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Fast coalescence of post-Newtonian Supermassive Black Hole Binaries in real galaxies

We present the results of theoretical modeling of supermassive black hole binary (SMBHB) mergers using direct 2 -body simulations with a Hermite integration scheme. The BH's gravitational interaction is described based on the post-Newtonian (𝒫𝒩 terms) approximation up to the 3.5𝒫𝒩 terms. We carry out a large set of runs using a parametric description of SMBHB orbits. The final time of the SMBHs gravitational coalescence is parametrized as a function of initial eccentricity e_0 and mass ratio q of the binary. We carry out detailed tests of our coding. We tested our 𝒫𝒩 terms against the analytic prescription described at the theoretical works in middle 60th. The gravitational radiation polarization amplitudes h and h from the SMBHBs merging process are also analyzed. Based on our numerical work we estimate the expected merging time for a list of selected potential SDSS SMBHBs. Our results show that the merging time is a strong function of the assumed initial eccentricities and fall within the range of thousands years.

ШВИДКЕ ЗЛИТТЯ ПОСТНЬЮТОНІВСЬКИХ ПОДВІЙНИХ НАДМАСИВНИХ ЧОРНИХ ДІР У РЕАЛЬНИХ ГАЛАКТИКАХ, Соболенко М. О., Берцик П. П., Шпурзем Р., Купі Г. — Приводяться результати теоретичного моделювання злиття подвійних надмасивних чорних дір за допомогою прямого 2-тільного моделювання з ермітівською схемою інтегрування. Гравітаційна взаємодія чорних дір описується постньютонівським наближенням до 3.5𝒫𝒩-терму. На основі параметричного опису орбіт ПНЧД отримано великий набір моделей.

Кінцевий час гравітаційного злиття ПНЧД параметризовано як функцію початкового ексцентриситету e_0 та відношення мас q подвійної. Проведено детальне тестування нашого коду. Ми порівнювали PN-терми з аналітичним описом у теоретичних дослідженнях середини 1960-х рр. Проаналізовано амплітуду поляризованого гравітаційного випромінювання h та h під час злиття ПНЧД. З використанням нашого числового коду оцінено очікуваний час злиття для списку вибраних потенційних SDSS ПНЧД. Наші результати показують, що час злиття досягає тисяч років та є строгою функцією обраного початкового ексцентриситету.

БЫСТРОЕ СЛИЯНИЕ ПОСТНЬЮТОНОВСКИХ ДВОЙНЫХ СВЕРХМАССИВНЫХ ЧОРНЫХ ДЫР В РЕАЛЬНЫХ ГАЛАКТИКАХ, Соболенко М. А., Берцик П. П., Шпурзем Р., Купи Г. — Представлены результаты теоретического моделирования слияния двойных сверхмассивных чёрных дыр с помощью прямого 2-тельного моделирования с эрмитовской схемой интегрирования. Гравитационное взаимодействие чорных дыр описывается постньютоновским приближением до 3.5PN-терма. На основе параметрического описания орбит ДСМЧД получен большой набор моделей. Конечное время гравитационного слияния ДСМЧД параметризовано как функция начального эксцентрисета e_0 и отношения масс q двойной. Проведено детальное тестирование нашего кода. Сравнивались PN-термы с аналитическим описанием в теоретических исследованиях средины 1960-х гг. Проанализирована амплитуда поляризованного гравитационного излучения h и h во время слияния ДСМЧД. С использованием нашего численного кода оценено ожидаемое время слияния для списка выбранных потенциальных SDSS ДСМЧД. Наши результаты показывают, что время слияния достигает тысяч лет и является строгой функцией выбранного начального эксцентриситета.

INTRODUCTION

The formation and evolution of galaxies and their SMBHs are connected in several ways. This relation can be found already at the early phases of protogalaxies formation [64], also at the later stages of hierarchical CDM cosmology [15, 31, 63] and also during the stages of different galaxy mergers [36, 42, 51]. One of the most simple and plausible channel of the SMBH mass growth is an accumulation of the BH's mass during host-galaxy mergers. Gas accretion can significantly increase the mass of BHs during "wet" merging that triggers star formation [3, 14, 25, 45, 57, 61]. Stellar accretion can also increase BH masses even in "dry" merging during the formation the giant elliptical galaxies [5, 44, 46, 67, 71]. The M — relation, that shows a connection between the mass of the SMBH and the mass of the central bulge of their host galaxies [30], we assume is evidence for such a scenario. The fact that the distribution of the most luminous and massive active galactic nuclei peaks at higher redshifts also support this idea [34]. SMBHBs inside merging galaxies could be one of the most powerful sources of gravitational waves (GW), which can be detected by the Pulsar Timing Array (PTA) or future space-based missions, such as LISA/eLISA, DESIGO/BBO [1, 33, 69]. The dynamical evolution of SMBHBs in the center of a merged stellar system can be traditionally divided in three phases [4].

(I) Two BHs can form a pair inside the merging host galaxy due to dynamical friction in the stellar background. Then these components sink into the centre of the stellar distribution. SMBHBs start to be "hard" when the length of the semimajor axis of the binary reaches the value:

$$a \approx a_h \approx \frac{G\mu}{4\sigma^2} \approx \frac{2.7\,\mathrm{pc}}{1+q}\left(\frac{m_2}{10^8 M_\odot}\right)\left(\frac{\sigma}{200\,\mathrm{km/s}}\right)^{-2}, \tag{1}$$

where G is a gravitational constant, mass of the BH's is $m_2 \le m_1$, mass ratio is $q = m_2/m_1$, $\mu = m_1 m_2/(m_1 + m_2)$ is a reduced mass, total mass is M_{tot} = $m_1 + m_2$. This means that the binding energy per unit mass $|E|/M_{tot} = G\mu/2a$ exceeds $\sim\sigma^2$ (the ambient stellar velocity dispersion) [50].

(II) Due to the slingshot interaction mechanism the binary can continue to harden via three-body scattering of single stars. If star's orbit intersects with the SMBHB orbit, a complex three-body interaction can eventually lead to the "ejection" of the star. This "ejected" star carries away energy and angular momentum from the binary (see references in [49, 68, 70]).

But if we assume spherical symmetry, the loss cone of the binary BH system can be depleted by the slingshot mechanism before this [2, 27]. Therefore the system hardening time can be more than the Hubble time [52]. This is the so called "final parsec problem" which can be solved in *N*-body simulations assuming a more realistic stellar particle distribution in a rotating system [6, 39], oblate / triaxial potential [29, 38, 60] or some combination of these configurations.

(III) At the third stage the components sink toward to the separation when GW emission begins to be efficient. Finally, the binary inspirals down to the coalescence, emitting a strong GW signal. For such a merger the two SMBHs have to reach a critical separation in a time shorter than the Hubble time (few Gyr):

$$a_{GW} \approx 2\times 10^{-3} f(e)^{1/4} \frac{q^{1/4}}{(1+q)^{1/2}} \left(\frac{M_{tot}}{10^6 M_\odot}\right)^{3/4} \mathrm{pc}, \tag{2}$$

where $f(e) = [1 + (73/24)e^2 + (37/96)e^4](1-e^2)^{-7/2}$ is a function of the binary eccentricity e [58, 59].

To estimate the SMBHBs real merging times, we need to make our calculations with the real speed of light values. Such *N*-body simulations are already available in the literature (for example [38, 40]). But on the real merging galaxy scale such simulations require a lot of computing resources.

In this paper we propose a slightly different approach. We perform simulations for different sets of parameters with various “parametric” values of light speed (for example see [9]). To explore the connection between the real merging time T_{mrg}, the total mass M_{tot} of the SMBHB and initial separation R between the BHs we estimate a scaling between the merging time T_{mrg} and the speed of the light c assuming the dependence between these parameters.

NUMERICAL METHODS AND INITIAL CONDITIONS

Some numerical details. For the two BHB dynamical orbit integration, we use the publicly available φGPU* [7, 8] with a 4[th] order Hermite integrator and block hierarchical individual time step scheme. This Hermite scheme requires us to know the acceleration and its first time-derivative, called *jerk*. Because we use this Hermite scheme for our $\mathcal{PN}$ runs, we need to include the $\mathcal{PN}$ corrections also to the acceleration and *jerk* terms. In the φGPU code we use the generalized “Aarseth” type criteria for the time step definition [53]:

$$\Delta t = \eta_p \left(\frac{A^{(1)}}{A^{(p-2)}} \right)^{1/(p-3)}, \tag{3}$$

where

$$A^{(k)} = \sqrt{|\mathbf{a}^{(k-1)}||\mathbf{a}^{(k+1)}| + |\mathbf{a}^{(k)}|^2}. \tag{4}$$

Here, $\mathbf{a}^{(k)}$ is the k^{th} derivative of acceleration, p is the order of the integrator, η_p is the accuracy parameter. For a 4[th]-order Hermite scheme the timestep looks like:

$$\Delta t = \eta_4 \frac{A^{(1)}}{A^{(2)}}, \tag{5}$$

where

$$A^{(1)} = \sqrt{|\mathbf{a}^{(0)}||\mathbf{a}^{(2)}| + |\mathbf{a}^{(1)}|^2}, \quad A^{(2)} = \sqrt{|\mathbf{a}^{(1)}||\mathbf{a}^{(3)}| + |\mathbf{a}^{(2)}|^2}. \tag{6}$$

For all our runs we use the $\eta_4 = 0.018$ parameter.

Post-Newtonian formalism. We use a post-Newtonian formalism in the 2-body code for calculating the relativistic binary systems dynamics. The results for up to 2$\mathcal{PN}$ and even up to 2.5$\mathcal{PN}$ equations of binary motion in harmonic coordinates were obtained by Damour and Deruelle [17—20, 24]. For the 3$\mathcal{PN}$ and 3.5$\mathcal{PN}$ terms we can use two different ways of computation. One of the possibilities is to use the ADM-Hamiltonian formalism of general relativity [22, 54—56]. Physically equivalent results [21, 23] can be obtained from the post-Newtonian iteration [11], when we compute the equation of motion directly (instead of via a Hamiltonian) in harmonic coordinates.

The equation of motion is a power series of $1/c$, where n-$\mathcal{PN}$ is proportional to $(v/c)^{2n}$. Schematically, one can write the correction for acceleration during the motion of object in binary system as [19, 65]:

* ftp://ftp.mao.kiev.ua/pub/berczik/phi-GPU/

$$\mathbf{a}_{NoSpin} \quad \mathbf{a}_N \quad \frac{1}{c^2}\mathbf{a}_{1\mathcal{PN}} \quad \frac{1}{c^4}\mathbf{a}_{2\mathcal{PN}} \quad \frac{1}{c^5}\mathbf{a}_{2.5\mathcal{PN}} \quad \frac{1}{c^6}\mathbf{a}_{3\mathcal{PN}} \quad \frac{1}{c^7}\mathbf{a}_{3.5\mathcal{PN}} \quad O\ \frac{1}{c^8}\ , \tag{7}$$

where $\mathbf{a}_N$ is the classical Newtonian acceleration; $\mathbf{a}_{1\mathcal{PN}}$, $\mathbf{a}_{2\mathcal{PN}}$, $\mathbf{a}_{3\mathcal{PN}}$ are the non dissipative terms which “conserve” the energy of the system. The $\mathbf{a}_{2.5\mathcal{PN}}$, $\mathbf{a}_{3.5\mathcal{PN}}$ are the dissipative terms which “carry out” energy from the system due to GW emission. We apply all $\mathcal{PN}$ corrections up to order $O(1/c^8)$, so the 3.5$\mathcal{PN}$ correction is the highest order that we take into account. To compare our results with the analytical solution from classical articles [58, 59] we use the code just with the single 2.5$\mathcal{PN}$ term.

Similar to the equation of motion in the centre of mass frame [10] the acceleration for one particle can be written in the following form:

$$\mathbf{a}_{NoSpin} \quad \frac{d\mathbf{v}}{dt} \quad \frac{GM}{r^2}[(1 \quad \mathcal{A})\mathbf{n} \quad \mathcal{B}\mathbf{v}], \tag{8}$$

where $r \quad |\mathbf{r}|$ is the separation between particles, $\mathbf{r} \quad \mathbf{r}_1 \quad \mathbf{r}_2$ is the position of the particles, $\mathbf{n} \quad \mathbf{r}/r$ is the normalized relative position vector, $\mathbf{v} \quad \mathbf{v}_1 \quad \mathbf{v}_2$ is the relative velocity. The functions $\mathcal{A}$ and $\mathcal{B}$ contain different orders of the $\mathcal{PN}$ approximation (similar to Eq. (7)).

For example the first $\mathcal{PN}$ correction term is given by:

$$A_{1\mathcal{PN}} \quad \frac{5Gm_1}{r} \quad \frac{4Gm_2}{r} \quad \frac{3}{2}(\mathbf{n}\ \mathbf{v}_2)^2 \quad \mathbf{v}_1^2 \quad 4(\mathbf{v}_1\ \mathbf{v}_2) \quad 2\mathbf{v}_2^2\ , \tag{9}$$

$$B_{1\mathcal{PN}} \quad 4(\mathbf{n}\ \mathbf{v}_1) \quad 3(\mathbf{n}\ \mathbf{v}_2). \tag{10}$$

Detailed references and the complete description of the problem can be found in works such as [9, 10, 43]. The complete equations in post-Newtonian formalism up to 3.5$\mathcal{PN}$ are given also in [10].

Adding the spin terms into the equation of motion we can describe as:

$$\mathbf{a}_{Spin} \quad \mathbf{a}_{NoSpin} \quad \frac{1}{c^3}\mathbf{a}_{1.5\mathcal{PN},SO} \quad \frac{1}{c^4}\mathbf{a}_{2\mathcal{PN},SS} \quad \frac{1}{c^5}\mathbf{a}_{2.5\mathcal{PN},SO}, \tag{11}$$

where $\mathbf{a}_{1.5\mathcal{PN},SO}$ and $\mathbf{a}_{2.5\mathcal{PN},SO}$ are the spin-orbit coupling terms, $\mathbf{a}_{2\mathcal{PN},SS}$ is the spin-spin coupling term (for example [26]). Now one can write the full equation (like Eq. (7)):

$$\begin{aligned}\mathbf{a}_{Spin} \quad \mathbf{a}_N \quad \frac{1}{c^2}\mathbf{a}_{1\mathcal{PN}} \quad \frac{1}{c^3}\mathbf{a}_{1.5\mathcal{PN},SO} \quad \frac{1}{c^4}(\mathbf{a}_{2\mathcal{PN}} \quad \mathbf{a}_{2\mathcal{PN},SS})\\ \frac{1}{c^5}(\mathbf{a}_{2.5\mathcal{PN}} \quad \mathbf{a}_{2.5\mathcal{PN},SO}) \quad \frac{1}{c^6}\mathbf{a}_{3\mathcal{PN}} \quad \frac{1}{c^7}\mathbf{a}_{3.5\mathcal{PN}} \quad O\ \frac{1}{c^8}\ ,\end{aligned} \tag{12}$$

where the full expression for $\mathbf{a}_{1.5\mathcal{PN},SO}$ and $\mathbf{a}_{2.5\mathcal{PN},SO}$ can be found in [26], for $\mathbf{a}_{2\mathcal{PN},SS}$ can be found in [66]. The value of the physical spin is chosen from the the next expression:

$$S^{true} \quad \frac{Gm^2}{c}, \tag{13}$$

where the value of is [0, 1]. At the centre of the binary mass frame we have the spin $\mathbf{S} \quad \mathbf{S}_1 \quad \mathbf{S}_2$. We use two body dynamics and spin-spin and spin-orbit coupling just for calculation of the first order of the gravitational waveform constraint (e.g. [41]):

$$h^{ij} \quad \frac{4G}{Dc^4} \quad v^i v^j \quad \frac{GM}{r} n^i n^j \quad , \tag{14}$$

where $Q^{ij} \quad 2(v^i v^j \quad GMn^i n^j / r)$ is the usual quadrupole term (second time derivatives of the mass quadrupole moment tensor) and D is the luminosity distance. Choosing the virtual detector orientation so that as the coordinate axes coincide with the source frame, we can describe the two-dimensional matrix with only two independent elements:

$$h_{ij} \quad \begin{matrix} h & h \\ h & h \end{matrix} \quad . \tag{15}$$

From h_{ij} we can obtain the amplitude of polarization h and h [12, 16, 62].

Initial conditions and description of model. We assume that the two point masses which represents our BHs with masses m_1 and m_2 are placed at positions Y_1 and Y_2 on the Y axis (see Fig. 1). For our analyses we choose the natural coordinate system of the two bodies, connected by the centre of mass of the system. The initial orbital velocity of the two point masses we chose so that the XY plane contains the full orbit. The initial separation between the components we defined as $R \quad |Y_1| \quad |Y_2|$. We also set the BH's mass ratio $q \quad m_1 / m_2$. We assume that $m_1 \quad m_2$. We also fix the total BH system mass $M_{tot} \quad m_1 \quad m_2$. The Keplerian motion of the two bodies can be fully

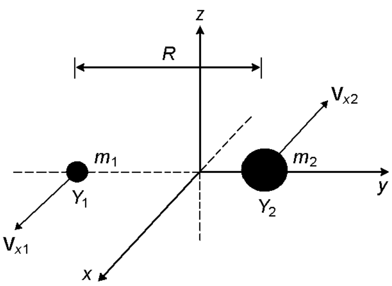

Fig. 1. Configuration of the system with two BHs

described by two main orbital parameters: the semimajor axis a and eccentricity e. We can write the binding energy of the binary system:

$$|E| \quad \frac{Gm_1 m_2}{2a} \quad \frac{G \quad M_{tot}}{2a} \, , \tag{16}$$

where $\quad m_1 m_2 / M_{tot}$ is the reduced mass. We also fix as a parameter the binary initial orbital eccentricity e_0. The initial setup of the particles we show in Figure 1. For further calculation we assume the normalization $R = 1$ and $M_{tot} \quad m_1 \quad m_2 = 1$.

We use the N-body (NB) or called Hénon units [32] where we also accept $G = 1$ and set the mass units M and length units R to unity*. Therefore the physical values of mass, length, energy, velocity and time will be in the form:

$$[M] = M, \qquad [L] = R, \tag{17}$$

$$[E] = \frac{GM^2}{R}, \tag{18}$$

$$[V] \quad \frac{GM}{R}^{\;1/2}, \quad [T] \quad \frac{R^3}{GM}^{\;1/2}. \tag{19}$$

Consequently the light speed c in N-body units is:

$$c \quad \frac{c_0}{V} \quad c_0 \quad \frac{GM}{R}^{\;1/2} \quad 14213 \quad \frac{M}{10^8 M_\odot}^{\;1/2} \quad \frac{R}{10^3\,\mathrm{pc}}^{\;1/2}, \tag{20}$$

where c_0 is the light speed in physical units.

DISCUSSION

Scaling routine between merging time T_{mrg} and "parametric" speed of the light was made for all models from Table 1 (for example see Fig. 2 for system with parameters $M_{tot} = 1$ [NB], $q = 0.5$, $R = 1$ [NB], $e_0 = 0.25$). Based on our post-Newtonian formalism (Eqs (7)-(12)) we can theoretically expect the relationship between merging time (which is directly proportional to the energy losses in our post-Newtonian formalism) and the light speed:

$$T_{mrg\,5} \quad b\,c^5, \quad T_{mrg\,5\ 7} \quad d\,c^5 \quad p\,c^7, \tag{21}$$

where b, d and p are the coefficients of the scaling. As we can see from Fig. 2 the difference between the two merging times are negligible. So, in this paper we use the $T_{mrg\,5}$ as a basic approximation for the binary merging time T_{mrg}.

Table 1. **The scale factor *b* from Eq. (21) for various mass ratio *q* and initial eccentricity e_0 (separation for each system R = 1 [NB] and total mass M_{tot} =1 [NB])**

e_0	*b*					
	$q = 1$	$q = 0.5$	$q = 0.333$	$q = 0.25$	$q = 0.2$	$q = 0.02$
0.00	7.863E-02	8.827E-02	1.043E-01	1.218E-01	1.397E-01	8.611E-01
0.25	2.578E-02	2.900E-02	3.437E-02	4.027E-02	4.639E-02	3.375E-01
0.50	5.584E-03	6.280E-03	7.440E-03	8.716E-03	1.004E-02	7.244E-02
0.75	4.648E-04	5.225E-04	6.186E-04	7.243E-04	8.339E-04	6.003E-03
0.95	1.893E-06	2.126E-06	2.514E-06	2.938E-06	3.383E-06	2.425E-05
0.99	8.146E-09	9.123E-09	1.076E-08	1.255E-08	1.441E-08	1.023E-07

* http://en.wikipedia.org/wiki/N-body_units

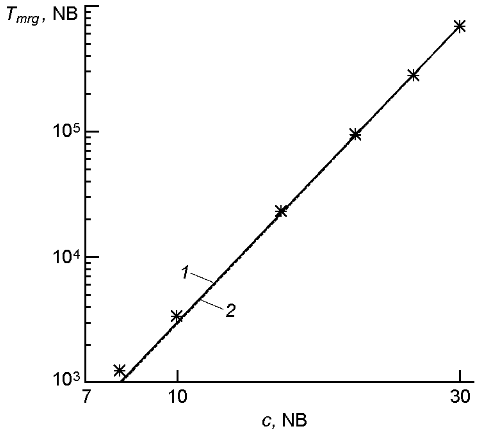

Fig. 2. Relation between the merging time T_{mrg} and light speed c for system from Table 1 (line *1* — $T_{mrg} = bc^5$, *2* — $T_{mrg} = dc^5 + pc^7$, stars — simulation). Initial eccentricity e_0 = 0.25 and mass ratio q = 0.5

We study the evolution of systems with various mass ratios and initial eccentricities, i. e. with various orbits. We use the following sets of the parameters: q = 1, 0.5, 0.333, 0.25, 0.2, 0.02 and e = 0.00, 0.25, 0.50, 0.75, 0.95, 0.99.

We apply the scaling factors from Table 1 to find the real merging times T_{mrg} (in physical units) where the physical light speed is c = 2.99792458 10^8 m/s.

We apply the above described "c-scaling procedure" for a wide range of physical parameters for masses ($10^6 M_\odot \quad M_{tot} \quad 5\,10^9 M_\odot$) and the initial separation between the BHs ($10^{\ 3} \quad R \quad 10^2$ pc). For each individual model we estimate the relation between the merging time T_{mrg}, separation between the BHs R and total mass M_{tot} of the SMBHB (Fig. 3, 4). For example using Fig. 3 for system with $M_{tot} = 10^9 M_\odot$, q = 0.5, R = 10 pc, e_0 = 0.5 merging time T_{mrg} 1700 years.

In a real cosmological merging scenario we expect that the SMBHBs merger does not evolve in isolation. High resolution cosmological numerical simulations (see references in [28, 37, 48]) show us that SMBHB mergers typically need to meet the next large galaxy in a time scale of 1-2 Gyr. If we assume the existence of a SMBH in this third galaxy too, in this case our binary BH is transformed to a triple BH system. Extensive direct N-body simulations of system with three BHs show that such a configuration is highly unstable [1, 13]. So, we assume that if in a time scale of 1-2 Gyr our original BHB system does not merge, the possibility of such a merger becomes very unlikely. In Figures 3, 4 we show the 1 Gyr merging time as the solid black lines for the different initial eccentricities.

For some fixed time this relation can be written in the form:

$$R \quad 10\ (M_{SMBHB,6})^{3/4}.$$

We found that with the rise of initial eccentricity e_0 the merging time T_{mrg} of the system decreases. This behavior is valid for mass ratios from q = = 1 to q = 0.2 and even for extremal q = 0.02. The general conclusion from our set of runs is that the lower initial eccentricity (circular) orbits generally

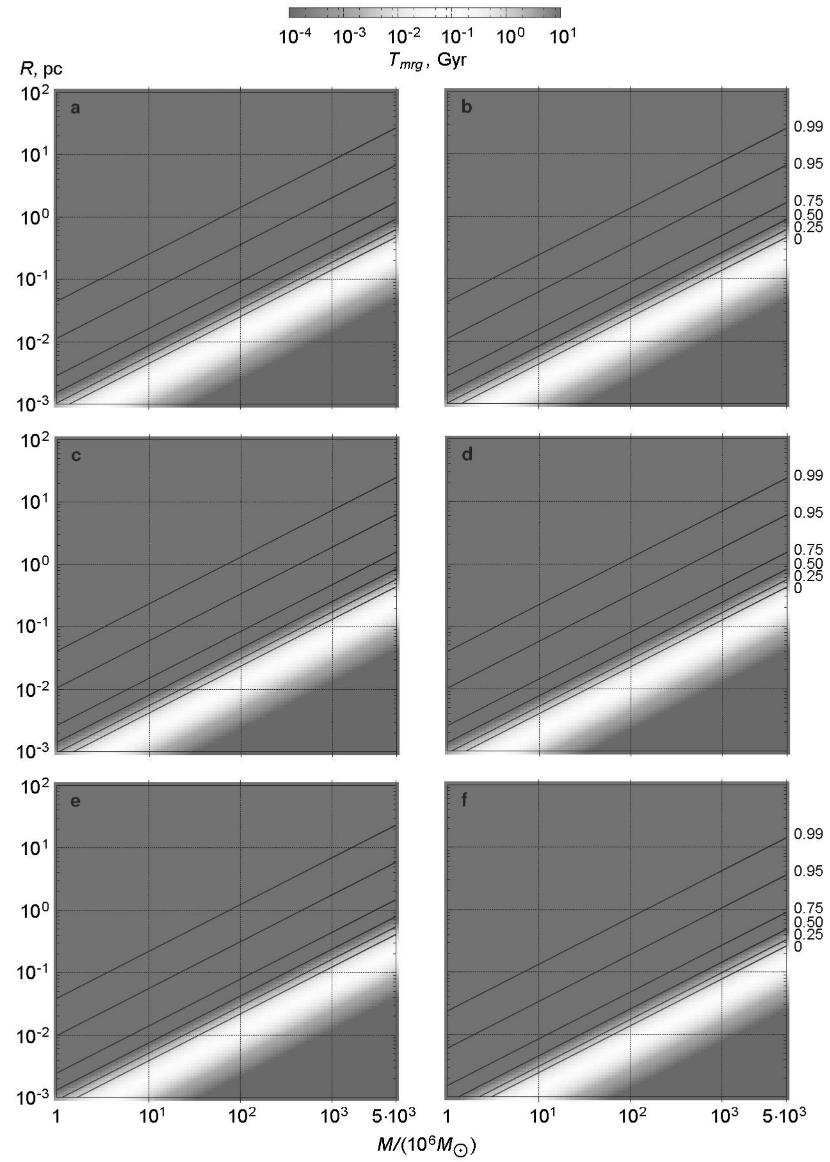

Fig. 3. The color coded final merging time T_{mrg} of SMBHB as a function of total mass and initial separation of the binary. Each separate plot shows the merging time evolution for the specific mass ratio of the binary: $q = 1$ (*a*), 0.5 (*b*), 0.333 (*c*), 0.25 (*d*), 0.2 (*e*), 0.02 (*f*). On each plots we indicate the 1 Gyr merging time line as a function of the initial eccentricity e_0 of the binary. Colored gamma for value $e_0 = 0.00$

have a longer merging time. For higher mass ratios even the eccentric orbits become more stable.

Comparison of the simulation results and theoretical work [58, 59] (which includes in the expressions only for the 2.5$\mathcal{PN}$ term) is shown in (Fig. 5, 6). For this numerical test we use the parameters $M_{tot} = 2$ [NB], $q =$ $= 1$, $R = 1$ [NB], $e_0 = 0.7$, $c = 15$ [NB] and we also include only the 2.5$\mathcal{PN}$ term. Our test simulations show that the numerical model behaves very similar to the theoretical curve.

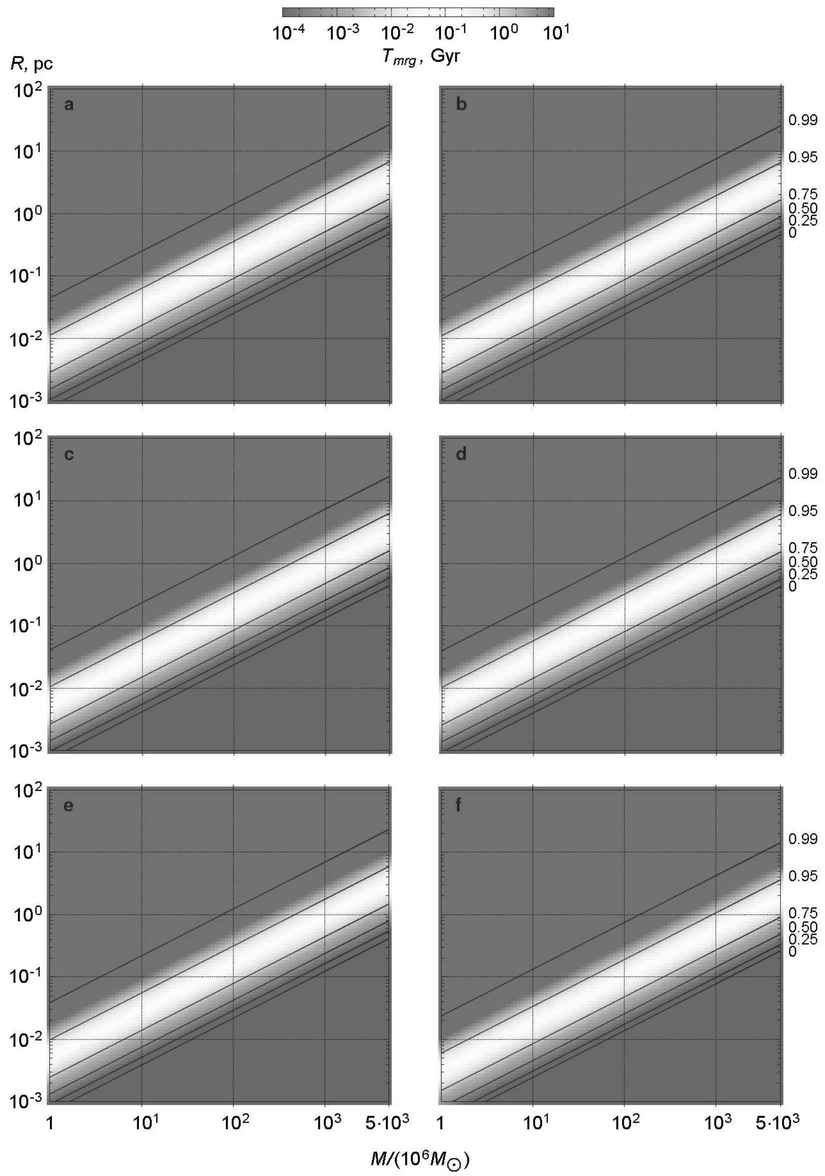

Fig. 4. The color coded final merging time T_{mrg} of SMBHB as a function of total mass and initial separation of the binary. Each separate plot shows the merging time evolution for the specific mass ratio of the binary: $q = 1$ (*a*), 0.5 (*b*), 0.333 (*c*), 0.25 (*d*), 0.2 (*e*), 0.02 (*f*). On each plots we indicate the 1 Gyr merging time line as a function of the initial eccentricity e_0 of the binary. Colored gamma for value $e_0 = 0.95$

For obtaining the GW constraints, for the selected test case ($M_{tot} = 10^8 M_{\odot}$, $q = 0.5$, $R = 0.01$ pc, $e_0 = 0.95$, $\mathbf{S}_1 = [0, 0, 1]$, $\mathbf{S}_2 = [0, 0, 1]$), we use the spin-spin and spin-orbit coupling which was described above [12]. In Fig. 7 we show the first periastron passes for h and h . In Fig. 8 we see the waveform during inspiraling just for h polarization (the h looks similar). In Table 2 we present the GW frequencies for BHs with typical masses and binary system orbital parameters.

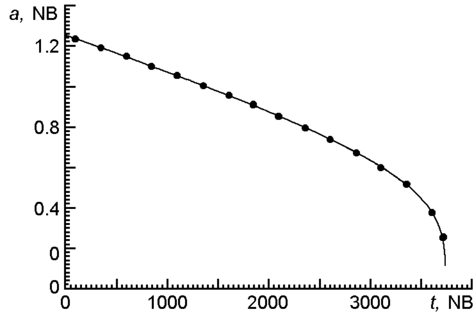

Fig. 5. Comparison the simulation's evolution (dots) of the semimajor axis *a* with analytical results (line) for a system with following initial parameters: $M_{tot} = 2$ [NB], $q = 1$, $R = 1$ [NB], $e_0 = 0.6$, $c = 15$ [NB] with just turning on 2.5𝒫𝒩

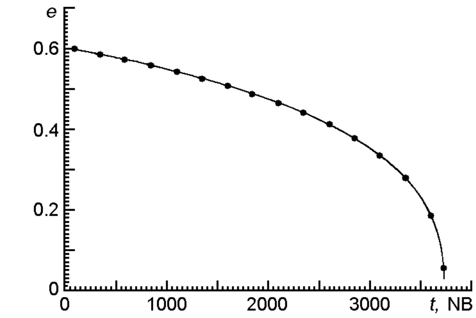

Fig. 6. Comparison the simulation's evolution (dots) of the eccentricity *e* with analytical results (line) for a system with following initial parameters: $M_{tot} = 2$ [NB], $q = 1$, $R = 1$ [NB], $e_0 = 0.6$, $c = 15$ [NB] with just turning on 2.5𝒫𝒩

***Table 2.* The GW frequency for BHs with typical masses M_{tot} and system parameters $q = 0.5$, $e_0 = 0.95$, $R = 0.01$ pc, S_1=[0, 0, 1], S_2=[0, 0, 1]**

$M_{tot}/M_\odot$	R (r_S)	T_{mrg}, yr	T_{orb}, s	, Hz
10^9	104	0.6	866925	1.15
10^8	1045	631.5	55175	18.1
10^7	10451	655148.3	5933	169

***Table 3.* Configurations of the systems for SDSS objects from [35] ($q = 1$)**

SDSS ID	z	log ($M_{tot}/M_\odot$)	$r_{\max}$, mpc
J075700.70+424814.5	1.17	9.1311	20
J002444.11+003221.4	0.40	9.5618	102
J004918.98+002609.4	1.94	9.3148	96
J161609.50+434146.8	0.49	8.1696	21
J093502.54+433110.7	0.46	9.3425	181
J032223.02-000803.5	0.62	8.2827	32
J095656.42+535023.2	0.61	8.2944	127

Using our well tested 𝒫𝒩-routine we estimate the possible BHBs merging time for the set of SDSS objects [35]. The main parameters of the binary BHs we present in Table 3. We estimate binary BH expected merging times assuming different eccentricities (e_0 = 0.00 — 0.99) of the orbits except J1201, for which we $e_0 = 0.3$. Also we calculated the merging time for the serendipitously discovered SDSS J120136.02 + 3000305.5 ($z = 0.146$) with system parameters $M_{tot} = 1.08\ \ 10^7 M_\odot$, $q = 0.08$, $r_{\max} = 1.3$ mpc, $e_0 = 0.3$ [47].

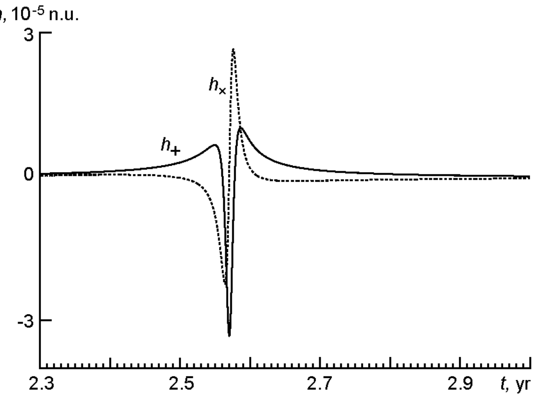

Fig. 7. Simulated strain from a *GW* during the first periastron for a system with $M_{tot} = 10^8 M_\odot$ (q = 0.5), $e_0 = 0.95$, $R = 0.01$ pc, $\mathbf{S}_1 = [0, 0, 1]$, $\mathbf{S}_2 = [0, 0, 1]$

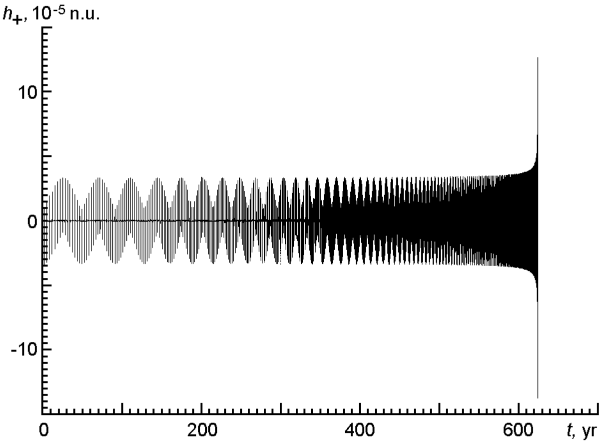

Fig. 8. Strain for a system with parameters $M_{tot} = 10^8 M_\odot$ ($q = 0.5$), $e_0 = 0.95$, $R = 0.01$ pc, $\mathbf{S}_1 = [0, 0, 1]$, $\mathbf{S}_2 = [0, 0, 1]$

Table 4. **Expected merging time T_{mrg} for SMBHBs for the selected SDSS objects as the function of the eccentricities**

e_0	T_{mrg}, yr						
	J0757	J0024	J0049	J1616	J0935	J0322	J0956
0.00	2.139E+04	6.906E+05	2.916E+06	1.767E+07	2.997E+07	4.390E+07	9.929E+09
0.25	6.796E+03	2.213E+05	9.385E+05	5.714E+06	9.672E+06	1.420E+07	3.214E+09
0.50	1.439E+03	4.738E+04	2.023E+05	1.241E+06	2.095E+06	3.086E+06	7.004E+08
0.75	1.221E+02	3.873E+03	1.668E+04	1.040E+05	1.745E+05	2.591E+05	5.920E+07
0.90	9.528E-01	6.635E+00	5.014E+02	4.195E+02	7.413E+02	1.046E+03	2.453E+05
0.99	9.255E-01	6.435E+00	7.780E+00	2.960E+00	1.946E+01	4.893E+00	1.007E+03

As we can see from Table 4 some of the selected SDSS objects have a quite short merging time even for moderately large eccentricities e_0 0.75. Almost all of the selected objects (except one J0956) have expected merging times only a few years for initial eccentricities $e_0 = 0$. However J1201 has an estimated $T_{mrg} = 3.27$ Myr, that is not such a gratifying result. Hopefully our merging time predictions can be tested with the larger SDSS4 observational catalogues, which are right now in a phase of observation.

CONCLUSION

In our study we analyze the dynamical behavior of SMBHBs. We use a highly accurate direct 2-body code where we apply the additional $\mathcal{PN}$ terms up to 3.5$\mathcal{PN}$ for calculation of the gravitational forces which act on the BHs and spin-spin and spin-orbit coupling for calculation of GW constraints. As the main result we obtain the resulting merging time T_{mrg} for a large set of initial mass ratios q of the BBH, initial masses, initial separations and orbital eccentricities e_0. This data we present as a set of color coded 3-D plots. We also make the original results presented on these plots for different mass ratios q and initial eccentricities e_0 publicly available*. Our $\mathcal{PN}$ treatment was extensively tested and the $\mathcal{PN}$ routines itself we also make publicly available via the same link above. In our high order direct 2-body implementation we use not only the $\mathcal{PN}$ accelerations but also the first derivatives of this accelerations. Our BHBs test calculations show that for BH masses in range $M_{tot} = (10^6 — 10^9)M_\odot$ with a fixed initial separation $R = 0.01$ pc and initial eccentricity $e_0 = 0.95$ the GW frequencies are well inside the LISA sensitivity band (Table 2) [9]. We use our $\mathcal{PN}$ routines to approximate the expected merging time for the selected sample SDSS SMBHBs [35]. Our results show that for significantly large eccentricities the expected merging time for these objects are in the range of years.

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* ftp://ftp.mao.kiev.ua/pub/sobolenko/

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