a better term, we call a $\boldsymbol{T}$*-hemiresolvent on* U any function $R\colon\ M \to \mathcal{P}(U^X)$ satisfying the conditions (R0) and (8). It is said to be finitary if it, moreover, satisfies the following particular case of (R3): every $R(x,t)$ has a finite support. The main result of this note is then stated as follows.

Theorem. Suppose that R is a finitary $\boldsymbol{T}$-hemiresolvent on some U. Then R can uniquely be extended to a finitary $\boldsymbol{T}$-resolvent on U.

Proof. Assume that R satisfies the supposition. We shall show that then R gives rise to a $\boldsymbol{T}$-shaped multi-algebra and that its resolvent is the required extension of R.

It was noticed in [1] that only the particular cases (7) and (8) of (R2) were actually used in proofs (and that (R2) is, in fact, equivalent to the conjunction of (7) and (8)). An inspection of the relevant proofs in Sect. 3 of [1] shows that even (7) may be got around in them. Indeed, in both places where (7) was referred to in the proof of Lemma 4(a,b) (see p. 27 and p. 28), the job could be done by help of (8). For example, the application of (7) in the first line of the proof of Lemma 4(b) can be eliminated as follows:

$$R(y,t) = R(y,[t/z]z) = C_z(R(z,t) \cap R(y,z)) = C_z(R(y,z) \cap R(z,t)).$$

It was only the proof of Lemma 6 where the use of (7) was essential and where also (R1a) and (R2a) were used. Thus, Lemma 4, Corollary 5 and Lemma 7 apply also to hemiresolvents. The construction of the multialgebra $Alg(R)$ in the last subsection of Sect. 3 and the proof that it is $\boldsymbol{T}$-shaped are independent of Lemma 6 and pass off successfully. At last, (11) shows that the resolvent of $Alg(R)$ is an extension of R.

To demonstrate that the extension is unique, suppose that R is the restriction of two different $\boldsymbol{T}$-resolvents, R_1 and R_2, on U. Then, by Theorem 3, there are different multi-algebras $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ in $\mathcal{V}(\boldsymbol{T})$ such that R_i is the resolvent of $\boldsymbol{U}_i$. This contradicts to (*) above. □

The author is thankful to the anonymous referee for a few useful remarks.

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References

[1] J. Cīrulis, *Multi-algebras from the viewpoint of algebraic logic*, Algebra Discrete Math., N.**1**, 2003, 20–31.

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Received by the editors: 16.10.2003
and final form in 24.05.2004.

Journal Algebra Discrete Math.