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(λ, μ)-fuzzy interior ideals of ordered Γ-semigroups

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Abstract. For all $\lambda, \mu \in [0, 1]$ such that $\lambda < \mu$, we first introduced the definitions of (λ, μ)-fuzzy ideals and (λ, μ)-fuzzy interior ideals of an ordered Γ-semigroup. Then we proved that in regular and in intra-regular ordered semigroups the (λ, μ)-fuzzy ideals and the (λ, μ)-fuzzy interior ideals coincide. Lastly, we introduced the concept of a (λ, μ)-fuzzy simple ordered Γ-semigroup and characterized the simple ordered Γ-semigroups in terms of (λ, μ)-fuzzy interior ideals.

1. Introduction and preliminaries

The formal study of semigroups began in the early 20th century. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis.

Γ-semigroups were first defined by Sen and Saha [14] as a generalization of semigroups and studied by many researchers, for example [1, 2, 5, 6, 8, 9, 3, 12, 15, 16, 17, 18].

The concept of fuzzy sets was first introduced by Zadeh [24] in 1965 and then the fuzzy sets have been used in the reconsideration of classical mathematics. Recently, Yuan [23] introduced the concept of fuzzy subfield with thresholds. A fuzzy subfield with thresholds λ and μ is also called a (λ, μ)-fuzzy subfield. Yao continued to research (λ, μ)-fuzzy normal

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subfields, (λ, μ)-fuzzy quotient subfields , (λ, μ)-fuzzy subrings and (λ, μ)-fuzzy ideals in [19, 20, 21, 22].

In this paper, we studied (λ, μ)-fuzzy ideals in ordered Γ-semigroups. This can be seen as an application of [22] and as a generalization of [7, 11, 13].

Let $S = \{x, y, z, ...\}$ and $\Gamma = \{\alpha, \beta, \gamma, ...\}$ be two non-empty sets.

An ordered Γ-semigroup $S_\Gamma = (S, \Gamma, \leq)$ is a poset $(S, \leq)$ such that there exists a mapping $S \times \Gamma \times S \to S$ (images of (a, α, b) to be denoted by $a\alpha b$), such that, for all $x, y, z \in S$, $\alpha, \beta, \gamma \in \Gamma$, we have

(1) $(x\beta y)\gamma z = x\beta(y\gamma z)$.

(2) $x \leq y \Rightarrow \begin{cases} x\alpha z \leq y\alpha z \\ z\alpha x \leq z\alpha y. \end{cases}$

Note that an ordered semigroup is a special ordered Γ-semigroup with $\Gamma = \{\circ\}$, i.e., Γ is a set with one element.

Let $(S, \circ, \leq)$ be an ordered semigroup. A nonempty subset A of S is called a left (respectively, right) ideal of S if (1) $S \circ A \subseteq A$ (respectively, $A \circ S \subseteq A$); (2) $a \in A$, $b \in S$, $b \leq a \Rightarrow b \in A$. A is called an ideal of S if it is both a left and a right ideal of S .

If $(S, \Gamma, \leq)$ is an ordered Γ-semigroup, and A is a subset of S, we denote by $(A]$ the subset of S defined as follows:

$$(A] = \{t \in S | t \leq a \text{ for some } a \in A\}.$$

Given an ordered Γ-semigroup S, a fuzzy subset of S (or a fuzzy set in S) is an arbitrary mapping $f : S \to [0, 1]$, where $[0, 1]$ is the usual closed interval of real numbers. For any $t \in [0, 1]$, f_t is defined by $f_t = \{x \in S | f(x) \geq t\}$.

For each subset A of S, the characteristic function f_A is a fuzzy subset of S defined by

$$f_A(x) = \begin{cases} 1, & \text{if} \quad x \in A \\ 0, & \text{if} \quad x \notin A. \end{cases}$$

In the following, we will use S, S_Γ or $(S, \Gamma, \leq)$ to denote an ordered Γ-semigroup.

In the rest of this paper, we will always assume that $0 \leq \lambda < \mu \leq 1$.

We will use $a \vee b$ to denote $\max\{a, b\}$ and $a \wedge b$ to stand for $\min\{a, b\}$. Note that $([0, 1], \wedge, \vee)$ is a distributive lattice.

2. (λ, μ)-fuzzy ideals and (λ, μ)-fuzzy interior ideals

In this section, we first introduce the concepts of (λ, μ)-fuzzy ideals and (λ, μ)-fuzzy interior ideals of an ordered Γ-semigroup. Then we show that every (λ, μ)-fuzzy ideal is a (λ, μ)-fuzzy interior ideal.

Definition 1. A fuzzy subset f of an ordered Γ-semigroup S is called a (λ, μ)-fuzzy right ideal of S if

(1) $f(x\alpha y) \vee \lambda \geq f(x) \wedge \mu$ for all $x, y \in S$, $\alpha \in \Gamma$ and

(2) If $x \leq y$, then $f(x) \vee \lambda \geq f(y) \wedge \mu$ for all $x, y \in S$.

A fuzzy subset f of S is called a (λ, μ)-fuzzy left ideal of S if

(1) $f(x\alpha y) \vee \lambda \geq f(y) \wedge \mu$ for all $x, y \in S$, $\alpha \in \Gamma$ and

(2) If $x \leq y$, then $f(x) \vee \lambda \geq f(y) \wedge \mu$ for all $x, y \in S$.

A fuzzy subset f of S is called a (λ, μ)-fuzzy ideal of S if it is both a (λ, μ)-fuzzy right and a (λ, μ)-fuzzy left ideal of S.

Definition 2. If $(S, \Gamma, \leq)$ is an ordered Γ-semigroup, a nonempty subset A of S is called an interior ideal of S if

(1) $S\Gamma A\Gamma S \subseteq A$ and

(2)If $a \in A, b \in S$ and $b \leq a$, then $b \in A$.

Definition 3. If $(S, \Gamma, \leq)$ is an ordered Γ-semigroup, a fuzzy subset f of S is called a (λ, μ)-fuzzy interior ideal of S if the following assertions are satisfied:

(1) $f(x\beta a\gamma y) \vee \lambda \geq f(a) \wedge \mu$ for all $x, a, y \in S$, $\beta, \gamma \in \Gamma$ and

(2) If $x \leq y$, then $f(x) \vee \lambda \geq f(y) \wedge \mu$.

Theorem 1. *Let $(S, \Gamma, \leq)$ be an ordered Γ-semigroup, Then f is a (λ, μ)-fuzzy interior ideal of S if and only if f_t is an interior ideal of S for all $t \in (\lambda, \mu]$.*

Proof. Let f be a (λ, μ)-fuzzy interior ideal of S, $\forall t \in (\lambda, \mu]$ and $\forall \beta, \gamma \in \Gamma$.

First of all, we need to show that $x\beta a\gamma y \in f_t$, for all $a \in f_t$, $x, y \in S$.

From $f(x\beta a\gamma y) \vee \lambda \geq f(a) \wedge \mu \geq t \wedge \mu = t$ and $\lambda < t$ we conclude that $f(x\beta a\gamma y) \geq t$, that is $x\beta a\gamma y \in f_t$.

Then, we need to show that $b \in f_t$ for all $a \in f_t, b \in S$ such that $b \leq a$.

From $b \leq a$ we know that $f(b) \vee \lambda \geq f(a) \wedge \mu$ and from $a \in f_t$ we have $f(a) \geq t$. Thus $f(b) \vee \lambda \geq t \wedge \mu = t$. Notice that $\lambda < t$, we conclude that $f(b) \geq t$, that is $b \in f_t$.

Conversely, let f_t be an interior ideal of S for all $t \in (\lambda, \mu]$.

If there are $x_0, a_0, y_0 \in S$, such that $f(x_0\beta a_0\gamma y_0) \vee \lambda < t = f(a_0) \wedge \mu$, then $t \in (\lambda, \mu], f(a_0) \geq t$ and $f(x_0\beta a_0\gamma y_0) < t$. That is $a_0 \in f_t$ and

$x_0\beta a_0\gamma y_0 \notin f_t$. This is a contradiction with that f_t is an interior ideal of S. Hence $f(x\beta a\gamma y)\vee\lambda \geq f(a)\wedge\mu$ holds for all $x,a,y\in S$.

If there are $x_0,y_0\in S$ such that $x_0\leq y_0$ and $f(x_0)\vee\lambda < t = f(y_0)\wedge\mu$, then $t\in(\lambda,\mu]$, $f(y_0)\geq t$ and $f(x_0)<t$, that is $y_0\in f_t$ and $x_0\notin f_t$. This is a contradiction with that f_t is an interior ideal of S. Hence if $x\leq y$, then $f(x)\vee\lambda\geq f(y)\wedge\mu$. □

Theorem 2. *Let $(S,\Gamma,\leq)$ be an ordered Γ-semigroup and f a (λ,μ)-fuzzy ideal of S, then f is a (λ,μ)-fuzzy interior ideal of S.*

Proof. Let $x,a,y\in S$, $\beta,\gamma\in\Gamma$, Since f is a (λ,μ)-fuzzy left ideal of S and $x,a\gamma y\in S$, we have that

$$f(x\beta(a\gamma y))\vee\lambda\geq f(a\gamma y)\wedge\mu \tag{1}$$

Since f is a (λ,μ)-fuzzy right ideal of S, we have that

$$f(a\gamma y)\vee\lambda\geq f(a)\wedge\mu \tag{2}$$

From (1) and (2) we know that $f(x\beta a\gamma y)\vee\lambda = (f(x\beta(a\gamma y))\vee\lambda)\vee\lambda \geq (f(a\gamma y)\wedge\mu)\vee\lambda = (f(a\gamma y)\vee\lambda)\wedge(\mu\vee\lambda)\geq f(a)\wedge\mu$. □

3. (λ,μ)-fuzzy interior ideals of regular/intra-regular ordered Γ-semigroups

We prove here that in regular and in intra-regular ordered Γ-semigroups the (λ,μ)-fuzzy ideals and the (λ,μ)-fuzzy interior ideals coincide.

Definition 4. An ordered Γ-semigroup $(S,\Gamma,\leq)$ is called regular if for all $a\in S$ there exists $x\in S$ such that $a\leq a\beta x\gamma a$, for all $\beta,\gamma\in\Gamma$.

Definition 5. An ordered Γ-semigroup $(S,\Gamma,\leq)$ is called intra-regular if for all $a\in S$ there exists $x,y\in S$ such that $a\leq x\beta a\gamma a\delta y$, for all $\beta,\gamma,\delta\in\Gamma$.

Theorem 3. *Let $(S,\Gamma,\leq)$ be a regular ordered Γ-semigroup and f a (λ,μ)-fuzzy interior ideal of S, then f is a (λ,μ)-fuzzy ideal of S.*

Proof. Let $x,y\in S$, then $f(x\beta y)\vee\lambda\geq f(x)\wedge\mu$, for all $\beta\in\Gamma$.

Indeed, since S is regular and $x\in S$, there exist $z\in S$ such that $x\leq x\beta z\gamma x$, for all $\beta,\gamma\in\Gamma$. Thus we have that $x\beta y\leq(x\beta z\gamma x)\beta y = (x\beta z)\gamma x\beta y$. So

$$f(x\beta y)\vee\lambda\geq f((x\beta z)\gamma x\beta y)\wedge\mu \tag{3}$$

for f is a (λ, μ)-fuzzy interior ideal. Again since f is a (λ, μ)-fuzzy interior ideal of S, we have

$$f((x\beta z)\gamma x\beta y) \vee \lambda \geq f(x) \wedge \mu. \tag{4}$$

From (3) and (4) we have that $f(x\beta y) \vee \lambda = (f(x\beta y) \vee \lambda) \vee \lambda \geq (f((x\beta z)\gamma x\beta y) \wedge \mu) \vee \lambda = (f((x\beta z)\gamma x\beta y) \vee \lambda) \wedge (\mu \vee \lambda) \geq f(x) \wedge \mu$, and f is a (λ, μ)-fuzzy right ideal of S.

In a similar way, we can prove that f is a (λ, μ)-fuzzy left ideal of S.

Thus f is a (λ, μ)-fuzzy ideal of S. □

Theorem 4. *Let $(S, \Gamma, \leq)$ be an intra-regular ordered semigroup and f a (λ, μ)-fuzzy interior ideal of S, then f is a (λ, μ)-fuzzy ideal of S.*

Proof. Let $a, b \in S$, then $f(a\beta b) \vee \lambda \geq f(a) \wedge \mu$, for all $\beta \in \Gamma$.

Indeed, since S is intra-regular and $a \in S$, there exist $x, y \in S$ such that $a \leq x\beta a\gamma a\delta y$. Then $a\beta b \leq (x\beta a\gamma a\delta y)\beta b$.

Since f is a (λ, μ)-fuzzy interior ideal of S, we have that $f(a\beta b) \vee \lambda = (f(a\beta b) \vee \lambda) \vee \lambda \geq (f(x\beta a\gamma a\delta y\beta b) \wedge \mu) \vee \lambda = (f(x\beta a\gamma a\delta y\beta b) \vee \lambda) \wedge (\mu \vee \lambda)$.

Again since f is a (λ, μ)-fuzzy interior ideal of S, we have $f(x\beta a\gamma a\delta y\beta b) \vee \lambda = f((x\beta a)\gamma a\delta(y\beta b)) \vee \lambda \geq f(a) \wedge \mu$.

Thus we have that $f(a\beta b) \vee \lambda \geq f(a) \wedge \mu$, and f is a (λ, μ)-fuzzy right ideal of S.

In a similar way we can prove that f is a (λ, μ)-fuzzy left ideal of S.

Therefore, f is a (λ, μ)-fuzzy ideal of S. □

Remark 1. From previous theorems we know that in regular or intra-regular ordered Γ-semigroups the concepts of (λ, μ)-fuzzy ideals and (λ, μ)-fuzzy interior ideals coincide.

4. (λ, μ)-fuzzy simple ordered Γ-semigroups

In this section, we introduce the concept of (λ, μ)-fuzzy simple ordered Γ-semigroups and characterize this type of ordered Γ-semigroups in terms of (λ, μ)-fuzzy interior ideals.

Definition 6. An ordered Γ-semigroup S is called simple if it does not contain proper ideals, that is, for any ideal $A \neq \emptyset$ of S, we have $A = S$.

Definition 7. An ordered Γ-semigroup S is called (λ, μ)-fuzzy simple if for any (λ, μ)-fuzzy ideal f of S, we have $f(a) \vee \lambda \geq f(b) \wedge \mu$, for all $a, b \in S$.

Remark 2. In [11], Kehayopulu and Tsingelis studied $(0,1)$-fuzzy simple ordered semigroup (They called it fuzzy simple ordered semigroup. see Definition 3.1 of [11]).

Sardar, Davvaz and Majumder researched $(0,1)$-fuzzy simple ordered Γ-semigroup , which was called fuzzy simple ordered Γ-semigroup in their paper [13].

Theorem 5. *Let S be an ordered Γ-semigroup, then S is (λ,μ)-fuzzy simple if and only if for any (λ,μ)-fuzzy ideal f of S, if $f_t \neq \emptyset$, then $f_t = S$, for all $t \in (\lambda,\mu]$.*

Proof. Assume that S is is (λ,μ)-fuzzy simple. For any (λ,μ)-fuzzy ideal f of S, suppose that $f_t \neq \emptyset$. We need to prove that $x \in f_t$ for all $x \in S$, where $t \in (\lambda,\mu]$.

Since $f_t \neq \emptyset$, we can suppose that there exists $y \in f_t$, that is $f(y) \geq t$.

So $f(x) \vee \lambda \geq f(y) \wedge \mu \geq t \wedge \mu = t$.

Notice that $\lambda < t$, we have that $f(x) \geq t$, that is $x \in f_t$.

Conversely, for any (λ,μ)-fuzzy ideal f of S, suppose that $f_t = S$, for all $t \in (\lambda,\mu]$. We need to prove that $f(a) \vee \lambda \geq f(b) \wedge \mu$, for all $a, b \in S$.

If there exist $a_0, b_0 \in S$, such that $f(a_0) \vee \lambda < t = f(b_0) \wedge \mu$, then $t \in (\lambda,\mu]$, $f(a_0) < t$ and $f(b_0) \geq t$. Thus $a_0 \notin f_t = S$. This is a contradiction.

So $f(a) \vee \lambda \geq f(b) \wedge \mu$ holds, for all $a, b \in S$. □

Proposition 1. *Let S be an ordered Γ-semigroup and f a (λ,μ)-fuzzy right ideal of S, then $I_a = \{b \in S | f(b) \vee \lambda \geq f(a) \wedge \mu\}$ is a right ideal of S for every $a \in S$.*

Proof. Let $a \in S$, then $I_a \neq \emptyset$ since $a \in I_a$.

(1) Let $b \in I_a$ and $s \in S$, then $b\beta s \in I_a$ for any $\beta \in \Gamma$. Indeed, since f is a (λ,μ)-fuzzy right ideal of S and $b, s \in S$, we have

$$f(b\beta s) \vee \lambda \geq f(b) \wedge \mu. \tag{5}$$

Since $b \in I_a$, we have that

$$f(b) \vee \lambda \geq f(a) \wedge \mu. \tag{6}$$

From (5) and (6) we conclude that $f(b\beta s) \vee \lambda = (f(b\beta s) \vee \lambda) \vee \lambda \geq (f(b) \wedge \mu) \vee \lambda = (f(b) \vee \lambda) \wedge (\mu \vee \lambda) \geq f(a) \wedge \mu$. So $b\beta s \in I_a$.

(2) Let $b \in I_a$ and $S \ni s \leq b$, then $s \in I_a$. Indeed, since f is a (λ,μ)-fuzzy right ideal of S , $s, b \in S$ and $s \leq b$, we have

$$f(s) \vee \lambda \geq f(b) \wedge \mu. \tag{7}$$

Since $b \in I_a$, we have

$$f(b) \vee \lambda \geq f(a) \wedge \mu. \tag{8}$$

From (7) and (8) we obtain that $f(s) \vee \lambda = (f(s) \vee \lambda) \vee \lambda \geq (f(b) \wedge \mu) \vee \lambda = (f(b) \vee \lambda) \wedge (\mu \vee \lambda) \geq f(a) \wedge \mu$. So $s \in I_a$. □

Similarly, we have

Proposition 2. *Let S be an ordered Γ-semigroup and f a (λ, μ)-fuzzy left ideal of S, then $I_a = \{b \in S | f(b) \vee \lambda \geq f(a) \wedge \mu\}$ is a left ideal of S for every $a \in S$.*

By the previous two propositions we have

Proposition 3. *Let S be an ordered Γ-semigroup and f a (λ, μ)-fuzzy ideal of S, then $I_a = \{b \in S | f(b) \vee \lambda \geq f(a) \wedge \mu\}$ is an ideal of S for every $a \in S$.*

Lemma 1. *Let S be an ordered Γ-semigroup and $\emptyset \neq I \subseteq S$, then I is an ideal of S if and only if the characteristic function f_I is a (λ, μ)-fuzzy ideal of S.*

Proof. "$\Rightarrow$"

Suppose I is an ideal of S. For any $x \in S$, two cases are possible:

(1) $x \in I$. In this case, $x\gamma y \in I$ for any $\gamma \in \Gamma$ and $y \in S$. This is because I is an ideal of S.

Thus $f_I(x\gamma y) = f_I(x) = 1$ and so $f_I(x\gamma y) \vee \lambda \geq f_I(x) \wedge \mu$.

Similarly, we have $f_I(y\gamma x) \vee \lambda \geq f_I(x) \wedge \mu$.

So f_I is a (λ, μ)-fuzzy ideal of S.

(2) $x \notin I$. In this case, $f_I(x) = 0$. So $f_I(x\gamma y) \vee \lambda \geq f_I(x) \wedge \mu$ and $f_I(y\gamma x) \vee \lambda \geq f_I(x) \wedge \mu$ hold. Thus f_I is a (λ, μ)-fuzzy ideal of S.

"$\Leftarrow$"

Conversely, suppose that f_I is a (λ, μ)-fuzzy ideal of S. Then $f_I(x\gamma y) \vee \lambda \geq f_I(x) \wedge \mu$ and $f_I(y\gamma x) \vee \lambda \geq f_I(x) \wedge \mu$.

Set $x \in I$, we need to show that $x\gamma y \in I$ and $y\gamma x \in I$ for any $\gamma \in \Gamma$ and $y \in S$.

Since $x \in I$, we have that $f_I(x) = 1$, so $f_I(x\gamma y) \vee \lambda \geq \mu$ and $f_I(y\gamma x) \vee \lambda \geq \mu$. Note that $\lambda < \mu$, we have that $f_I(x\gamma y) \geq \mu$ and $f_I(y\gamma x) \geq \mu$. Thus $f_I(x\gamma y) = 1$ and $f_I(y\gamma x) = 1$. That is $x\gamma y \in I$ and $y\gamma x \in I$ for any $\gamma \in \Gamma$ and $y \in S$. □

Theorem 6. *An ordered Γ-semigroup S is simple if and only if it is (λ, μ)-fuzzy simple.*

Proof. Suppose S is simple, let f be a (λ,μ)-fuzzy ideal of S and $a,b\in S$. By previous propostion, the set I_a is an ideal of S. Since S is simple, we have $I_a=S$. Then $b\in I_a$, from which we have that $f(b)\vee\lambda\geq f(a)\wedge\mu$. Thus S is (λ,μ)-fuzzy simple.

Conversely, suppose S contains proper ideals and let I be such ideal of S. By the previous lemma, we know that f_I is a (λ,μ)-fuzzy ideal of S. We have that $S\subseteq I$. Indeed, let $x\in S$. Since S is (λ,μ)-fuzzy simple, $f_I(x)\vee\lambda\geq f_I(b)\wedge\mu$ for all $b\in S$. Now let $a\in I$. Then we have $f_I(x)\vee\lambda\geq f_I(a)\wedge\mu=1\wedge\mu=\mu$. Notice that $\lambda<\mu$, we conclude that $f_I(x)\geq\mu$, which implies that $f_I(x)=1$, that is $x\in I$. Thus we have that $S\subseteq I$, and so $S=I$. We get a contradiction. □

Lemma 2. *An ordered Γ-semigroup S is simple if and only if for every $a\in S$, we have $S=(S\Gamma a\Gamma S]$.*

Proof. It is easy from Lemma 1.2 of [10] or from Theorem 1.1 of [4]. □

Theorem 7. *Let S be an ordered Γ-semigroup, then S is simple if and only if for every (λ,μ)-fuzzy interior ideal f of S, we have $f(a)\vee\lambda\geq f(b)\wedge\mu$, for all $a,b\in S$.*

Proof. Suppose S is simple. Let f be a (λ,μ)-fuzzy interior ideal of S and $a,b\in S$. Since S is simple and $b\in S$, by the previous lemma, we have that $S=(S\Gamma b\Gamma S]$. Since $a\in S$, we have that $a\in(S\Gamma b\Gamma S]$. Then there exist $x,y\in S$ and $\beta,\gamma\in\Gamma$ such that $a\leq x\beta b\gamma y$. Since $a,x\beta b\gamma y\in S, a\leq x\beta b\gamma y$ and f is a (λ,μ)-fuzzy interior ideal of S, we have that

$$f(a)\vee\lambda\geq f(x\beta b\gamma y)\wedge\mu. \tag{9}$$

Since $x,b,y\in S$ and f is a (λ,μ)-fuzzy interior ideal of S, we have that

$$f(x\beta b\gamma y)\vee\lambda\geq f(b)\wedge\mu. \tag{10}$$

From (9) and (10) we conclude that $f(a)\vee\lambda=(f(a)\vee\lambda)\vee\lambda\geq(f(x\beta b\gamma y)\wedge\mu)\vee\lambda=(f(x\beta b\gamma y)\vee\lambda)\wedge(\mu\vee\lambda)\geq f(b)\wedge\mu$.

Conversely, Suppose that for every (λ,μ)-fuzzy interior ideal f of S, we have $f(a)\vee\lambda\geq f(b)\wedge\mu$, for all $a,b\in S$.

Now let f be any (λ,μ)-fuzzy ideal f of S, then it is a (λ,μ)-fuzzy interior ideal of S. So we have $f(a)\vee\lambda\geq f(b)\wedge\mu$, for all $a,b\in S$. Thus S is (λ,μ)-fuzzy simple by its definition. And from the previous theorem, we conclude that S is simple. □

As a consequence we have

Theorem 8. *For an ordered* Γ*-semigroup* S*, the following are equivalent:*

(1) S *is simple.*

(2) $S = (S\Gamma a\Gamma S]$ *for every* $a \in S$.

(3) S *is* (λ, μ)*-fuzzy simple.*

(4) For every (λ, μ)*-fuzzy interior ideal* f *of* S*, we have* $f(a) \vee \lambda \geq f(b) \wedge \mu$*, for all* $a, b \in S$.

5. Conclusion and further research

In this paper, we generalized results of [11, 13]. We introduced (λ, μ)-fuzzy ideals and (λ, μ)-fuzzy interior ideals of an ordered Γ-semigroup and studied them. When $\lambda = 0$ and $\mu = 1$, we meet ordinary fuzzy ideals and fuzzy interior ideals. So we can say that (λ, μ)-fuzzy ideals and (λ, μ)-fuzzy interior ideals are more general concepts than fuzzy ones.

In [22], Yao gave the definition of (λ, μ)-fuzzy bi-ideals in semigroups. One can study (λ, μ)-fuzzy bi-ideals in ordered Γ-semigroups. We would like to explore this in next papers.

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References

[1] S. Chattopadyay, *Right inverse* Γ*-semigroup*, Bull. Calcutta Math. Soc., 78, 1986, pp.180-186.

[2] S. Chattopadyay, *Right Orthodox* Γ*-semigroup*, Southeast Asian Bull. Math., 29, 2005, pp.23-30.

[3] R. Chinram, *On quasi-*Γ*-ideals in* Γ*-semigroups*, ScienceAsia, 32, 2006, pp.351-353.

[4] R. Chinram, K. Tinpun, *A note on minimal bi-ideals in ordered* Γ*-semigroups*, International Mathematical Forum, 4(1), 2009, pp.1-5.

[5] T.K. Dutta, N.C. Adhikari, *On* Γ*-semigroup with right and left unities*, Soochow J. Math., 19(4), 1993, pp.461-474.

[6] T.K. Dutta, N.C. Adhikari, *On prime radical of* Γ*-semigroup*, Bull. Calcutta Math. Soc., 86(5), 1994, pp.437-444.

[7] Y. Feng, P. Corsini, (λ, μ)-fuzzy version of ideals, interior ideals, quail-ideals and bi-ideals, Journal of Applied Mathematics, Volume 2012, Article ID 425890, 7 pages, doi:10.1155/2012/425890.

[8] K. Hila, *On regular, semiprime and quasi-reflexive* Γ*-semigroup and minimal quasi-ideals*, Lobachevskii Journal of Mathematics, 29, 2008, pp.141-152.

[9] K. Hila, *On some classes of le-Γ-semigroup and minimal quasi-ideals*, Algebras Groups Geom., 24, 2007, pp.485-495.

[10] A. Iampan, M. Siripitukdet, *On minimal and maximal ordered left ideals in po-Γ-semigroups*, Thai Journal of Mathematics, 2(2), 2004, pp.275-282.

[11] N. Kehayopulu, M. Tsingelis, *Fuzzy interior ideals in ordered semigroups*, Lobachevskii Journal of Mathematics, Vol.21, 2006, pp.65-71.

[12] N.K. Saha, *On Γ-semigroup II*, Bull. Calcutta Math. Soc., 79, 1987, pp.331-335.

[13] S. Sardar, B. Davvaz, S. Majumder, *A study on fuzzy interior ideals of Γ-semigroups*, Computers and Mathematics with Applications, 60, 2010, pp.90-94.

[14] M.K. Sen, N.K. Saha, *On Γ-semigroup I*, Bull. Calcutta Math. Soc., 78, 1986, pp.180-186.

[15] M.K. Sen, S. Chattopadhyay, *Semidirect product of a monoid and a Γ-semigroup*, East-West J. Math., 6, 2004, pp.131-138.

[16] M.K. Sen, A. Seth, *On po-Γ-semigroups*, Bull. Calcutta Math. Soc., 85, 1993, pp.445-450.

[17] M.K. Sen, N.K. Saha, *Orthodox Γ-semigroups*, Internat. J. Math. Math. Soc., 13, 1990, pp.527-534.

[18] M. Uckun, M.A. Ozturk, Y.B. Jun, *Intuitionistic fuzzy sets in Γ-semigroups*, Bull. Korean Math. Soc., 44(2), 2007, pp.359-367.

[19] B. Yao, *(λ, μ)-fuzzy normal subfields and (λ, μ)-fuzzy quotient subfields*, The Journal of Fuzzy Mathematics, Vol.13, No.3, 2005, pp.695-705.

[20] B. Yao, *(λ, μ)-fuzzy subrings and (λ, μ)-fuzzy ideals*, The Journal of Fuzzy Mathematics, Vol.15, No.4, 2007, pp.981-987.

[21] B. Yao, *Fuzzy Theory on Group and Ring*, Beijing:Science and Technology Press (in Chinese), 2008.

[22] B. Yao, *(λ, μ)-fuzzy ideal in semigroups*, Fuzzy Systems and Mathematics, Vol.23, No. 1, 2009, pp123-127.

[23] X. Yuan ,C. Zhang ,Y. Ren, *Generalized fuzzy groups and many-valued implications*, Fuzzy Sets and Systems, 138, 2003, pp.205-211.

[24] L.A. Zadeh *Fuzzy sets*, Inform. Control, 8, 1965, pp.338-353.

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