

**To the Fiftieth Anniversary
of the Department
of Algebra and Mathematical Logic
of Kyiv Taras Shevchenko University**

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History of algebraic research in Kyiv University starts in 1902 when Professor D. Grave who was a pupil of the St. Petersburg mathematical school started his work at Kyiv University. After moving to Kyiv Professor Grave began intensive research in the modern at that time branches of mathematics such as group theory, Galois theory, theory of algebraic numbers. In 1910, the famous algebraic seminar of Grave was organized, where such prominent mathematicians as M.P.Kravchuk, O.Yu.Schmidt, B.M.Delone, M.G.Chebotarev, A.M.Ostrovsky and others studied and made first steps in Mathematics. But in the late thirties of the last century Kyiv algebraic school was practically destroyed as a result of political repressions, and its leading representative, academician M.P.Kravchuk died in a concentration camp. Before the World War II, Kyiv completely lost its role of a leading algebraic centre.

A renewal of algebraic research in Kyiv University began in 1955, when the well-known algebraist, Professor of Berlin University Lev Kaloujnine came to Kyiv. Lev Arkadievich Kaloujnine was born in Moscow in 1914. In 1923 his family moved to Germany. He studied at Hamburg and Berlin Universities and later at the Sorbonne, in Paris. He attended lectures of I.Schur, E.Schmidt, E.Cartan and other prominent mathematicians. During World War II L.Kaloujnine was imprisoned in a concentration camp in France. After the war, he resumed mathematical research and in 1948 defended his Doctoral Thesis. In 1951 L.Kaloujnine became Professor of Berlin University. After moving to Kyiv he was appointed Professor of Kyiv University and began teaching at the Department of Algebra, Mathematical Analysis and Probability Theory.

In 1959 a new Department of Algebra and Mathematical Logic was established in Kyiv University. Initially the only faculty of this department

were Professor L.Kaloujnine (head of the department) and Assistant Professor Tsilya Oleksandrivna Shub. Scientific interests of L.Kaloujnine were very wide, but his most well-known results were obtained by him in Group Theory. The theory of wreath products, developed by him in collaboration with M.Krasner found wide applications in Group Theory and in other parts of algebra. The Kaloujnine-Krasner theorem about embedding of extensions in wreath products is well-known and is included in most textbooks on Group Theory. A new direction of research — stability theory — was based on the theorem of Kaloujnine on stable groups. His results about structure of Sylow p -subgroups of symmetric groups and their generalizations are gems of the group theory and they find applications in various branches of algebra.

In the sixties of the last century new Assistant Professors of the Department were appointed: V.A.Vyshensky, V.I.Belichkov, V.V.Kirichenko and Yu.A.Drozd. From 1965 till 1969, V.S.Charin was working as Professor of the Department (in 1970 he became the head of the Department of Mathematical Foundations of Cybernetics at Kyiv University). At that time, V.M.Glushkov, O.A.Letichevskiy and V.G.Bodnarchuk (Institute of Cybernetics of National academy of Sciences) got involved in the teaching, as well as L.N.Nazarova, A.V.Roiter and I.D.Ivanyuta (Institute of Mathematics of NAS).

The research of scientists of the Department of that period covered several branches of Group Theory and Topological Algebra. Under the influence of L.Kaloujnine they began research in Mathematical Linguistic, Automata Theory and Abstract Galois Theory.

A new branch of modern Algebra — Representation Theory — began to develop starting from 1961. A seminar under guidance of A.V.Roiter started its work, where some students of the University, in particular V.V.Kirichenko, Yu.A.Drozd and S.A.Kruglyak participated. Most of the results obtained by the participants of the seminar were devoted to the theory of integer-valued representations and structure of orders in semisimple algebras. The results from the theory of genera of integer-valued representations (A.V.Roiter, Yu.A.Drozd), theory of Bassian and quasibassian orders (A.V.Roiter, Yu.A.Drozd, V.V.Kirichenko), conditions for representation finiteness (A.V.Roiter, Yu.A.Drozd) largely determined further research in this part of algebra. The origin and development of a quite new area, the theory of matrix problems was also closely related to this seminar. It became a powerful tool for the study of structure of representation categories, as well as in lots of other questions.

In 1963 a specialized Physics and Mathematics boarding school affiliated to the Kyiv State University began its work in Kyiv. The faculty of the Department of Algebra and Mathematical Logic was an active

part in its foundation. A very important role here was played by Docent V.A.Vyshenskiy who created curriculum for this school, taught some courses, especially Algebra, managed mathematical circles and trained schoolchildren for mathematical competitions. At that time V.A.Vyshenskiy became lecturer for Ukrainian Television School on Mathematics, where he was teaching during 25 years mathematical courses by correspondence and managing mathematical television school for secondary school students.

In 1970, under political pressure of the administration of the University, L.A.Kaloujnine was forced to resign his position of the head of the Department and was appointed the position of Professor of the Department. S.T.Zavalo was appointed head of the Department, a pupil of the known algebraist A.G.Kurosh from the Moscow algebraic school. S.T.Zavalo also became the Dean of the Faculty of Mechanics and Mathematics where he was working till 1980.

In the early 70's of the last century, V.I.Sushchanskiy, V.V.Sergeichuk and O.G.Ganyushkin joined the faculty of the Department. Research in the theory of permutation groups, universal algebras, representation theory and structure ring theory continued at that time. L.A.Kaloujnine and V.I.Sushchanskiy studied wreath products of groups, they also began to study systematically some operations on permutation groups. V.I.Sushchanskiy applied wreath products of infinite sequences of permutation groups to construct some new examples of Burnside type groups, i.e., infinite periodic finitely generated groups. The constructions which he proposed were the most known group theoretic results obtained by the specialists of the Department at that time. S.T.Zavalo studied admissible groups with operators.

At that time L.A.Kaloujnine together with his pupils V.O.Ustimenko and M.X.Klin developed the method of invariant relations in the theory of permutation groups. Using this method, L.A.Kaloujnine and M.X.Klin constructed the first infinite series of maximal primitive subgroups of symmetric groups, V.O.Ustimenko investigated maximality of the exponentiations of symmetric groups. He also obtained important results on the Tits–Kantor conjecture about maximality of linear groups over finite fields. A technique of calculation in V -rings was developed and applied to solve various combinatorial problems: how to construct strongly regular graphs, symmetric block diagrams, metrical relations diagrams and how to classify Boolean functions (V.O.Ustimenko, M.X.Klin, V.I.Sushchanskiy, O.G.Ganyushkin).

Under influence of theory of integer-valued representations, a new branch of Algebra — Theory of Matrix Problems — started its development. A number of important results were obtained here by the members of the Department. In particular, Yu.A.Drozdz developed a method of matrix

reduction applicable to a wide class of problems. It allowed him to prove the extended conjecture of Brauer–Thrall. One of the fundamental results of representation theory of associative algebras is the famous Drozd’s theorem on dichotomy “tameness-wildness” for free boxes and finite dimensional associative algebras. He gave also (together with V.M.Bondarenko) a criterion of representation tameness for group algebras of finite groups. Yu.A.Drozd obtained also some applications of matrix technique to multiplicative theory of ideals and to several parts of ring theory.

V.V.Sergeichuk developed the technique of involutive matrix problems, which allowed him to classify systems of linear operators and bilinear forms. S.A.Ovsienko obtained a series of deep results about relations between matrix problems and integer-valued quadratic forms. The technique of matrix problems was applied to classification of finite p -groups and inverse semigroups (V.V.Sergeichuk and V.V.Plakhotnik). A natural continuation of some works on orders was the research of V.V.Kirichenko (together with his pupils O.G.Zavadsky and N.M.Gubareni) in structural ring theory. V.V.Kirichenko introduced the notion of a quiver for several classes of rings. Using the notion of the quiver of a semiperfect ring he obtained structure theorems for right Noetherian semichain rings.

In 1980, Yu.A.Drozd became the head of the Department of Algebra and Mathematical Logic. In the eighties, the faculty of the Department changed considerably: V.O.Ustimenko and S.A.Ovsienko, who worked earlier in research laboratories of the University, were promoted to Docents. V.V.Bavula and V.M.Futorny joined the faculty as Assistant Professors. V.V.Kirichenko was appointed the head of Department of Geometry of Kyiv University.

Topics of the research were changing at that time. Categorical, geometrical and combinatorial methods were applied to study Representation Theory, transformation groups and Ring Theory. Yu.A.Drozd with his pupils and S.A.Ovsienko started a sequence of papers, where representations of Lie algebras and Lie groups were studied. They investigated new classes of representations: weight modules, Gelfand-Zetlin modules, generic representations of some “mixed” linear groups, etc. In particular, some results of V.M.Futorny about generalized Verma modules over classical simple and affine Lie algebras became classical since then. V.V.Kirichenko obtained new important results about structure of generalized serial and quasifrobenius rings. V.I.Sushchanskiy solved a series of problems in theory of factorized groups using some original combinatorial and group-theoretic constructions. V.O.Ustimenko pointed out new methods how to build extremal incidence diagrams and geometries. V.V.Bavula founded theory of generalized Weyl algebras and developed new methods for studying simple infinite dimensional algebras.

In the nineties, close scientific relations were established between the faculty of the Department and several scientific groups from other countries. Researchers of the Department gave talks on scientific seminars, took part in joint research projects, obtained many grants for joint research. This contributed to further expansion of the research program at the Department of Algebra and Mathematical Logic. Such branches of Algebra as Algebraic Geometry, theory of singularities and vector bundles, Geometric Group Theory, Algebraic Topology and theory of transformation semigroups became the objects of research of the members of the Department.

Let us describe shortly the results obtained at the Department during the period of the last fifteen years.

Yu.A.Drozd developed (in collaboration with German mathematicians) the theory of Cohen–Macaulay modules over curve and surface singularities, studied vector bundles over projective curves, obtained classification of stable homotopy types of polyhedra in some new cases, described quadratic and some classes of cubic modules. Jointly with S.A.Ovsienko he proved that a locally finite dimensional matrix problem has the same representation type as its factor by the free action of a torsion-free group.

V.I.Sushchanskiy together with his pupils obtained a series of important results about automorphism groups of trees, diagonal inductive limits of symmetric groups, groups of infinite unitriangular matrices over finite fields. He is one of the founders of the theory of groups and semigroups of automata mappings, which is widely applied in Algebra, Ergodic Theory, Holomorphic Dynamics, Spectral Theory and Differential Geometry.

V.V.Kirichenko founded a new direction in ring theory connected with applications of geometric methods in structure theory of rings and modules. It allowed him to describe various important classes of rings, especially such as semichain, hereditary, semidistributive. Using these methods he solved the known problem of Skorniyakov about structure of rings over which all finitely generated modules are semichain. Further development of these methods made clear relations between different mathematical objects such as partially ordered sets, nonnegative doubly stochastic and Gorenstein matrices.

O.G.Ganyushkin together with pupils studied the structure of nilpotent subsemigroups in the classical transformation semigroups. In a series of joint papers with V.S.Mazorchuk he introduced and investigated structure of factor-powers of transformation semigroups and obtained essentially new results in combinatorial semigroup theory.

S.A.Ovsienko applied the technique of the A_∞ -categories to representation theory. A wide generalization of the notion of quasihereditary algebras and Ringel duality was given using these methods. He also studied

the category of Gelfand–Zetlin modules over general linear Lie algebras and proved locally finiteness of blocks of this category. V.I.Bekker obtained a list of faithful categories of Schurian vector spaces, built the Auslander–Reiten quivers for wide classes of Schurian vector space categories, obtained together with V.V.Bavula a tameness criterion for weight and generalized weight modules over a wide class of generalized Weyl algebras.

O.O.Bezushchak obtained (together with V.I.Sushchanskiy) a characterization of isometry groups of generalized metrics of Baer type in terms of generalized wreath products. She also described the normal structure of isometry groups of Baer metrics and conjugacy classes in these groups. The growth of the semigroup of partial automorphisms of the integer line was characterized.

V.M.Futorny constructed quantum analogues of generalized Verma modules for some parabolic subalgebras in simple Lie algebras and in affine Lie algebras. He also constructed generalized Verma modules over toroidal Lie algebras.

V.S.Mazorchuk obtained an embedding criterion for generalized Verma modules over semisimple Lie algebras, described supports of simple modules over generalized Virasoro algebras described the structure of parabolic analogues of the well-known category \mathcal{O} , gave an abstract definition of the Enright transformation and applied it to study of this category.

V.V.Nekrashevich developed the theory of non-amenability for uniformly bounded metric spaces, established the shift equivalency of nets and subgroups of finite index in finitely generated non-amenable groups and bi-Lipschitz equivalency of quasi-isometric non-amenable groups. He also investigated the dynamics of automorphism actions of rooted tree on its boundary and described conjugacy classes of the full automorphism groups of trees (together with his teacher V.I.Sushchanskiy).

A.S.Oliynyk proved that most (in the sense of Baer category) finitely generated subsemigroups of the semigroup of automata transformations are free, discovered series of free subsemigroups. He also constructed examples of free groups of finite automaton permutations and (together with V.I.Sushchanskiy) infinite unitriangular matrices, found out an embedding of free products of finite groups into groups of finite state automorphisms of rooted trees.

N.S.Golovashchuk classified nonsingular definite weakly positive integer quadratic forms and (using the technique of coverings) obtained a finiteness criterion for some classes of matrix problems. M.V.Zeldich obtained some quantitative characteristics of nonsingular definite weakly positive integer quadratic forms and solved the problem of A.V.Roiter about ρ -exact partially ordered sets. G.M.Kudryavtseva investigated trans-

formation semigroups, established an isomorphism criterion for nilpotent subsemigroups of the semigroup of partial automorphisms of a vector space.

Ya.V.Lavrenyuk characterized automorphisms of several classes of automorphism groups of rooted trees, classified diagonal inductive limits of symmetric groups. S.V.Popovich studied representations of $*$ -groups and obtained (together with Yu.S.Samoilenko and S.A.Kruglyak) a solution of the spectral problem in case of Dynkin diagrams.

In 1993, a scientific council was established at the Faculty of Mechanics and Mathematics for defending doctoral dissertations in Algebra, Mathematical Logic, Number Theory, Discrete Mathematics and Theory of Algorithms. In 1995, V.M.Futorny defended his doctoral dissertation (he is now Professor of the University of São Paulo, Brazil), shortly afterwards another Assistant Professor of the Department, V.V.Bavula, defended doctoral dissertation (he is now Professor of the University of Sheffield, Great Britain). In the late nineties Docents V.V.Sergeichuk and V.O.Ustimenko, who also defended doctoral dissertations at this council, changed the university, (V.V.Sergeichuk is now a leading scientific researcher of the Institute of Mathematics of NAS of Ukraine, V.O.Ustimenko was appointed to Professor of the University of Maria Curie Skłodowska, Lublin, Poland).

In 1998, V.I.Sushchanskyi became the head of the Department of Algebra and Mathematical Logic. The faculty changed in the late nineties: new Assistant Professors O.O.Bezushchak, G.M.Kudryavtseva and A.S.Oliynyk joined the faculty, V.S.Mazorchuk defended his doctoral dissertation and left the Department (he is now Professor of the University of Uppsala, Sweden). Victor Bekkert went abroad, he is now Docent of the University Belo Horizonte, Brazil.

Starting from 2004 till now, A.P.Petyravchuk, who was working earlier at the Department of Geometry, is the head of the Department. In 2006, Assistant Professor V.V.Nekrashevich, a pupil of V.I.Sushchanskyi, defended his doctoral dissertation and moved to the USA, where he works at Texas A&M University. The same year S.A.Ovsienko also defended doctoral dissertation, he is now Professor of the Department of Algebra and Mathematical Logic. A research group is functioning at the Department, which consist of Senior Scientific Researchers Ya.V.Lavrenyuk, N.S.Golovaschchuk and M.V.Zeldich. Last year Ya.V.Lavrenyuk defended doctoral dissertation devoted to the geometric group theory.

The faculty of the Department now consists of nine members, and has not changed during the last years, although the list of courses taught by the Department is constantly getting wider. The faculty of the Department teaches the following general courses: “Linear Algebra”, “Discrete

Mathematics”, “Algebra”, “Number theory”, “Mathematical Logic”, “Applied Algebra”, “History of Mathematics”. Besides, the following courses are taught for senior students: “Algebraic Geometry and Its Applications in Cryptology”, “Combinatorial Analysis”, “Fundamentals of Information Security” and many special courses in different branches of Algebra and related disciplines — from Representation Theory, Group and Semigroup Theory to C^* -algebras and Topological Dynamics.

Every year from 4 to 8 leading specialists of the Institute of Mathematics, University Kyiv–Mohila Academy and other scientific organizations give special courses for students studying Algebra and Mathematical Logic. For many years Yu.S.Samoilenko and V.M.Bondarenko, Yu.V.Bodnarchuk, Ya.P.Sysak, V.V.Lybashenko, S.F.Kolyada, V.V.Sergeichuk teach here. Yu.A.Drozd, who became in 2006 the head of the Department of Algebra of the Institute of Mathematics of NAS of Ukraine, also continues giving lectures at the University.

The main scientific directions of the Department are the following: Representation Theory (S.A.Ovsienko, N.S.Golovashchuk, O.Yu.Drozd-Korolyova, together with Yu.A.Drozd and V.M.Bondarenko), Geometric Group Theory, Groups and Semigroups of automata transformations (A.S.Oliynyk, Ya.V.Lavrenyuk and E.V.Bondarenko), Group and Semigroup theory (O.G.Ganyushkin, O.O.Bezushchak, E.A.Kochubinska), Theory of Lie Algebras and Associative Algebras (A.P.Petravchuk, S.A.Ovsienko).

Note that research in Geometric Group Theory in Kyiv University was founded in the middle 90’s of the last century when R.I.Grigorchuk, a leading specialist in this area of algebra, gave a course for students and researches of the Faculty. A tradition arose then to organize every year in December an extended meeting “At the End of the Year” of the Algebraic Seminar of Kyiv University. Many former members of the Department and their colleagues from other countries come here to give their talks in various branches of Algebra.

Fifty years is a long time for any organization, especially for such a small one as a Department of a University. During all years the heads of the Department were maintaining traditions that were established many years ago and at the same time updated the courses taught at the Department and broadened research directions. This may be the reason of such vitality of the Department. Though its faculty has changed considerably during the last ten years, its traditions are kept and multiplied, and the Department has developed its own style.