ELECTROMAGNETIC MODEL OF GAS DISCHARGE IN LONG TUBE OF SLIGHTLY VARYING RADIUS

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The aim of this research is the construction of the electromagnetic model of the stationary state gas discharges in the diffusion regime in rather long narrow tubes of slightly variable radius in order to obtain plasma density axial distribution. In the framework of this model it was carried out the study of the external magnetic field value on the axial structure of gas discharge in slightly tapered and slightly divergent metal waveguides. The modeling has shown the difference of plasma density axial distribution for the cases of waveguide with constant and varying radius.

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INTRODUCTION

The modern technologies require plasma sources with the specified plasma density value and uniformity in axial direction [1]. A characteristic feature of microwave gas discharges sustained by surface waves (SW) is that they are sustained by eigenwaves of discharge system. So, one of the approaches to describe stationary plasma density distribution in rather long discharges is so called electromagnetic model. Such model consists of the detailed equations that describe the wave propagation and the model equation that describes the discharge features [2]. Such models were widely used to describe the axial structure of stationary state gas discharge in long discharge chambers [1-3].

The application of metal waveguides with slight variation of the waveguide radius along the discharge is one of the possible mechanisms of controlling the axial distribution of plasma density [4]. The aim of this work is the studying the influence of variable radius of metal waveguides on the properties of the discharge that is sustained by the symmetric and dipolar modes. The choice of these modes was stipulated by its availability for discharge sustaining [1, 2].

1. BASIC EQUATIONS

It was considered the diffusion regime of the discharge in long discharge structure. The studied wave propagates along the waveguide that consists of plasma column of radius R_{pl}, which is enclosed by the cylindrical metal wall of radius R. The vacuum gap ($R_{pl} < r < R$) separates the plasma column from waveguide metal wall. External steady magnetic field $\vec{B}_0 = (0, 0, B_0)$ is directed along the axis of the structure. Plasma was considered in the hydrodynamic approach as a cold weakly absorbing media with constant effective collisional frequency $\nu = \omega$, where ω is the wave frequency [4]. The case when only one mode with the specified azimuthal wave number is excited in the discharge structure is considered. It was supposed that all geometric, plasma and wave parameters slightly vary in axial direction on the distances of wavelength order, so the WKB approach can be used to obtain the solution of the equations [5]. The solutions of the system of Maxwell equations can be found in the form

$$E, B_{r,\varphi,z}(r,\varphi,z) = E, B_{r,\varphi,z}(r,z)\,\mathrm{e}^{-i\omega t + im\varphi + \int\limits_{z_0}^{z} k_3(z')\,dz'}, \quad (1)$$

here k_3 is the axial wavenumber, E, B – amplitude of electric and magnetic wave fields, respectively. Changing the value of variable A along the discharge at the distances of the order of wavelength is small compared to the magnitude of this variable, therefore $A^{-1}\dfrac{\partial A}{\partial z} << k_3$, where symbol A denotes E, B, k_3 or n. All terms of order $O\left(\dfrac{1}{k}\dfrac{\partial}{\partial z}\ln\left(A\right)\right)$ are neglected in the result system of equations [5]. In such case axial wave components in plasma region have the form

$$\begin{cases} E_z^{pl}(r) &= \sum\limits_{s=1}^{2} C_s I_m(k_s r), \\ B_z^{pl}(r) &= i\dfrac{\varepsilon_1}{\varepsilon_2 k k_3}\sum\limits_{s=1}^{2} C_s F_s I_m(k_s r), \end{cases} \quad (2)$$

where $F_s = \varepsilon_3(k_3^2 - k^2\varepsilon_1)\varepsilon^{-1} - k_s^2$, $k_s^2 = p_1 \pm \sqrt{p_1^2 - p_2}$, $p_1 = [(\varepsilon_1 + \varepsilon_3)(k_3^2 - k^2\varepsilon_1) + k^2\varepsilon_2^2](2\varepsilon_1)^{-1}$, $p_2 = \varepsilon_3[(k_3^2 - k^2\varepsilon_1)^2 - k^4\varepsilon_2^2]$, $\varepsilon_{1,2,3}$ are the components of permittivity tensor of magnetized collisional plasma [6] and C_s – field constants. Other wave field components in plasma region have the form

$$\begin{cases} E_r^{pl} = -\dfrac{i}{k_3}\sum\limits_{s=1}^{2}\dfrac{k^2\varepsilon_3 + k_s^2}{k_s} C_s I_m^{'}(k_s r) + i\dfrac{m}{r}\dfrac{\varepsilon_1}{k_3\varepsilon_2}\sum\limits_{s=1}^{2}\dfrac{F_s}{k_s^2} C_s I_m(k_s r), \\ E_\varphi^{pl} = -\dfrac{\varepsilon_1}{k_3\varepsilon_2}\sum\limits_{s=1}^{2}\dfrac{F_s}{k_s} C_s I_m^{'}(k_s r) + \dfrac{m}{r}\dfrac{1}{k_3}\sum\limits_{s=1}^{2}\dfrac{k^2\varepsilon_3 + k_s^2}{k_s^2} C_s I_m(k_s r), \\ B_r^{pl} = \dfrac{\varepsilon_1}{k\varepsilon_2}\sum\limits_{s=1}^{2}\dfrac{F_s}{k_s} C_s I_m^{'}(k_s r) - \dfrac{m}{r} k\varepsilon_3 \sum\limits_{s=1}^{2}\dfrac{C_s}{k_s^2} I_m(k_s r), \\ B_\varphi^{pl} = -ik\varepsilon_3\sum\limits_{s=1}^{2}\dfrac{C_s}{k_s} I_m^{'}(k_s r) + i\dfrac{m}{r}\dfrac{\varepsilon_1}{k\,\varepsilon_2}\sum\limits_{s=1}^{2}\dfrac{F_s}{k_s^2} C_s I_m(k_s r), \end{cases} \quad (3)$$

where $I_m^{'}$ – modified Bessel function of the first order, and a stroke denotes the derivative by argument.

Taking into account boundary conditions on waveguide metal wall, one can present the axial wave field components in vacuum region

$$\begin{cases} E_z^V(r) = AG(\psi r), \\ H_z^V(r) = BQ(\psi r), \end{cases}, \tag{4}$$

where $\psi^2 = k_3^2 - k^2$, A and B – constants, and functions $G(\psi r)$ and $Q(\psi r)$ are equal to

$$\begin{cases} G(\psi r) = \dfrac{K_m(\psi r)}{K_m(\psi R)} - \dfrac{I_m(\psi r)}{I_m(\psi R)}, \\ Q(\psi r) = \dfrac{K_m(\psi r)}{K_m^{'}(\psi R)} - \dfrac{I_m(\psi r)}{I_m^{'}(\psi R)}, \end{cases} \tag{5}$$

where K_m – modified Bessel functions of the second kind. Then other wave field components in vacuum region have the following form

$$\begin{cases} E_r^V = \dfrac{1}{\psi^2}\left\{k\dfrac{m}{r}BQ(\psi r) - i\psi k_3 AG(\psi r)\right\}, \\ E_\varphi^V = \dfrac{1}{\psi^2}\left\{k_3\dfrac{m}{r}AG(\psi r) + i\psi kBQ'(\psi r)\right\}, \\ B_r^V = -\dfrac{1}{\psi^2}\left\{k\dfrac{m}{r}AG(\psi r) + i\psi k_3 BQ'(\psi r)\right\}, \\ B_\varphi^V = \dfrac{1}{\psi^2}\left\{k_3\dfrac{m}{r}BQ(\psi r) - i\psi kAG'(\psi r)\right\}. \end{cases} \tag{6}$$

Using the continuity of tangential wave field components at the plasma-vacuum interface ($r = R_{pl}$) one can obtain the local dispersion equation

$$\frac{(Z_1 - Y_1 + e_1)(Z_2 - \varepsilon_3 Y_2 + h_2)}{(Z_1 - Y_2 + e_2)(Z_2 - \varepsilon_3 Y_1 + h_1)} = \frac{F_2}{F_1}, \tag{7}$$

where $Z_1 = \dfrac{k}{\psi}\dfrac{Q'(\psi R_{pl})}{Q(\psi R_{pl})}$, $Z_2 = \dfrac{k}{\psi}\dfrac{G'(\psi R_{pl})}{G(\psi R_{pl})}$,

$$Y_s = \frac{k}{k_s}\frac{I_m^{'}(k_s R_{pl})}{I_m(k_s R_{pl})},\; e_s = \frac{m}{R_{pl}}\frac{k^3\varepsilon_2}{\varepsilon_1 F_s}\left[\frac{\varepsilon_3}{k_s^2} - \frac{1}{\psi^2}\right],$$

$$h_s = \frac{m}{R_{pl}}\frac{\varepsilon_1}{k\varepsilon_2}F_s\left[\frac{1}{k_s^2} - \frac{1}{\psi^2}\right].$$

Using the continuity of tangential wave field components at the plasma-vacuum interface ($r = R_{pl}$) it is possible to find field constants A, B, $C_{1,2}$

$$A = \frac{E_z(R_{pl})}{G(\psi R_{pL})},\; B = i\frac{\varepsilon_1}{\varepsilon_2 k k_3}\sum_{s=1}^{2} F_s C_s \frac{I_m(k_s R_{pl})}{Q(\psi R_{pl})},$$

$$C_1 = -\frac{Z_2 - \varepsilon_3 Y_2 + h_2}{\varepsilon_3[Y_2 - Y_1] - [h_2 - h_1]}\frac{E_z(R_{pl})}{I_m(k_1 R_{pl})}, \tag{8}$$

$$C_2 = \frac{Z_2 - \varepsilon_3 Y_1 + h_1}{\varepsilon_3[Y_2 - Y_1] - [h_2 - h_1]}\frac{E_z(R_{pl})}{I_m(k_2 R_{pl})}.$$

The electromagnetic model that describes the stationary state of gas discharge except the equations for the wave includes the energy balance equation along the discharge and the model equation that connects wave power absorbed per unit length of discharge with local plasma density [1, 2]. The axial component of wave energy flux has the following form:

$$S_z = \frac{c}{8\pi}\mathrm{Re}\{2\pi\int_0^{R_{pl}}(E_r^{pl*}B_\varphi^{pl} - E_\varphi^{pl*}B_r^{pl})rdr + 2\pi\int_{R_{pl}}^{R}(E_r^{V*}B_\varphi^{V} - E_\varphi^{V*}B_r^{V})rdr\}, \tag{9}$$

where asterisk denotes complex conjugate.

Due to electron collisions with neutral particles some part of wave energy is absorbed in plasma and is consumed to excitation, ionization, heating of the neutral gas and other. It is possible to write down the energy that is absorbed per unit length of the discharge

$$Q = i\frac{\omega}{8}\left(\varepsilon_{ji}^* - \varepsilon_{ij}\right)\int_0^{R_{pl}} E_i^* E_j \; r\,dr. \tag{10}$$

The equation of wave energy balance along the discharge can be written as [2]:

$$\frac{dS_z(z)}{dz} + Q = 0. \tag{11}$$

Here $S_z(z)$ denotes such SW energy flux that is necessary for sustaining of plasma column from coordinate z up to the end of the discharge. Thus the end of the discharge is determined as a value of axial coordinate where $S_z = 0$. It is also necessary to use one more equation that connects wave power absorbed per unit length of discharge Q with local plasma density n_0. Such equation is determined by the kinetics of the discharge and can be written as [2]:

$$Q = Q_\beta N^{1+\beta}, \tag{12}$$

where $N = \omega_{pe}^2\omega^{-2}$ – dimensionless plasma density, Q_β – constant of proportionality that does not depend on axial coordinate. The parameter β is determined by the regime of the discharge. For the diffusion (free fall) gas discharge regime $\beta = 0$. This can be explained by the fact that in the case when plasma is produced mainly due to one step ionization when ionization velocity is proportional to n_0, therefore $Q \sim n_0$ [2].

2. MAIN RESULTS

The SW that sustains the discharge is the eigenwave of discharge structure on the whole length of the plasma column. So, the possibility of axial plasma source parameters variation is mainly determined by dispersion properties of the wave sustaining the discharge. Such circumstances determined the detailed study of the dispersion properties of the electromagnetic SW with azimuth wavenumbers $m = 0, \pm 1$, that widely used for discharge sustaining. The results of the solution of the local dispersion equation (7) for different external magnetic field values ($\Omega = \omega_{ce}\omega^{-1}$, ω_{ce} – electron cyclotron frequency) are presented on the Figs. 1-3.

The increase of the external magnetic field value leads to the increase of the dimensionless wave frequency $\omega\omega_{pe}^{-1}$ for the wave with $m = 0,1$ in the whole range of the existence (see Figs. 1, 2). The wave with $m = -1$ possesses different behavior – the dimensionless frequency increased with the increase of the external magnetic field value when axial wavenumber is rather high ($k_3R_{pl} \geq 4$) and decreased when $k_3R_{pl} < 4$ (see Fig. 3).

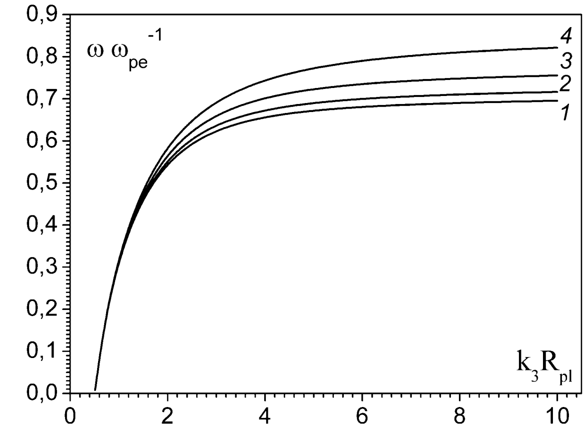

Fig. 1. The solutions of the equation (7) for $m = 0$ *mode when* $\eta = R / R_{pl} = 1.5$ *,* $\sigma = R_{pl}\omega / c = 0.5$ *. Numbers of curves correspond to different* Ω *values: 1 –* $\Omega = 0.2$ *; 2 –* $\Omega = 0.4$ *; 3 –* $\Omega = 0.6$ *; 4 –* $\Omega = 0.8$

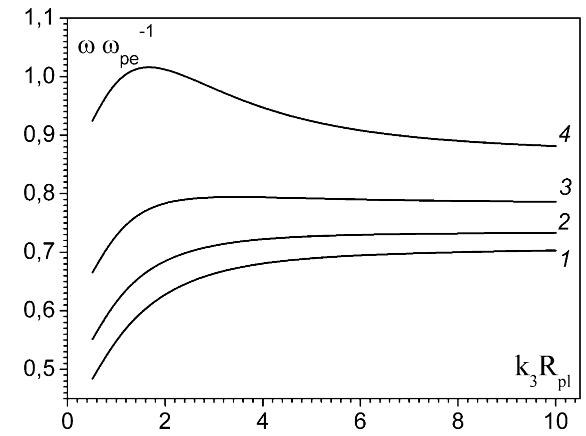

Fig. 2. The solutions of the equation (7) for $m = 1$ *mode. Parameters and numbering of the curves are the same as for the Fig. 1*

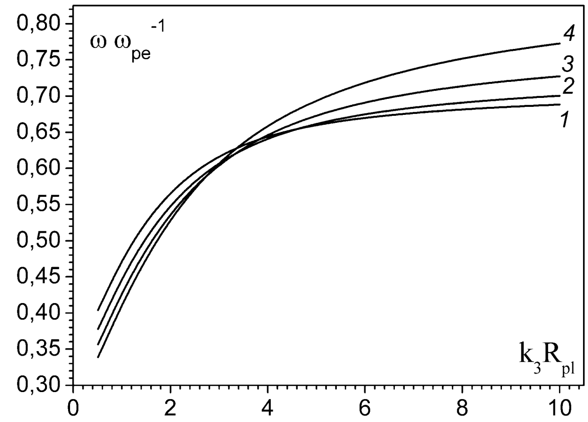

Fig. 3. The solutions of the equation (7) for $m = -1$ *mode. Parameters and numbering of the curves are the same as for the Fig. 1*

It is supposed that the eigenwave frequency ω is fixed and is determined by the wave generator. So, changing of the dimensionless parameter ω / ω_{pe} occurs due to the slight variation of plasma density along the discharge structure (through the value of ω_{pe}). Thus, it is possible to estimate maximum possible reachable plasma density value ($N = \omega_{pe}^2 \omega^{-2}$) in the steady state of the discharge that is sustained by the chosen mode. It is shown that the SW sustained discharges possess the maximum plasma density for the mode with $m = 0$.

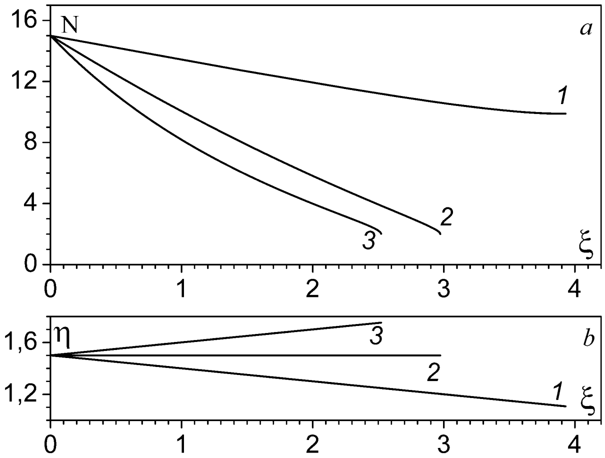

Fig. 4. Axial structure of the SW sustained discharge for $m = 0$ *(a) and the normalized radius* η *variation (b);* $\sigma = 0.5$ *,* $\Omega = 0.2$ *,* $\eta = 1.5$

The plasma density axial profiles for the discharges that are sustained by the studied modes are presented on the Figs. 4-6.

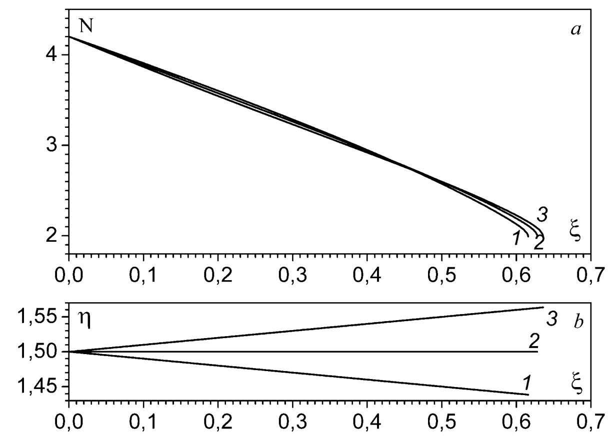

Fig. 5. Axial structure of the SW sustained discharge for $m = 1$ *(a) and the normalized radius* η *variation (b);* $\sigma = 0.5$ *,* $\Omega = 0.2$ *,* $\eta = 1.5$

The value of dimensionless axial coordinate $\xi = \nu z / (\omega R_{pl})$ where the total wave energy flux in plasma and vacuum become equal to zero was chosen as the dimensionless discharge length L (z – axial coordinate that is measured from the generator up to the end of discharge).

The dimensionless plasma density N axial profile in the discharge sustained by the $m = 0, \pm 1$ modes are presented on the Figs. 4,a; 5,a; 6,a. The corresponded normalized radius η variation is represented on Figs. 4,b; 5,b; 6,b. The study was carried out for $\eta = 1.5$ near the wave generator. It were studied the linearly decreased (line 1) and linearly increased (line 3) profiles. For comparison the results when η is steady along the discharge are also presented (line 2).

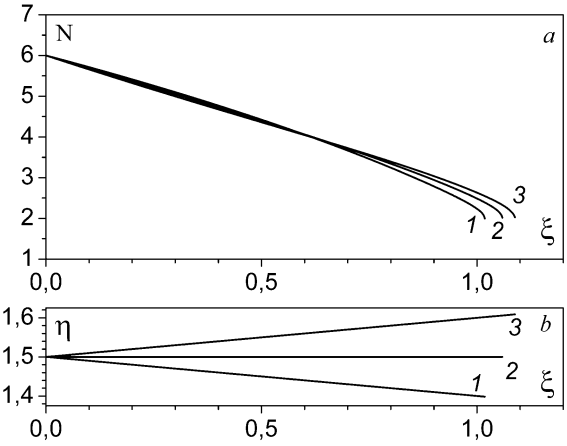

Fig. 6. Axial structure of the SW sustained discharge for $m=-1$ (a) and the normalized radius η variation (b); $\sigma=0.5$, $\Omega=0.2$, $\eta=1.5$

It was shown that slight variation of the waveguide radius along the discharge makes effect on the plasma density axial gradient and on the length of the discharge. The maximum influence was found for the discharges sustained by the $m=0$ mode (see Fig. 4). In the case of decreasing along the discharge waveguide radius the plasma density axial gradients are much smaller (line 1) than when it is steady (line 2) and increased (line 3). Also, the dimensionless plasma density values for the case of decreasing radius along the discharge are rather bigger than for the other cases.

The axial structure of the discharges that are sustained by the modes with $m=\pm1$ are much stable with respect to the waveguide radius variation (see Figs. 5, 6). It is also necessary to mention that small increase of waveguide radius along the discharge leads to small increase of plasma density at the end of discharge as compared with steady and decreasing radius of the waveguide that is opposite to the case of the discharges sustained by the $m=0$ mode.

CONCLUSIONS

It was constructed the electromagnetic model of steady state gas discharge in long tube of slightly varying radius. Within this model it was studied the influence of slightly varying radius on the axial structure of the discharges sustained by SW with $m=0,\pm1$ that widely used for discharge sustaining. It was shown that variable radius of the waveguide metal enclosure affects much greatly on the axial structure of the discharges sustained by the SW with $m=0$. The discharges sustained by the SW with $m=\pm1$ are much stable with respect to slight charging of the waveguide radius.

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ЭЛЕКТРОМАГНИТНАЯ МОДЕЛЬ ГАЗОВОГО РАЗРЯДА В ДЛИННОЙ ТРУБКЕ ПЕРЕМЕННОГО РАДИУСА

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Целью исследования является построение электромагнитной модели стационарных состояний газовых разрядов, протекающих в диффузионном режиме в длинных узких трубках со слабо изменяющимся радиусом для изучения аксиального профиля плотности плазмы. В рамках модели проведено исследование влияния внешнего магнитного поля на аксиальную структуру газового разряда в слабо сужающихся и расширяющихся металлических волноводах. Получены аксиальные профили плотности плазмы для волноводов с постоянным и изменяющимся радиусом.

ЕЛЕКТРОМАГНІТНА МОДЕЛЬ ГАЗОВОГО РОЗРЯДУ В ДОВГІЙ ТРУБЦІ ЗМІННОГО РАДІУСА

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Метою дослідження є побудова електромагнітної моделі стаціонарних станів газових розрядів, що протікають в дифузійному режимі в довгих вузьких трубках зі змінним радіусом для дослідження аксіального профілю густини плазми. У рамках моделі проведено дослідження впливу зовнішнього магнітного поля на аксіальну структуру газового розряду в металевих хвилеводах, що слабко звужуються або розширюються. Отримано аксіальні профілі густини плазми для хвилеводів з постійним і змінним радіусом.