



Mykola Komarnytskyi was born in 1948 in the village Komarnykyi of L'viv region. In 1966 he entered the Faculty of Mechanics and Mathematics of L'viv Ivan Franko University. After graduating from this university in 1971 he was assigned to the Department of Algebra of the Institute of Physics and Mechanics in L'viv, wherefrom he was called up for military service for two years. Returning from the army, Mykola Komartyttskyi worked from 1974 until 1979 as an engineer at the Department of Algebra of the Institute of Applied Problems of Mechanics and Mathematics did got at the same time his graduate study at this Institute. At 1979 he got Candidate of Sciences (Ph.D.) degree.

In February 1979 he was selected as an assistant of the chair of higher mathematics at the L'viv University. From 1980 he worked as an associate professor of the chair of geometry, and during 1987 – 1992 he held the chair of algebra and topology. In the period of 1992 – 1998 Mykola Komarnytskyi was again an associate professor of the chair of algebra and topology. During 1993 – 1995 he was at the position of senior researcher to accomplish his habilitation. In 1998 he earned the degree of Doctor of Sciences. From 1998 he is a professor of the chair of algebra and topology, and from 2002 the head of this chair. In 1992 – 1993 and in 1996 – 1999 he was the deputy dean of the Department of Mechanics and Mathematics.

Mykola Komarnytskyi was selected the first chairman of the L'viv Society of Logicians founded in 1999. He is a member of Editorial Boards of five scientific journals published in Ukraine and a member of the Council of Experts on mathematics of the Higher Certification Commission of Ukraine. He was awarded by the Lviv regional administration prize and the rank "Excellence of the Education in Ukraine".

Mykola Komarnytskyi carries on a very considerable scientific and organizational activity: he manages the City Seminar on algebra, supervises and doctoral and candidate theses, works in organizing committees of many international scientific conferences.

During his work at the Institute of the Applied Problems of Mechanics and Mathematics and at the Department of Mechanics and Mathematics of the L'viv National University Mykola Komarnytskyi proved himself to be an experienced teacher and an active and highly skilled working researcher. He gave courses in algebra, number theory, discrete mathematics, logic, algebra and geometry, as well as various special courses and special seminars. He was advisor of several Candidate (Ph.D.) theses (Vovk R.V., 1997; Tushnickiy I.Ya., 1999, Zelisko G.V., 2002, Melnyk I., 2008). Mykola Komarnytskyi is the author of about 100 scientific publications, and is highly respected by the international scientific community.

Already being a student, Mykola Komarnytskyi showed the ability to clearly explain to others what he had learned and his love to mathematics. His students and collaborators consider him as one of the best lecturers. He is a noted scientist recognized in the world for his considerable contributions to many branches of algebra, and one of leading experts in algebra and logic in Ukraine.

Mykola Komarnytskyi has considerable scientific achievements in many areas of modern algebra: theory of rings and modules (especially, the theory of radicals and torsions and in the theory of elementary divisor rings), model theory, categorical logic. He has got very interesting results about axiomatization of important classes of rings and modules. This topic was first studied by Eklöf and Sabbah. They proved in 1971 that the axiomatizability of the class of injective modules is equivalent to the fact that the basic ring is noetherian, and that the classes of semihereditary rings and Prüfer rings are axiomatizable, while the classes of noetherian rings and principal ideals rings are not so. Mykola Komarnytskyi investigated the axiomatization of the so called *V-rings*, that is of associative rings R with unit such that all simple left and all simple right R -modules are injective. In particular, he solved the Cozzens-Faith problem on ultrapowers of principal ideal domains and described the lattice of left ideals in an ultraproduct of a family of left Bezout domains.

In addition, he found necessary and sufficient conditions for axiomati-

zability of the radical class of any radical in the category of modules over a Dedekind domain in terms of divisibility of radical modules. Among other results of Mykola Komarnytskyi in this direction, we should mention the proof of axiomatizability of the class of noncommutative Prüfer rings (in collaboration with Ivanna Melnyk). His joint results with G. Zelisko on the structure of endomorphism rings of ultrapowers of modules and his description of torsion-theoretical spectra of ultrapowers of a countable family of principal ideal V -domains play an important role in the ring theory.

In categorical logic Mykola Komarnytskyi studied several categories of ring objects in an elementary topos in order to detect the existence of their model-companions and proved that the category of geometric fields with supports in the Sierpinski topos has a model-companion, namely, the theory of algebraically closed objects which are geometric fields.

The problem of diagonalization of matrices is classical, its origin is the Gauss theorem which states that any matrix over a field is equivalent to the diagonal matrix with 1 and 0 on the diagonal. The first results on the diagonalization of matrices over integers were obtained by G. Smith in 1861. He proved that each matrix with integer coefficients can be reduced by elementary transformations to a diagonal form such that every diagonal element is a divisor of the next one (such diagonal form is often called the *Smith form*). Later the Smith theorem was generalized to various classes of rings. In particular, Dickson and van der Warden extended it to certain classes of commutative and noncommutative Euclidean rings, and Teichmüller extended the Smith theorem to noncommutative principal ideal rings.

All these results had been obtained before Irving Kaplansky introduced the notion of *elementary divisor ring*, that is such a ring that every matrix over it can be transformed, multiplying it by invertible ones (from both sides), to a diagonal matrix such that every diagonal element is a complete divisor of the next one. I. Kaplansky proved that over an elementary divisor ring every finitely presented module decomposes into a direct sum of cyclic modules. For commutative rings the inverse holds, namely, if every finitely presented module over a ring decomposes in the direct sum of cyclic modules, then the ring is an elementary divisor ring. For non-commutative rings this inverse statement is no more true, and Warfield stated the problem to find an internal characterization of rings R such that every finitely presented R -module decomposes into a direct sum of cyclic submodules.

Mykola Komartytskyi proposed a partial solution to this problem, introducing a new class of rings, called *almost invariant elementary divisor ring*. They are such rings that every matrix over it can be transformed

to a diagonal form, where all diagonal entries, except maybe the last one, are invariant elements and each of them is a left divisor of the next one. Just as in Kaplansky case, one only has to check this property for 1×2 , 2×2 and 2×1 matrices. The result on decomposability of finitely presented modules into direct sums of cyclic ones remains true for such rings too.

Also Mykola Komarnytskyi investigated the diagonal reduction of matrices over distributive Bezout domains and showed, together with B. Zabavsky, that an elementary divisor distributive Bezout domain is a duo-domain.

Most of known classes of elementary divisor rings depend essentially on the chain conditions of ideals. The first example of elementary divisor ring without stabilization of chain of ideals was discovered by Wedderburn as early as in 1915; it is the ring of analytic functions. In more abstract form this example allowed Hellmer to introduce a new class of elementary divisor rings, which are called *adequate rings*. Mykola Komarnytskyi showed that if every ideal of a commutative Bezout domain R is transfinite nilpotent, then R is adequate, so an elementary divisor ring. He also applied the obtained results to simplification of formulas of the first order theory of modules.

Mykola Komarnytskyi is also a leading specialist in the theory of differential rings. His deep ideas allowed to prove important results about existence and properties of infinite primary factorizations of ideals and modules over non-noetherian rings under certain natural restrictions. It became a starting point of a new general theory of primary factorizations of differential ideals and submodules of differential modules. Moreover, the lattices of differential torsions and radicals in the category of the differential modules over special differential rings were characterized, and it allowed to obtain same basic properties of rings of linear differential operators with coefficients in such differential rings; in particular, to prove that differential operators over differentially closed and universal differential rings have the Wedderburn property.

Mykola Komarnytskyi also has got interesting results on differential preradicals and differential pretorsions and torsions in the category of differential modules over various differential rings, and discovered many deep properties of such objects. In particular, the conditions under which differentiations can be extended from a differential module to its bicommutators have been obtained.

At present the scientific interests of Mykola Komarnytskyi concern model-theoretical characterizations of differential-injective and differential-projective modules, properties of ultraproducts of differential rings, a description of differential spectra of direct products of differentially simple

rings. He plans to investigate the differential rings with abelian ring of Li differentiations, as well as differentiations of certain near-rings, and to describe properties and the structure of some types ideally differential rings and modules.

Certainly, we could only mention here a part of scientific achievements of Mykola Komarnytskyi. He is effectively working in many areas of modern algebra, and will do still more scientific discoveries.

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