ELECTRON ENERGY PROBABILITY FUNCTION AND DUST CHARGE IN THE TEMPORAL AFTERGLOW OF A PLASMA WITH LARGE DUST DENSITY

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Analytical expressions for the electron energy probability function (EEPF) in an argon plasma afterglow with large dust density, which are obtained from the homogeneous Boltzmann equation for different steady-state EEPFs (including both Maxwellian and Druyvesteyn distributions at electron energies larger than the dust-surface potential), are presented. The casewhen the rate for electron-neutral momentum-transfer collisions is independent of the electron energy is considered. It is analyzed how the EEPF shape depends on the afterglow time and the decay time of dust charge. It is also found how the decay time of dust charge depends on the decay time of effective electron temperature and that of electron density. The conditions when the energy derivative of the EEPF may be positive are obtained.

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INTRODUCTION

Dusty plasmas have been extensively studied for several decades [1-3]. The plasmas have been analyzed in glow and afterglow regimes. However, in most theoretical studies of dusty plasma afterglows [4,5], the hydrodynamic approach and Maxwellian electron energy probability functions (EEPFs) have been adopted. Meantime, in practice, the EEPF in afterglow plasmas is usually non-Maxwellian [6, 7], that can affect the electron and ion densities, the effective electron temperature, the reaction rates, the dust charging, the ion/electron fluxes to the processed surface, as well as plasma chemistry [8]. Therefore, determination of the EEPF profile is important for applications of dusty plasmas in different technologies, including nanotechnologies.

At present, there are only a few studies devoted to the EEPF in plasma afterglows with high dust density [9-12]. In [9] and [10], the influence of nanoparticles on an argon discharge plasma afterglow with large metastable atom density was studied using particle-in-cell with Monte Carlo-collisions simulations. Analytical results on the EEPF in a dusty plasma afterglow were presented in our recent works [11, 12]. However, in the previous studies it was not analyzed in detail how the EEPF shape depends on the decay time for dust charge, as well as how the decay time for dust charge is connected with the decay times for effective electron temperature and electron density. In this paper, we study these important questions.

1. MAIN EQUATIONS AND ASSUMPTIONS

Consider a plasma in argon consisting of electrons, singly-charged positive ions with number densities n_e and n_i, respectively, and negatively charged dust particles with density n_d, radius a_d and charge Z_d (in units of electron charge e). The plasma is assumed to be quasineutral, i.e., $n_e + n_d|Z_d| = n_i$. We also assume that the ions have Maxwellian distribution with temperature T_i, but the EEPF F is in general non-Maxwellian and satisfies the homogeneous Boltzmann equation [6]:

$$\frac{\partial F(u,t)}{\partial t} - \frac{2e}{3m_e\sqrt{u}}\frac{\partial}{\partial u}\left[\frac{u^{3/2}}{\nu_m}E^2\frac{\partial F(u,t)}{\partial u}\right] \approx S(F), \quad (1)$$

where u is the electron energy (in eV), t is the time, m_e is the electron mass, E is an external electric field sustaining the plasma, and $S(F) = S_{ea}(F)+S_{ed}(F)$. Here, the terms $S_{ea}(F)$ and $S_{ed}(F)$ describe the electron-atom and electron-dust collisions, respectively, ν_m is the effective rate of momentum transfer for collisions of electrons with neutrals and dust particles. In the case of a plasma afterglow, $E = 0$ and the second term on the left-hand side of equation (1) vanishes.

We consider afterglow times larger than $<\nu_{1\Sigma}(u)>^{-1}$, where $\nu_{1\Sigma}(u)$ is the frequency of all inelastic electron-atom collisions. The angular brackets denote the energy-averaged values. It is also assumed that the electron energy loss in elastic electron-atom collisions is larger than that in elastic collisions of electrons with dust particles.

In this case, equation (1) can be simplified to [12]

$$\frac{\partial F(u,t)}{\partial t} - \frac{1}{\sqrt{u}}\frac{\partial}{\partial u}\left[\delta\nu(u)u^{3/2}F(u,t)\right] + F(u,t)\nu_{ed}^c(u,t) = 0, \quad (2)$$

where $\nu_{ed}^c(u,t) = \alpha[1-\varphi_s(t)/u]\sqrt{u}$ for $u > \varphi_s(t)$, and $\nu_{ed}^c = 0$ for $u < \varphi_s(t)$. $\varphi_s(t) = e^2|Z_d(t)|/a_d$, $\delta = 2m_e/m_i$, m_i is the ion mass, $\alpha = n_d\pi a_d^2\sqrt{2\times1.6\times10^{-12}/m_e}$ with n_d in cm^{-3}, a_d in cm^2, and m_e in g. ν is the rate for electron-neutral momentum-transfer collisions. Here, it is assumed that the time-dependencies for dust charge $Z_d(t)$ and dust surface potential $\varphi_s(t)$ are exponential: $Z_d(t) = Z_{d0}\exp(-t/\gamma)$, $\varphi_s(t) = \varphi_{s0}\exp(-t/\gamma)$, where $\varphi_{s0} = \varphi_s(t=0)$, $Z_{d0} = Z_d(t=0)$, and γ is the time characterizing the dust charge decrease in afterglow.

To calculate the EEPF in a dusty plasma afterglow, one has to know the dust charge at t = 0 and its decay time γ. The dust charge is found using the orbit-motion- limited theory [1], accounting for the non-Maxwellian character of the EEPF as well as ion-neutral collisions in the sheath of the dust particle. The decay time γ is a parameter here.

We assume that the profile of the EEPF for $u > \varphi_s(t)$ at t = 0 corresponds to the case when the velocity space is isotropic, and [13]

$$F(u,0) = A_1 \exp(-A_2 u^x), \qquad (3)$$

where $A_1$and A_2 are constants, x = 1 and x = 2 for Maxwellian and Druyvesteyn electron energy distributions, respectively.

In solving equation (2) analytically by the method of characteristics [14], we also assume that the rate for electron-neutral momentum-transfer collisions is independent of the electron energy.

2. ANALYTICAL AND NUMERICAL RESULTS

Using the method of characteristics, one can find analytical solutions of equation (2) for the EEPF in the energy regimes $u < \varphi_s(t)$ and $u > \varphi_s(t)$ [12]:

$$F(u,t) = A_1 \exp\Big[3\delta\nu t/2 - A_2 u^x e^{x\delta\nu t} + \frac{2\alpha_1\gamma}{2-\beta}\sqrt{\frac{\varphi_{s0}}{u}}e^{-\frac{t\delta\nu}{2}} - \frac{2\alpha_1\gamma}{\beta}\sqrt{\frac{u}{\varphi_{s0}}}e^{\frac{t\delta\nu}{2}} + \frac{4\alpha_1\gamma(1-\beta)}{\beta(2-\beta)}\left(\frac{u}{\varphi_{s0}}\right)^{\frac{1}{2(1-\beta)}} e^{\frac{\beta t}{2\gamma(1-\beta)}}\Big], \qquad (4)$$

$$F(u,t) = A_1 \exp\Big[3\delta\nu t/2 - A_2 u^x e^{x\delta\nu t} + \frac{2\alpha_1\gamma}{2-\beta}\left(e^{-t\delta\nu/2} - e^{-t/\gamma}\right)\sqrt{\frac{\varphi_{s0}}{u}} + \frac{2\alpha_1\gamma}{\beta}\left(1-e^{t\delta\nu/2}\right)\sqrt{\frac{u}{\varphi_{s0}}}\Big], \qquad (5)$$

where $\alpha_1 = n_d \pi a_d^2 \sqrt{2\varphi_{s0} \times 1.6\times10^{-12}/m_e}$, $\beta = \delta\nu\gamma$.

Note that the energy derivative of the EEPF in equation (4) can be positive. In fact, at t = 0, we have

$$\frac{\partial F(u,0)}{\partial u} = F(u,0)\times\Big[-A_2 x u^{x-1} + \frac{\alpha_1\gamma}{u}\Big(-\frac{1}{2-\beta}\sqrt{\frac{\varphi_{s0}}{u}} - \frac{1}{\beta}\sqrt{\frac{u}{\varphi_{s0}}} + \frac{2}{\beta(2-\beta)}\left(\frac{u}{\varphi_{s0}}\right)^{\frac{1}{2(1-\beta)}}\Big)\Big]. \qquad (6)$$

If the electron energy is close to φ_{s0}, we can write that $u/\varphi_{s0} = 1-\Delta$, where $\Delta << 1$, and

$$\frac{\partial F(u,0)}{\partial u} \approx F(u,0)\left\{-A_2 x u^{x-1} + \frac{\alpha_1\gamma\Delta}{u(\beta-1)}\right\}, \qquad (7)$$

where the second term in brackets is positive if $\beta > 1$. Therefore, for large dust radii and/or large dust densities, small u, and $\beta > 1$, the second term in the brackets of equation (7) can be larger than the absolute value of the first term. As a result, we have $\partial F(u,0)/\partial u > 0$.

Our numerical calculations also confirm that the derivative of F on energy may be positive if β>1, while $\partial F(u,0)/\partial u < 0$ if β>1 (Fig 1). The calculations in figure 1 were carried out for $\nu = \alpha(\langle u_0\rangle/u^*) n_a \sqrt{2e\langle u_0\rangle/m_e}$, were $\langle u_0\rangle$ = 3 eV, $\alpha = 1.59\times10^{-15}\,\mathrm{cm}^2$, u^*=11.55 eV, n_a is the argon atom density. It was assumed additionally that $n_e(0) = 10^9$ cm^{-3}, gas pressure P = 1 Torr, $a_d = 50$ nm and $n_d = 10^7$ cm^{-3}.

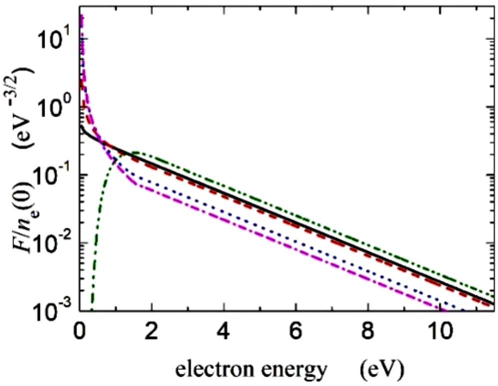

Fig. 1. The EEPF for t =0and different decay times of the dust charge: $\gamma = 0.1/\delta\nu$ (solid curve), $0.5/\delta\nu$ (dashed curve), $0.9/\delta\nu$ (dotted curve), $0.99/\delta\nu$ (dash-dotted curve), and $1.5/\delta\nu$ (dash-dot-dotted curve). The curves have been obtained assuming that the EEPF at $u > \varphi_{s0}$ is Maxwellian with temperature 2 eV

It was also found that the EEPF at $u > \varphi_s(t)$ and β>1 decreases with increasing γ. This agrees with our previous studies on the EEPF in afterglow plasmas for the case when the rate for electron-neutral momentum-transfer collisions is a function of the electron energy [12].

Note that the positive derivative $\partial F(u,t)/\partial u$ at β>1 may not be observed in experiments where the diffusion of electrons to the walls and/or the electron-electron collisions are important.

Next analyze how the decay time for dust charge in the plasma afterglow depends on the decay time for effective electron temperature τ_T and that for electron density τ_e. In this analysis, we will assume that

$$n_e(t) \approx n_{e0}\exp(-t/\tau_e),\ n_i(t) \approx n_{i0}\exp(-t/\tau_i),$$
$$T_{eff}(t) \approx T_{e0}\exp(-t/\tau_T),\ Z_d(t) = Z_{d0}\exp(-t/\gamma),$$

where $\tau_e \approx 1/\langle \nu_{ed}^c \rangle$, $\langle \nu_{ed}^c \rangle = \frac{1}{n_e}\int_0^\infty \nu_{ed}^c F\sqrt{u}du$ is the energy-averaged rate of collection of electrons by the dust particles. τ_i is the decay time for the ion density. n_{i0}, n_{e0}, T_{e0} and Z_{d0} are the ion density, electron density, effective electron temperature, and dust charge, respectively, at $t = 0$.

$T_{eff}(t) = (2/3)\int_0^\infty F(u,t)u^{3/2}du / n_e(t)$ is the effective electron temperature, $\tau_T \approx (\langle \nu_{ed}^{cT} \rangle + \langle \delta\nu^T \rangle)^{-1}$, where $\langle \nu_{ed}^{cT} \rangle = \int_0^\infty \nu_{ed}^c u^{3/2} F du / W$, $\langle \delta\nu^T \rangle = \int_0^\infty \delta\nu u^{3/2} F du / W$, and $W = (3/2) n_e T_{eff}$ is the average electron energy per unit volume.

The decay time for dust charge γ can be estimated by assuming that the EEPF is locally Maxwellian and the ion-neutral collisions in the dust sheath do not affect the dust charge. In this case, the ion current to a dust particle is

$I_i(t) \approx e n_i(t) a_d^2 (8\pi T_i / m_i)^{1/2} \left[1 + |Z_d(t)| e^2 / a_d T_i\right]$,

while the electron current is $I_e(t) \approx -e n_e(t) 4\pi a_d^2 \sqrt{\frac{T_{eff}(t)}{2\pi m_e}} \exp\left[\frac{-e^2|Z_d(t)|}{a_d T_{eff}(t)}\right]$. Using the expressions for the electron and ion currents and assuming $I_e(t) \approx -I_i(t)$, one can obtain

$$n_i(t)(T_i/m_i)^{1/2}\left[1 + \frac{|Z_d(t)|e^2}{a_d T_i}\right] \approx n_e(t)\left[T_{eff}(t)/m_e\right]^{1/2} \exp\left[\frac{-e^2|Z_d(t)|}{a_d T_{eff}(t)}\right]. \quad (8)$$

Next, we take the time-derivative from both sides of the quasineutrality equation ($n_e + n_d|Z_d| \approx n_i$). Assuming that the dust density is time-independent in the afterglow, we get for the decay time of the ion density

$$\tau_i \approx \left(\frac{n_{e0}}{n_{i0}\tau_e} + \frac{n_d|Z_{d0}|}{n_{i0}\gamma}\right)^{-1}. \quad (9)$$

Taking the time-derivative of equation (8) and accounting for equation (9), we obtain the decay time of dust charge in the plasma afterglow

$$\gamma = \left[\frac{n_d|Z_{d0}|/n_{i0} + \beta_i/(1+\beta_i) + \beta_e}{(n_d|Z_{d0}|/n_{i0})\tau_T/\tau_e + \beta_e + 0.5}\right]\tau_T \approx \left[\frac{n_d|Z_{d0}|/n_{i0} + 1 + \beta_e}{(n_d|Z_{d0}|/n_{i0})\tau_T/\tau_e + \beta_e + 0.5}\right]\tau_T, \quad (10)$$

where $\beta_e = e^2|Z_{d0}|/a_d T_{e0}$ and $\beta_i = e^2|Z_{d0}|/a_d T_i$.

For the case $(n_d|Z_{d0}|/n_{i0})\tau_T/\tau_e << 1$ and $1.5 < \beta_e < 5$ [15], one finds γ is slightly larger than τ_T. In this case, the condition $\beta > 1$ may be fulfilled and the energy derivative of EEPF at $u < \varphi_s(t)$ may be positive. If the dust density is sufficiently large ($\langle \nu_{ed}^{cT} \rangle \geq \langle \delta\nu^T \rangle$), the decay time of the dust charge is smaller than $1/\langle \delta\nu^T \rangle \sim 1/\delta\nu$, i.e., $\beta < 1$.

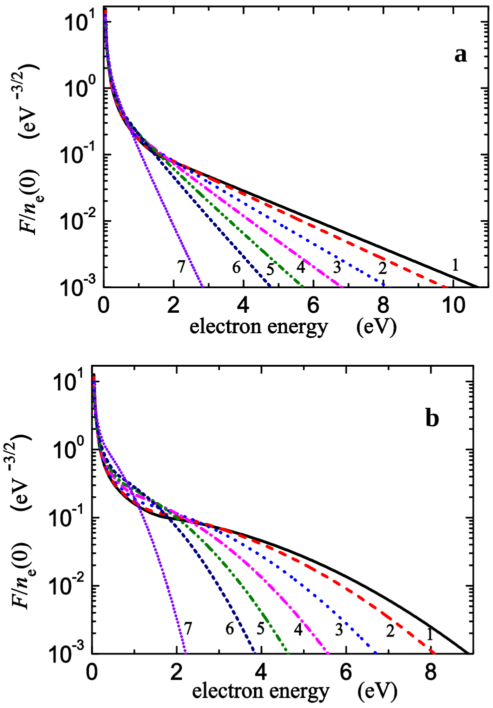

Fig. 2. The normalized EEPF for different afterglow times: $t = 0$ (curve 1), $0.1/\delta\nu$ (curve 2), $0.3/\delta\nu$ (curve 3), $0.5/\delta\nu$ (curve 4), $0.7/\delta\nu$ (curve 5), $0.9/\delta\nu$ (curve 6), and $1.5/\delta\nu$ (curve 7). The EEPFs in figures2 (a) and 2 (b) correspond to the initial Maxwellian and Druyvesteyn distributions for $u > \varphi_{s0}$, respectively. The results are obtained for $n_d = 5\times10^6$ cm^{-3} and $a_d = 50$ nm, $n_e(0) = 10^9$ cm^{-3}, P = 1 Torr and T_{eff} (t = 0) = 2eV

We also analyzed how the EEPF shape depends on time. In Fig. 2, the EEPFs at different afterglow times are given. One can see in Fig. 2 that the profile of the EEPF at $u > \varphi_s(t)$ changes only slightly in time, i.e., if at $t = 0$ the EEPF is Maxwellian, it tends to stay near Maxwellian during the afterglow, but with a new effective electron temperature that decreases with time, analogous to the dust-free case [16]. In our opinion, the preservation of the EEPF profile for $u > \varphi_s(t)$ is due to the fact that at large electron energies the term $A_2 u^x e^{x\delta\nu t}$ in equation(5) dominates over the third and fourth terms in the brackets of equation (5). Similarly to the dust-free case [16], in the $n_d \neq 0$case, the derivative $\partial F / \partial u$ at $\nu = \text{const}$ is always negative, independent on t and u.

CONCLUSIONS

Thus, we have presented analytical expressions for the EEPF in an argon plasma afterglow with large dust density. The expressions were obtained from the homogeneous Boltzmann equation for the electrons for different steady-state EEPFs, including both Maxwellian and Druyvesteyn distributions at electron energies larger than the dust-surface potential. The casewhen the rate for electron-neutral momentum-transfer collisions is independent of the electron energy has been considered. It has been analyzed how the EEPF shape depends on the afterglow time and the decay time of dust charge. It has also been found how the decay time of dust charge depends on the decay time of effective electron temperature and that of electron density. The conditions when the energy derivative of the EEPF may be positive have been obtained. The results here are relevant to many plasma applications [17, 18], especially gas discharge plasmas used for synthesis of novel nanomaterials.

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ФУНКЦИЯ РАСПРЕДЕЛЕНИЯ ЭЛЕКТРОНОВ ПО ЭНЕРГИИ И ЗАРЯД ПЫЛЕВЫХ ЧАСТИЦ В ПЛАЗМЕ ПОСЛЕСВЕЧЕНИЯ С БОЛЬШОЙ ПЛОТНОСТЬЮ ПЫЛЕВЫХ ЧАСТИЦ

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Представлены аналитические выражения для функции распределения электронов по энергии (ФРЭЭ) в аргоновой плазме с большой плотностью пылевых частиц в режиме послесвечения. Выражения получены из однородного уравнения Больцмана для электронов в случае разных начальных ФРЭЭ (включая распределения Максвелла и Дрюйвестейна при энергиях электронов больших чем потенциальная энергия поверхности пылевых частиц). Рассмотрен случай, когда частота упругих столкновений электронов с атомами аргона не зависит от энергии электронов. Проанализировано, как форма ФРЭЭ зависит от времени послесвечения плазмы и времени, характеризующего уменьшение заряда пылевых частиц. Найдено также, как время, характеризующее уменьшение заряда пылевых частиц, зависит от времен, характеризующих спадание эффективной температуры и плотности электронов. Найдены условия, когда производная ФРЭЭ по энергии может быть положительной.

ФУНКЦІЯ РОЗПОДІЛУ ЕЛЕКТРОНІВ ЗА ЕНЕРГІЄЮ ТА ЗАРЯД ПИЛОВИХ ЧАСТИНОК У ПЛАЗМІ, ЩО Є В РЕЖИМІ ПІСЛЯСВІТІННЯ ТА МАЄ ВИСОКУ ГУСТИНУ ПИЛОВИХ ЧАСТИНОК

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Представлено аналітичні вирази для функції розподілу електронів за енергією (ФРЕЕ) в аргоновій плазмі з великою густиною пилових частинок, що є в режимі післясвітіння. Вирази отримано з однорідного рівняння Больцмана для електронів з різними початковими ФРЕЕ (включно з розподілами Максвелла та Дрюйвестейна за енергій електронів, які є більшими за потенціальну енергію поверхні пилових частинок). Розглянуто випадок, коли частота пружних зіткнень електронів з атомами аргону не залежить від енергії електронів. Проаналізовано, як форма ФРЕЕ залежить від часу післясвітіння плазми та часу, що характеризує зменшення заряду пилових частинок. Знайдено також, як час, що характеризує зменшення заряду пилових частинок, залежить від часів, що характеризують спадання ефективної температури та густини електронів. Знайдено умови, коли похідна ФРЕЕ за енергією може бути додатною.