

Exciton condensation in quantum wells. Exciton hydrodynamics. The effect of localized states

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The hydrodynamic equations for indirect excitons in the double quantum wells are studied taking into account 1) a possibility of an exciton condensed phase formation, 2) the presence of pumping, 3) finite value of the exciton lifetime, 4) exciton scattering by defects. The threshold pumping emergence of the periodical exciton density distribution is found. The role of localized and free exciton states is analyzed in the formation of emission spectra.

Key words: *self-organization, quantum wells, excitons, phase transition*

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1. Introduction

Phase transitions in systems of unstable particles are specific examples of non-equilibrium phase transitions and processes of self-organization [1]. In such a system, particles are created by external sources and disappear due to different reasons. If there is an attractive interaction between the particles, they may create a condensed phase during their lifetime. A steady state may arise in a system if the number of the created particles in the unit time is equal to the number of the disappeared particles. This state is stationary, but it is not equilibrium. The following examples of such systems with unstable particles may be presented: 1) dielectric exciton liquid in crystals; 2) electron-hole liquid in semiconductors; 3) highly excited gas with many excited molecules; 4) vacancies and interstitials in a crystal created by nuclear irradiation; 5) quark gluon plasma and others. The finite value of the particles determine some peculiarities of phase transitions in such systems. The main peculiarities are as follows: a) a phase transition in a system of unstable particles may occur if the lifetime is larger than some critical value; b) in the presence of parameters at which two phases coexist, the sizes of the regions of condensed phases of unstable particles are restricted; c) there is strong spatial correlation between different regions of condensed phases, that is why periodical structures may arise.

The present paper is devoted to an investigation of self-organization processes of the exciton system in semiconductor quantum wells. The appearance of periodical dissipative structures in exciton systems at the irradiations greater than some critical value was shown in the work [2, 3]. Experimental observation of a periodical distribution of the exciton density was obtained in [4, 5] in a system of indirect excitons in semiconductor double quantum wells. An indirect exciton consists of an electron and hole separated over two wells by an electric field. Due to the damping of the electron-hole recombination, indirect excitons have a large lifetime which makes it possible to create a high concentration of excitons at small pumping and to study the manifestation of the effects of exciton-exciton interaction. The authors of the paper [4] observed the emission from a double quantum well on the basis of AlGaAs system in the form of periodically situated islands along the ring around the laser spot. In the paper [5], in which the excitation of a quantum well was carried out through a window in a metallic electrode, a periodical structure of the islands situated along the ring under the perimeter of the window was found in the luminescence spectrum. The islands were observed at a frequency that corresponds to the narrow line arising at a threshold value of pumping [6]. Afterwards, spatial structures of exciton density distributions

were observed in a single wide quantum well [7], in different types of electrodes that create a periodical potential [8] or have windows in the shape of a rectangle, two circles and others [9, 10].

The phenomenon of a symmetry loss and the creation of structures in the emission spectra of indirect excitons urged a series of theoretical investigations [11–18]. The main efforts were directed towards the verification of a fundamental possibility of the appearance of the periodicity of the exciton density distribution. A specific explanation of the experiment is presented in two works [11, 16] with respect to only one experiment [4]. The authors of the work [11] considered the instability that arises under the kinetics of level occupations by the particles with the Bose-Einstein statistics. Namely, the growth of the occupation of the level with zero moment should stimulate the transitions of excitons to this level. However, the density of excitons was found greater, and the temperature was found lower than these values observed in the experiments. In the paper [16] the authors did not take into account the screening between the charges at macroscopic distances.

Our model is based on the appearance of self-organization processes in non-equilibrium systems for excitons with attractive interaction shown in [2, 3]. Investigations performed in this model [19–25] gave the explanations of spatial structures and their temperature and pumping dependencies obtained in different experiments [4, 5, 8–10]. Theoretical approaches of the works [19–25] are based on the following assumptions.

1. There is an exciton condensed phase caused by the attractive interaction between excitons. The existence of attractive interaction between excitons is confirmed by the calculations of biexcitons [26–29], and by investigations of a many-exciton system [30]. Nevertheless, there is an experimental work [31], where the authors explain their experimental results by the existence of a repulsion interaction between excitons. These results come into conflict with our suggestion regarding the attractive interaction. We shall remove this contradiction in section 3.
2. The finite value of the exciton lifetime plays an important role in the formation of a spatial distribution of exciton condensed phases. As usual, the exciton lifetime significantly exceeds the duration of the establishment of a local equilibrium. However, it is necessary to take into account the finiteness of the exciton lifetime in the study of spatial distribution phases in two-phase systems, because the exciton lifetime is less than the time of the establishment of equilibrium between phases. The latter is determined by slow diffusion processes and is long.

Two approaches of the theory of phase transitions were used while developing the theory: the model of nucleation (Lifshits-Slyozov) and the model of spinodal decomposition (Cahn-Hillart). These models were generalized to the particles with the finite lifetime, which is important for interpretation of the experimental results. The involvement of Bose-Einstein condensation for excitons was not required in order to explain the experiments.

In the present paper, the hydrodynamic equation for excitons is investigated for the case of excitons being in a condensed phase. The appearance of an instability of the uniform distribution of the exciton density and the development of nonhomogeneous structures are studied. The effect of defects on spectral positions of the emission spectra of both gas and condensed phases is analysed as well.

2. Analysis of hydrodynamic equations of exciton condensed phase

Hydrodynamic equations of excitons were obtained and analysed in the work [32]. Hydrodynamic equations of excitons generalizing the Navier-Stokes equations that take into account the finite exciton lifetime, the pumping of exciton, the existence of an exciton condensed phase and the presence of defects were developed in [33]. In the paper, we make some analysis of these equations.

The system is described by the exciton density $n \equiv n(\vec{r}, t)$ and by the velocity of the exciton liquid $\vec{u} \equiv \vec{u}(\vec{r}, t)$. The equations for conservation of the exciton density and for the movement of exciton density

are basic for the exciton hydrodynamic equations.

$$\frac{\partial n}{\partial t} + \text{div}(n\vec{u}) = G - \frac{n}{\tau_{\text{ex}}}, \quad (2.1)$$

$$\frac{\partial mnu_i}{\partial t} = -\frac{\partial \Pi_{ik}}{\partial x_k} - \frac{mnu_i}{\tau_{\text{sc}}}, \quad (2.2)$$

where G is the pumping (the number of excitons created for unit time in unit area of the quantum well), τ_{ex} is the exciton lifetime, m is the exciton mass, Π_{ik} is the tensor of density of the exciton flux

$$\Pi_{ik} = P_{ik} + mnu_i u_k - \sigma'_{ik}, \quad (2.3)$$

where P_{ik} is the pressure tensor, σ'_{ik} is the viscosity tensor of tension. In the equation (2.2), we neglected the small momentum change caused by the creation and the annihilation of excitons.

Introducing coefficients of viscosity and using (2.1), equation (2.2) may be rewritten in the form

$$\rho \left[\frac{\partial u_i}{\partial t} + \left(u_k \frac{\partial}{\partial x_k} \right) u_i \right] = -\frac{\partial P_{ik}}{\partial x_k} + \eta \Delta u_i + (\zeta + \eta/3) \left(\frac{\partial}{\partial x_i} \right) \text{div} \vec{u} - \frac{\rho u_i}{\tau_{\text{sc}}}, \quad (2.4)$$

where $\rho = mn$ is the mass of excitons in the unit volume.

We assume that the state of the local equilibrium is realized and the state of the system may be described by free energy, which depends on a spatial coordinate. Let us present the functional of the free energy in the form

$$F = \int d\vec{r} \left[\frac{K}{2} (\vec{\nabla} n)^2 + f(n) \right]. \quad (2.5)$$

At the given presentation of free energy, the pressure tensor is determined by the formula [34]

$$P_{\alpha\beta} = \left[p - \frac{K}{2} (\vec{\nabla} n)^2 - Kn \Delta n \right] \delta_{\alpha\beta} + K \frac{\partial n}{\partial x_\alpha} \frac{\partial n}{\partial x_\beta}, \quad (2.6)$$

where $p = nf'(n) - f(n)$ is the equation of the state, p is the isotropic pressure.

Taking into account (2.6) and introducing coefficients of viscosity, we finally rewrite the equation (2.2) in the form

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} + \frac{1}{m} \frac{\partial}{\partial x_i} \left(-K \Delta n + \frac{\partial f}{\partial n} \right) + \nu \Delta u_i + (\zeta/m + \nu/3) \left(\frac{\partial}{\partial x_i} \right) \text{div} \vec{u} + \frac{u_i}{\tau_{\text{sc}}} = 0. \quad (2.7)$$

Equations (2.1), (2.7) are the equations of the hydrodynamics for an exciton system. It follows from the estimations, made in the work [32], that the terms with the viscosity coefficients are small and we shall neglect them.

In the case of a steady state irradiation, the equations (2.1) and (2.7) have the solution $n = G\tau$, $u = 0$. To study the stability of the uniform solution we consider, that the behavior of a small fluctuation of the exciton density and the velocity from these values are as follows: $n \rightarrow n + \delta n \exp[i\vec{k} \cdot \vec{r} + \lambda(\vec{k})t]$, $u = \delta u \exp[i\vec{k} \cdot \vec{r} + \lambda(\vec{k})t]$. Having substituted these expressions in equations (2.1), (2.7), we obtain, in the linear approximation with respect to fluctuations, the following expression

$$\lambda_{\pm}(\vec{k}) = \frac{1}{2} \left[-\left(\frac{1}{\tau_{\text{sc}}} + \frac{1}{\tau_{\text{ex}}} \right) \pm \sqrt{\left(\frac{1}{\tau_{\text{sc}}} - \frac{1}{\tau_{\text{ex}}} \right)^2 - \frac{4k^2 n}{m} \left(k^2 K + \frac{\partial^2 f}{\partial n^2} \right)} \right]. \quad (2.8)$$

It follows from (2.8) that both parameters $\lambda_{\pm}(\vec{k})$ have a negative real part at small and large values of vector \vec{k} and, therefore, the uniform solution of the hydrodynamic equation is stable. The instability with respect to a formation of nonhomogeneous structures arises at some threshold value of exciton density and at some critical value of the wave vector, when $\partial^2 f / \partial n^2$ becomes negative. The analysis of the equation (2.8) gives the following expression for the critical values of the wave vector k_c and the exciton density n_c

$$k_c^4 = \frac{m}{Kn_c \tau_{\text{sc}} \tau_{\text{ex}}}, \quad (2.9)$$

$$\frac{k_c^2 n_c}{m} \left(k_c^2 K + \frac{\partial^2 f(n_c)}{\partial n_c^2} \right) + \frac{1}{\tau_{\text{sc}} \tau_{\text{ex}}} = 0. \quad (2.10)$$

For stable particles ($\tau_{\text{ex}} \rightarrow \infty$), the equations (2.9), (2.10) give the condition $\partial^2 f / \partial n^2 = 0$, which is the condition for spinodal decomposition for a system in the equilibrium case.

Depending on parameters, the equations (2.1), (2.7) describe the ballistic and diffusion movement of the exciton system. The relaxation time τ_{sc} plays an important role in the formation of the exciton movement. Due to the appearance of nonhomogeneous structures, there exist exciton currents in a system ($\vec{j} = n\vec{u} \neq 0$) even under the uniform steady-state pumping. Excitons are moving from the regions having a small exciton density to the regions having a high density. In the present paper, we shall consider the spatial distribution of exciton density and exciton current in the double quantum well under steady-state pumping. In this case, the exciton current is small and we assume the existence of the following conditions

$$\frac{\partial u_i}{\partial t} \ll u_i / \tau_{\text{sc}}, \quad (2.11)$$

$$u_k \frac{\partial u_i}{\partial x_k} \ll u_i / \tau_{\text{sc}}. \quad (2.12)$$

Particularly, the equation (2.11) holds in the study of the steady-state exciton distribution. The fulfillment of equation (2.12) will be shown later following some numerical calculations.

Using the conditions (2.11) and (2.12), we obtain from equation (2.7) the value of the velocity \vec{u}

$$\vec{u} = -\frac{\tau_{\text{sc}}}{m} \vec{\nabla} \left(-K\Delta n + \frac{\partial f}{\partial n} \right). \quad (2.13)$$

As a result, the equation for the exciton density current may be presented in the form

$$\vec{j} = n\vec{u} = -M\nabla\mu, \quad (2.14)$$

where $\mu = \delta F / \delta n$ is the chemical potential of the system, $M = nD / \kappa T$ is the mobility, $D = \kappa T \tau_{\text{sc}} / m$ is the diffusion coefficient of excitons.

Therefore, the equation for the exciton density (2.1) equals

$$\frac{\partial n}{\partial t} = \frac{D}{\kappa T} (-Kn\Delta^2 n - K\vec{\nabla}n \cdot \vec{\nabla}\Delta n) + \frac{D}{\kappa T} \vec{\nabla} \cdot \left(n \frac{\partial^2 f}{\partial n^2} \vec{\nabla}n \right) + G - \frac{n}{\tau_{\text{ex}}}. \quad (2.15)$$

Just in the form of (2.15), we investigated a spatial distribution of the exciton density at exciton condensation using the spinodal decomposition approximation by choosing different dependencies f on n [21, 23, 25]. Thus, our previous consideration of the problem corresponds to the diffusion movement of hydrodynamic equations (2.1), (2.7). For the system under study, a condensed phase appears if the function $f(n)$ describes a phase transition. In the papers mentioned above, the examples of such dependencies were given. Here, we analyse another dependence $f(n)$, which is also often used in the theory of phase transitions

$$f = \kappa T n [\ln(n/n_a) - 1] + a \frac{n^2}{2} + b \frac{n^4}{4} + c \frac{n^6}{6}, \quad (2.16)$$

where a, b, c are constant values. Three last terms in the formula (2.16) are the main terms. They arise due to an exciton-exciton interaction and describe the phase transition. The first term was introduced in order to describe the system in a space, where the exciton concentration is small (this is important if such a region exists in a system). At an increase of the exciton density, the term $an^2/2$ manifests itself firstly. It contributes the an value to the chemical potential. In our system, the origin of this term is connected with the dipole-dipole interaction, which should become apparent at the beginning with the growth of the density due to its long-range nature. To estimate a for the dipole-dipole exciton interaction in double quantum well we may use the plate capacitor formula $an = 4\pi e^2 dn / \epsilon$, where d is the distance between the wells, ϵ is the dielectric constant. This formula is usually used to determine the exciton density from the experimental meaning of the blue shift of the frequency of the exciton emission with the rise of the density. It follows from the formula that $a = 4\pi e^2 d / \epsilon$. When the exciton density grows, the last two terms in (2.14) begin to play a role. The existence of a condensed phase requires that the value b should be negative ($b < 0$). For stability of a system, at large n , the parameter c should be positive

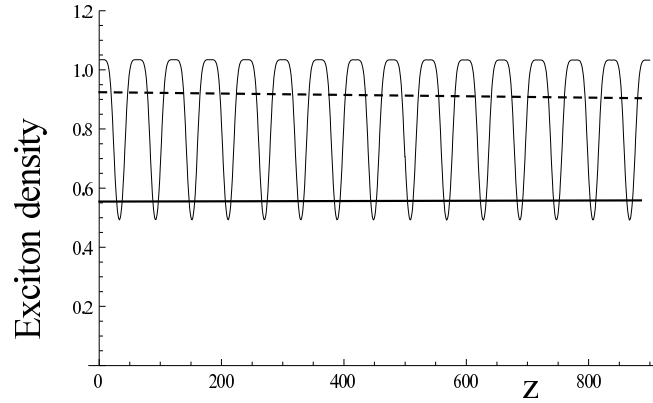


Figure 1. The spatial dependence of the exciton density at a different value of the pumping: for a continuous line $G = 0.0055$, for a periodical line $G = 0.008$, for a dashed line $G = 0.0092$. $D_1 = 0.03$, $b = -1.9$.

($c > 0$). It is assumed in the model that the condensed phase arises due to the exchange and Van der Waals interactions. The calculations show that in some region of distances between the wells, these interactions exceed the dipole-dipole repulsion.

Let us introduce dimensionless parameters: $\tilde{n} = n/n_0$, where $n_0 = (a/c)^{1/4}$, $\tilde{b} = b/(ac)^{1/2}$, $\tilde{\tau} = \tilde{\tau}/\xi$, where $\xi = (K/a)^{1/2}$ is the coherence length, $\tilde{t} = t/t_0$, where $t_0 = \kappa TK/(Dn_0a^2)$, $D_1 = \kappa T/(an_0)$, $\tilde{G} = Gt_0/n_0$, $\tilde{\tau}_{ex} = \tau/t_0$. As a result, the equation (2.15) is reduced to the form (hereinafter the symbol \sim will be omitted in the equation)

$$\frac{\partial n}{\partial t} = D_1 \Delta n - n \Delta^2 n + n \Delta n (1 + 3bn^2 + 5n^4) - \tilde{\nabla} n \cdot \tilde{\nabla} \Delta n + (\tilde{\nabla} n)^2 (1 + 9bn^2 + 25n^4) + G - \frac{n}{\tau_{ex}}. \quad (2.17)$$

The solutions of the equation (2.17) are presented in figure 1 for the one-dimensional case [$n(\vec{r}, t) \equiv n(z, t)$] for three values of the steady-state uniform pumping.

The solutions are obtained at the initial conditions $n(z, 0) = 0$ and the boundary conditions $n'(0, t) = n'(L, t) = n''(0, t) = n''(L, t) = 0$, where L is the size of a system. The periodical solution exists in some interval of the pumping $G_{c1} < G < G_{c2}$. At specified parameters, the periodical solution exists at $0.0055 < G < 0.0092$. Outside this region, the solution describes a uniform system: the gas phase at a low pumping and the condensed phase at a large pumping. The upper part of the periodical distribution corresponds to a condensed phase, the lower part corresponds to the gas phase. The size of the condensed phase increases with the change of the pumping from G_{c1} to G_{c2} . Figure 2 shows the spatial dependence of the exciton current calculated by the formula (2.14). The current equals zero in the centers of the condensed and gas phases and it has a maximum in the region of a transition from the condensed phase to the gas phase. Let us do some estimations. The results for the currents in figure 2 are presented in dimensionless units: $\tilde{j} = j/j_0$, where $j_0 = n_0 u_0$, $u_0 = (\tau_{sc} n_0 a)/(m\xi)$ is the unit of the velocity. The exciton density is presented in figure 1 in dimensionless units ($\tilde{n} = n/n_0$). It is seen in figure 1 that $\tilde{n} \sim 1$, and the magnitude of n is of an order of n_0 . Thus, for estimations we may assume that $n_0 a$ corresponds to the shift of the luminescence line with an increase of the exciton density, the magnitude of ξ is of the order of the size of the condensed phase. For the following magnitudes of parameters $\tau_{sc} = 10^{-11}$ s, $n_0 a = 2 \cdot 10^{-3}$ eV, $m = 2 \cdot 10^{-28}$ g, $\xi = 2 \cdot 10^{-4}$ cm, we obtain $u_0 \sim 10^6$ cm/s. According to calculations (see figure 2), the magnitudes of the current and the velocity are two orders of magnitude less than their units j_0 and u_0 , so the condition $u \sim 10^4$ cm/s takes place. In order to verify the fulfilment of the condition (2.12), let us suppose that $(\partial u_i)/(\partial x_k \sim u/l)$, where l is the period of a structure. It follows from experiments [4, 5] that $l \sim (5 \div 10)$ μm . Using these data we see that the condition (2.12) is very well satisfied. This condition is violated at $\tau_{sc} \geq 10^{-9}$ s. Therefore, the formation of nonuniform exciton dissipative structures in a double quantum well occurs due to the diffusion movement of excitons. To prove the main hydrodynamic equation (2.15), the last term in equation (2.2) is of importance. It describes the loss of the momentum

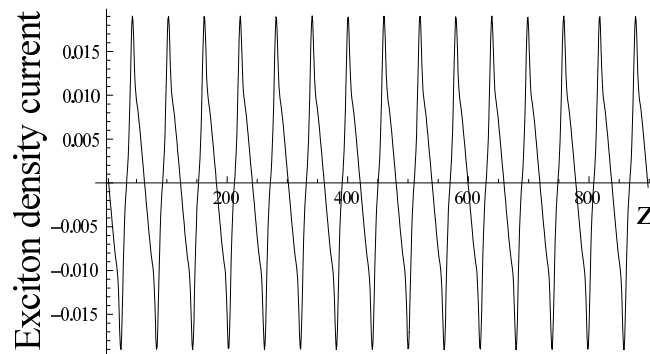


Figure 2. The spatial dependence of the exciton current at $G = 0.008$, $D_1 = 0.03$, $b = -1.9$.

due to the scattering of excitons by defects and phonons. It is this term that describes the processes that cause a decay of the exciton flux. From the viewpoint of a possibility of the appearance of superfluidity, the situation for excitons is more complicated than that for the liquid helium and for the atoms of alkali metals at ultralow temperatures. In the latter systems, the phonons (movement of particles) are an intrinsic compound part of the system spectrum, the interaction between phonons (particles) is the interaction between the atoms of a system and does not cause the change of the complete momentum of a system and its movement as a whole. Phonons and defects for excitons are external subsystems that brake the exciton movement. Therefore, to create the exciton superfluidity, it is needed that the value of τ_{sc} should grow significantly. This is possible for exciton polaritons that weakly interact with phonons; moreover, there is a certain experimental evidence on an observation of the polariton condensation [35]. For indirect excitons, the critical temperature of a superfluid transition is strongly lowered by inhomogeneities [36, 37]. Thus, the question regarding the possibility of the superfluidity existence for indirect excitons on the basis of AlGaAs system is open.

Thus, the peculiarities observed at large densities of indirect excitons may be explained by phase transitions in a system of particles having attractive interactions and by the finite value of the lifetime without an involvement of the Bose-Einstein condensation.

3. Distribution of excitons between localized and delocalized states

According to the experimental results [31], the frequency of the emission from the islands of a condensed phase, where the exciton density is large, is higher than the frequency of emission from the region between islands, in which the density is less. The authors made the conclusion [31] that the interaction between excitons is repulsive, and, therefore, the formation of a condensed phase by attractive interaction is impossible. This contradicts the main assumption of our works [19–25], though these works explain many experiments. Now, we remove this contradiction taking into account the presence of localized excitons.

The localized states arise due to the presence of residual donors, acceptors, defects, and inhomogeneous thickness of the wells. Their existence is confirmed by the presence of an emission at the frequencies less than the frequency of the exciton band emission and by broadening of exciton lines. At a low temperature and at a small pumping, the main part of the emission band consists of the emission from defect centers, while the part of the exciton emission grows with an increased pumping. Now, we consider the relation between the contribution to the emission band intensity from free excitons and from the excitons (pairs of electrons and holes) localized on the defects. We assume that the localized states are saturable, namely, every center may capture a restricted number of electron-hole pairs. In our calculations we assume that only a single excitation may be localized on a defect. There are no other localized excitations or they have a very low binding energy and are unstable. The dependence of the density of localized states on the energy was chosen in the exponential form, namely $\rho(E) = \alpha N_l \exp(\alpha E)$, where N_l is the density of the defect centers, E is the depth of the trap level. The exciton states (free and localized) are

distributed onto levels after the creation of electrons and holes due to an external irradiation and their subsequent recombination and relaxation. Since the time of relaxation is much less than the exciton lifetime, the distribution of excitation between free and localized states corresponds to the thermodynamical equilibrium state. In the considered model, we should obtain a distribution of electron-hole pairs, whose population on a single level may be changed from zero to infinity for $E > 0$ (for free exciton states) and from zero to one for $E < 0$ (for localized states). Formally, in the considered system, free excitons have Bose-Einstein statistics while localized excitations obey the Fermi-Dirac statistics. At a small exciton density, Bose-Einstein and Boltzmann statistics give similar results for free excitons, but the application of Fermi-Dirac statistics for localized states on a single level for one trap is important. The equation for energy distribution may be found from the minima of a large canonical distribution

$$w(n_k, n_i) = \exp\left(\frac{\Omega + N\mu - E}{\kappa T}\right), \quad (3.1)$$

where $N = \sum_i n_i + \sum_k n_k$, $E = \sum_i n_i E_i + \sum_{k,l} n_k E_k$, $n_i = 0, 1$, $n_k = 0, 1, \dots, \infty$, k is the wave vector of the exciton, l designates the singular levels. Parameter μ is the exciton chemical potential.

The distribution of excitons over free and localized levels is determined from the minimum of the functional (3.1). As a result, we obtain the following conditions for the mean values of the free exciton density n and the density of localized states n_L

$$n_{\text{ex}} = \frac{g\nu}{4\pi E_{\text{ex}} a_{\text{ex}}^2} \int_0^{\infty} \frac{dE}{\exp\left(\frac{E-\mu}{\kappa T}\right) - 1}, \quad (3.2)$$

$$n_L = \alpha N_l \int_{-\infty}^0 \frac{\exp(\alpha E) dE}{\exp\left(\frac{E-\mu}{\kappa T}\right) + 1}, \quad (3.3)$$

where $a_{\text{ex}} = (\hbar^2 \varepsilon) / (\mu_{\text{ex}} e^2)$ and $E_{\text{ex}} = (\mu_{\text{ex}} e^4) / (2\varepsilon^2 \hbar^2)$ are the radius and the energy of the exciton in the ground state in the bulk material, $g = 4$, μ_{ex} is the reduced mass of the exciton, ν is the ratio of the reduced and the total mass of the exciton. The chemical potential μ is determined from the condition

$$n_L + n = G\tau_{\text{ex}}, \quad (3.4)$$

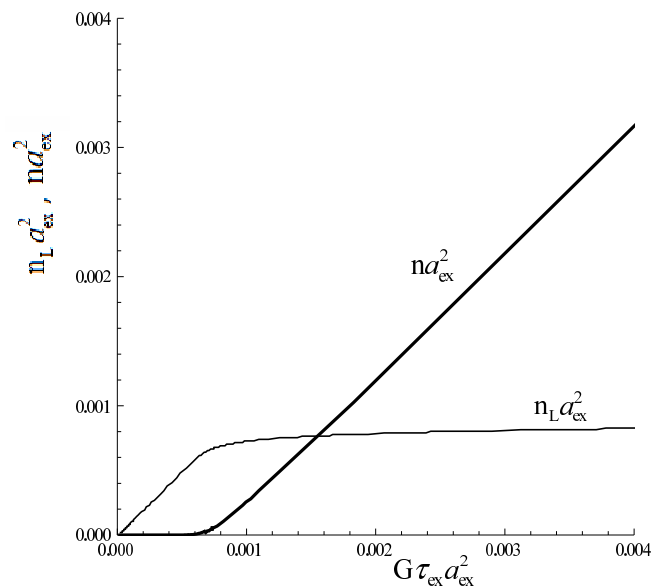


Figure 3. The dependence of the density of free (thick line) and trapped (thin line) excitons on the pumping. The parameters of the system: $T = 2K$, $N_l = 0.001/a_{\text{ex}}^2$, $\alpha = 300(\text{eV})^{-1}$.

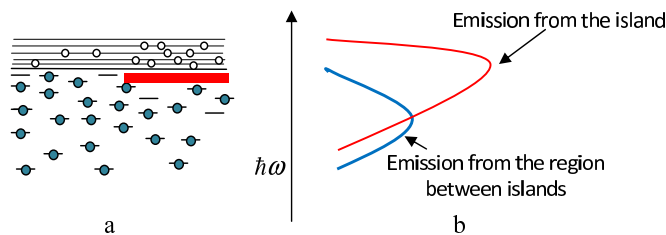


Figure 4. (Color online) The distribution of excitations in the traps and in the states of the exciton band. The thick line in figure 4 (a) corresponds to the energy per a single exciton in the condensed phase. On the right [figure 4 (b)], the upper line describes the whole emission from the island (the emission of both the condensed phase and trapped excitons), the low line describes the emission of the trapped excitons.

where $G\tau_{\text{ex}}$ is the whole number of excitation (free and localized) per unit surface.

The dependence of distribution of free and localized excitons on the pumping is presented in figure 3 as a function of the whole number of excitation presented in units of $1/a_{\text{ex}}^2$. Let the exciton radius be equal to 10 nm. Then, the concentration of the traps and the width of the distribution of trap levels, chosen under calculations of figure 3, are of the order of 10^9 cm^{-2} and 0.003 eV, correspondingly.

As it is seen in figure 3, the number of localized excitations at small pumping exceeds the number of free excitons and the emission band is determined by the emission from the traps. With an increase of pumping, the occupation of the trap levels becomes saturated. For the chosen parameters, the concentration of excitations under saturation is of the order of 10^9 cm^{-2} . The exciton density grows simultaneously with the saturation of the localized levels. As a result, the shortwave part of the emission band should increase with an increased pumping. When the exciton density becomes larger, the collective exciton effects begin to manifest themselves. The equations (3.2), (3.3) do not take into account the interactions between excitations, and special models and theories are needed to describe the collective effects. The appearance of a narrow line was observed in [6] with an increased pumping on the shortwave part of the exciton emission band. Simultaneously, patterns arise in the emission spectra. The narrow line appeared after the localized states become occupied. According to [6], this line is explained by the exciton Bose-Einstein condensation. According to our model [19, 22], the appearance of the islands corresponds to the condensed phase caused by the attractive interaction between excitons. The energy per a single exciton in the condensed phase is less than the energy of free excitons (the thick line in figure 4), but the gain of the energy under condensation of indirect excitons in AlGaAs system is less than the whole bandwidth, which are formed by the localized and delocalized states. Thus, the energy of photons emitted from the islands of a condensed phase is higher than the energy of photons emitted by traps (see figure 4). The excitons cannot leave the condensed phase (the islands) and move to the traps (to the states of lower energy) since the levels of the traps are already occupied. This may be the reason of the results obtained in [31], where the maximum of the frequency of emission from the islands is higher than the maximum frequency from the regions between the islands *in spite of the attractive interaction between the excitons*.

The qualitative results coincide with the results obtained in [33] using another method from the solution of kinetic equations for level distributions at some simple approximation for the probability transition between the levels. Similar behavior of distribution of free and trapped excitons is observed for another energy dependence of the density of localized states.

The results may be used to explain the intensity and temperature dependencies of the exciton emission of dipolar excitons in InGaAs coupled double quantum wells [38]. The authors observed a growth of the shortwave side of the band with an increased pumping.

4. Conclusion

Hydrodynamic equations are analyzed for excitons in a double quantum well. The equations take into account the presence of pumping, the finite value of the exciton lifetime and the possibility of a condensed phase formation in a phenomenological model. The equations describe the diffusion and the ballistic movement of an exciton system. It is shown that the spatial nonuniform structures, observed

experimentally in double wells on the basis of AlGaAs crystal, may be explained by hydrodynamic equations in the diffusion approximation. The effect of saturable localized states on spectral distribution of the emission from condensed and gas phases is obtained. The theory explains the features of experimental dependencies of the emission spectra from the condensed and gas phases.

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Конденсація екситонів в квантових ямах. Гідродинаміка екситонів. Вплив локалізованих станів дефектів

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Проведено аналіз рівнянь гідродинаміки екситонів у квантовій ямі. Рівняння враховують 1) можливість фазового переходу в системі, 2) присутність зовнішньої накачки, 3) скінчений час життя екситонів, 4) розсіяння екситонів на дефектах. Визначно порогову накачку утворення періодичного розподілу екситонної густини. Досліджується вплив локалізованих і вільних екситонів на формування спектрів випромінювання.

Ключові слова: *самоорганізація, квантові ями, екситони, фазовий перехід*