EIGENVALUES AND EIGENFIELDS OF A CORRUGATED GYROTRON CAVITY WITH CONDUCTING WALLS

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Electromagnetic analysis of a cylindrical gyrotron cavity with longitudinal wall corrugations is performed on the basis of the approximate surface impedance model (SIM) and the full-wave spatial harmonic method (SHM). The good convergence of SHM with respect to the number of spatial harmonics is shown. The perturbation approach is extended to a cylindrical corrugated cavity with finite wall conductivity. With this approach attenuation of TE cavity modes due to ohmic wall losses is investigated. For the $TE_{8,3}$ mode, as an example, the number of corrugations, which ensures reasonable accuracy of SIM, has been determined. For such number of corrugations, good agreement between SIM and SHM is demonstrated for mode eigenvalue, eigenfields and attenuation.

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INTRODUCTION

Millimeter and submillimeter gyrotrons are subject of much current interest due to widespread use in spectroscopy, medical technologies, material processing, space research and security systems. Among the gyrotron applications, electron-cyclotron heating of magnetically confined plasma in controlled thermonuclear fusion devices remains the chief application for more than half a century. For instance, heating system of the International Thermonuclear Experimental Reactor (ITER) requires more than twenty 170-GHz MW-class gyrotrons, which are now under development worldwide. Of special concern are gyrotrons operated at the second harmonics of cyclotron frequency because of the lower requirement on operating magnetic field. The weakness of such gyrotrons is the competition between operating gyrotron mode and parasitic modes excited at the first (fundamental) cyclotron harmonic. This unwanted effect shortens the operating region of harmonic gyrotrons, reduces their efficiency and output power.

To avoid harmonic mode competition, one has to suppress selectively the fundamental competing modes. This can be done with distributed longitudinal corrugations (slots) made on the cylindrical surface of a gyrotron cavity [1]. The effect of corrugations on cavity modes is frequency-dependent. For this reason, parameters of the corrugated wall can be selected in such a way as to increase losses of the fundamental (low-frequency) competing modes relative to those of the operating (high-frequency) mode.

Such beneficial effect of corrugations has been demonstrated in [1] on the basis of the approximate surface impedance model (SIM), which is commonly used in the analysis of RF structures with densely-spaced periodic corrugations. According to this model, the corrugated wall is approximated by a smooth cylindrical surface with averaged (effective) anisotropic impedance, which depends on the corrugation parameters and mode frequency. The widely accepted criterion of SIM validity is as follows [2]:

$$N > 2|m|, \qquad (1)$$

where N is the number of corrugations, m is the azimuth mode index.

In [1], the number of corrugations has been selected large enough ($N = 20$) to fulfill the condition (1) for the operating $TE_{8,9}$ mode ($m = 8$) under consideration. Despite this, results of SIM appear to be incorrect in this case. This has been demonstrated in [3] and is explained by the strong coupling between azimuthal space harmonics of the corrugated gyrotron cavity [4]. This coupling is ignored in [1]. Following [4], alternative criterion of SIM validity was used in [3]

$$N > |m| + \chi, \qquad (2)$$

where χ is the mode eigenvalue.

The mode eigenvalue χ always exceeds $|m|$. Therefore, when compared to (1), condition (2) is valid for a larger number of corrugations N. Reasonable accuracy of SIM calculations for the $TE_{8,9}$ mode of [1] was shown in [3] in the case of increased N subject to (2).

The $TE_{8,9}$ mode is high-order mode ($|m| << \chi$). For such modes, criterion (2) yields large number (about 45) of corrugations, which are difficult to fabricate. Therefore, of interest are modes with lower mode eigenvalues χ. An example is the $TE_{8,3}$ mode [5, 6]. Our purpose is to investigate the validity of the surface impedance model for this mode. For this purpose we will use the full-wave rigorous approach known as spatial harmonics method (SHM). In [3], this method was applied to study the eigenvalue and eigenfields of a gyrotron cavity made from the perfect electric conductor. However, it is known that for a corrugated gyrotron cavity the ohmic wall losses can be extremely high [1, 7] and thus can distinctly affect gyrotron performance [8]. For this reason, our purpose is also to take into account the finite conductivity of the gyrotron cavity and to investigate effect of the ohmic wall losses on attenuation of TE cavity modes.

1. MATHEMATICAL MODEL

Consider TE mode of a circular waveguide with longitudinal wall corrugations. The transverse cross-section of the waveguide is shown in Fig. 1. First, assume that the waveguide conductivity σ is infinitely high.

The components of the mode field are expressed in terms of the membrane function Ψ, which is proportional to $\exp\{-i\omega t + ik_z z\}$ and satisfies the wave (Helmholtz) equation:

$$\left(\Delta_\perp + k_\perp^2\right)\Psi = 0 \qquad (3)$$

with the Dirichlet boundary condition on the contour ∂S of the waveguide cross section:

$$\frac{\partial\Psi}{\partial n} = 0\,, \qquad (4)$$

where ω is the mode frequency, k_z is the longitudinal wavenumber, $k_\perp^2 = k^2 - k_z^2$, $k^2 = \omega^2\mu_0\varepsilon_0$, vector **n** is the outward normal to the contour ∂S.

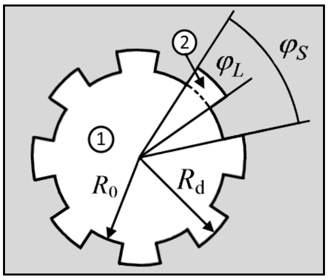

Fig. 1. Transverse cross section of a circular waveguide with longitudinal wall corrugations

To solve the eigenvalue problem (3) and (4), the full-wave spatial harmonics method [3, 4] is used. In this method, the waveguide cross-section is divided into two regions (see Fig. 1). The membrane function Ψ in regions 1 and 2 is represented as a superposition of the space Bloch and Fourier harmonics, respectively. The number of harmonics under consideration equals $2N_B$+1 for Bloch harmonics and N_F+1 for Fourier harmonics. The membrane function Ψ (the field component H_z) and its derivative $d\Psi/dr$ (the field component E_φ) must be continuous at the interface between adjacent regions. This condition yields characteristic equation for the mode eigenvalues $\chi = k_\perp R_0$ of the corrugated waveguide [3]. These eigenvalues can then be used to determine the membrane functions and the electromagnetic fields of TE modes. The results of SIM can be obtained in the extreme case of $N_B = N_F = 0$.

Let us next take into account the finite conductivity σ of the waveguide wall and its effect on the attenuation of TE guiding modes. Mode attenuation in the imperfectly conducting waveguide can be determined by the perturbation approach [1, 9], which requires the skin depth $\delta = \sqrt{2/\omega\mu_0\sigma}$ be much lower than the wavelength $\lambda = 2\pi/k$. According to this approach, the complex longitudinal wavenumber of TE mode is expressed as follows:

$$k_{z1}^2 \approx k_z^2 + \frac{(1-j)}{2}\delta\frac{\oint_{\partial S}\left(k^2|\Psi|^2 + \left(k_z^2/k_\perp^2\right)|\partial\Psi/\partial l|^2\right)dl}{\int_S |\Psi|^2 dS}\,. \qquad (5)$$

In the case of a circular waveguide with longitudinal wall corrugations the high accuracy of the perturbation approach was shown in [7]. However, investigations in [7] are based on the approximate surface impedance model and therefore need to be checked. In the next section, we will examine the results of SIM for eigenvalues, fields and mode attenuation of TE modes. As an example, $TE_{8,3}$ mode with frequency of about 400 GHz will be considered [5, 6].

2. COMPARATIVE STUDY OF SHM AND SIM

Consider a corrugated waveguide with the following parameters: N=20, R_0=0.215 cm, $\varphi_L/\varphi_S = 0.5$. Fig. 2 shows eigenvalue of the $TE_{8,3}$ mode ($m = 8$) as a function of the corrugation depth d. The well-known criterion (1) of SIM validity is fulfilled in the case of N=20. Despite this, results of SIM deviate widely from those followed from the rigorous spatial harmonics method, especially for large values of the corrugation depth d. This deviation is due to the coupling between spatial Bloch harmonics [3].

To validate the obtained results, the convergence of SHM must be clearly shown. Fig. 3 depicts the relative error in evaluation of the mode eigenvalue χ with respect to the number of spatial harmonics in use:

$$\zeta_\chi(N_h) = \frac{\chi(N_h+1) - \chi(N_h)}{\chi(N_h)}\,, \qquad (6)$$

where N_h=N_B=N_F.

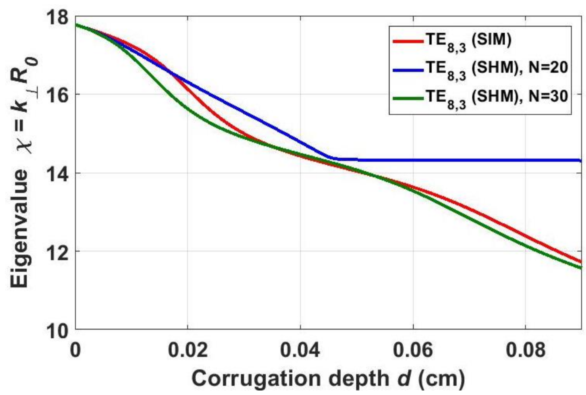

Fig. 2. The eigenvalue $\chi = k_\perp R_0$ as a function of the corrugation depth d for the $TE_{8,3}$ mode of a cylindrical corrugated waveguide

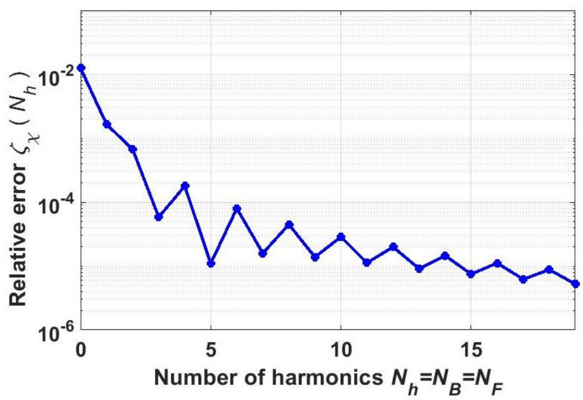

Fig. 3. Relative error of eigenvalue evaluation versus N_h for the $TE_{8,3}$ mode of a cylindrical waveguide with longitudinal wall corrugations (d=0.02 cm)

As expected, increase in number N_B and N_F of space harmonics improves the accuracy of SHM calculations. However, this makes the characteristic equation for TE modes more cumbersome and the numerical calculations more time-consuming. The reasonable choice is N_h = 2. For such number of space harmonics, relative error shown in Fig. 3 for SHM does not exceed $7 \cdot 10^{-4}$.

The good convergence of the SHM calculations for the field of the $TE_{8,3}$ mode can be clearly seen from Fig. 4, which shows the mismatches of the field components E_φ and H_z at the interface $r=R_0$ between regions 1 and 2 (see Fig. 1). Value in bracket shown in this figures (e.g. SHM (20)) denotes the number $N_h=N_B=N_F$ of space harmonics under consideration. The mismatches have the following form:

$$\delta_{E_\varphi}(\varphi) = \left| \frac{E_{\varphi 1}(R_0,\varphi) - E_{\varphi 2}(R_0,\varphi)}{E_{\varphi 1}(R_0,\varphi)} \right|, \qquad (7)$$

$$\delta_{H_z}(\varphi) = \left| \frac{H_{z1}(R_0,\varphi) - H_{z2}(R_0,\varphi)}{H_{z1}(R_0,\varphi)} \right|$$

and decrease with increasing N_h.

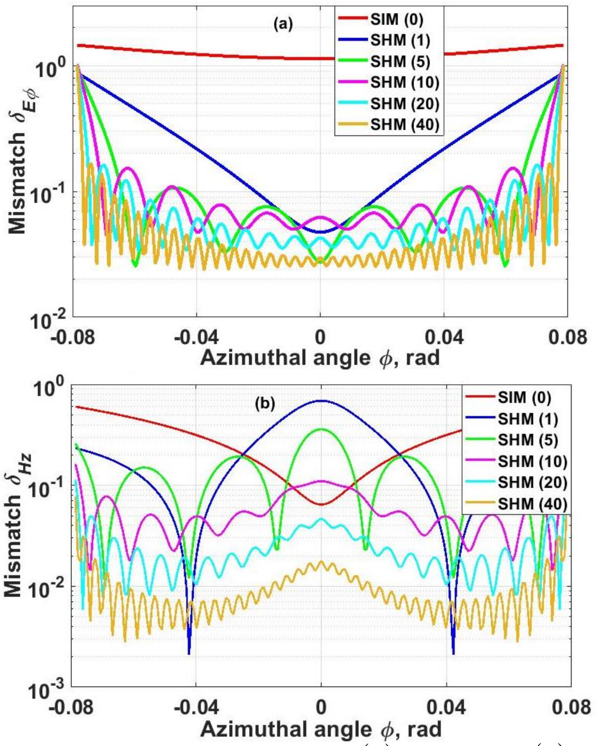

Fig. 4. The field mismatches $\delta_{E_\varphi}(\varphi)$ (a) and $\delta_{H_z}(\varphi)$ (b) at the groove aperture for different N_h (d=0.02 cm)

The validity of the surface impedance model can be expanded by increasing the number N of longitudinal corrugations [3]. This effect is shown in Fig. 5, where the mismatch between SIM and SHM results is presented:

$$\delta_\chi(N) = \left| \frac{\chi_{SHM}(N) - \chi_{SIM}}{\chi_{SIM}} \right|. \qquad (8)$$

It can be seen that for the $TE_{8,3}$ mode the eigenvalues followed from SHM and SIM are close enough as $N > 30$. Such number of corrugations is in agreement with improved criterion (2) of SIM validity. Fig. 2 shows the mode eigenvalues calculated by approximate and full-wave methods for N=30. The agreement between them is seen to be reasonable without regard to the depth d of corrugations.

Using the perturbation approach, we evaluate the complex longitudinal wavenumber (5) for the $TE_{8,3}$ mode of the corrugated gyrotron cavity with imperfectly conducting walls. The walls are assumed to be made of copper with reduced conductivity $2.9 \cdot 10^7$ S/m [1]. The attenuation of the $TE_{8,3}$ mode of the corrugated waveguide versus corrugation depth d is depicted in Fig. 6 for different number of corrugations.

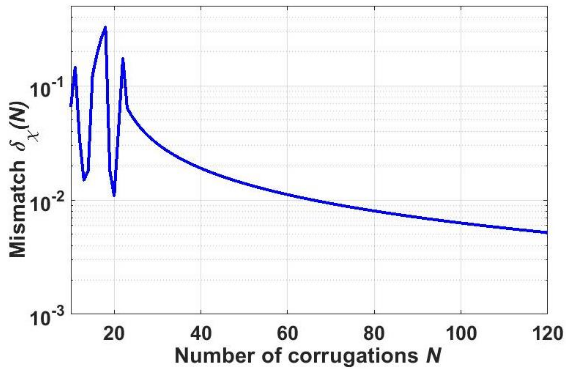

Fig. 5. The mismatch $\delta_\chi(N)$ for eigenvalue of the $TE_{8,3}$ mode of a corrugated waveguide versus the number of corrugations N (d=0.02 cm)

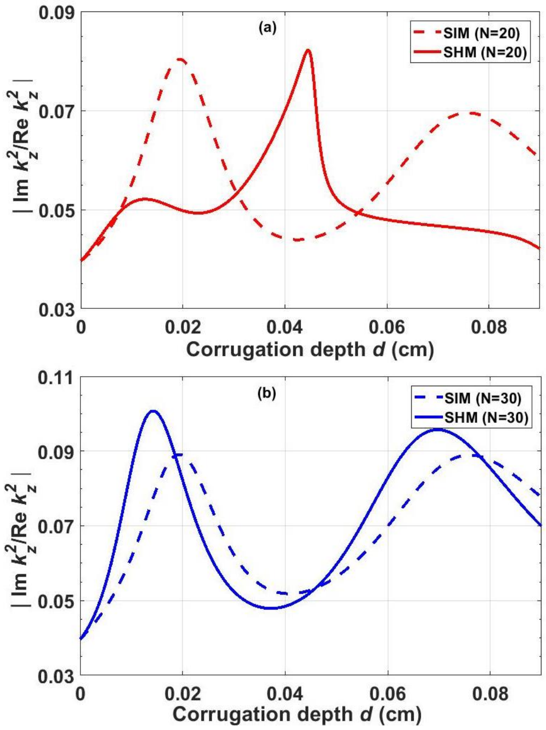

Fig. 6. Attenuation of the $TE_{8,3}$ mode of the imperfectly conducting corrugated waveguide versus the corrugation depth d for N=20 (a) and N=30 (b), Rek_z^2=15 cm^{-2}

From Fig. 6,a follows that the ohmic wall losses evaluated by SIM and SHM are completely different for the $TE_{8,3}$ mode of the corrugated cylindrical waveguide with $N = 20$ subject to (1). This is not particularly surprising, since in this case SIM incorrectly determines both the mode eigenvalue (see Fig. 2) and eigenfields (see Fig. 4), which affect the attenuation of the $TE_{8,3}$ mode (see (5)). The situation changes as the number of corrugations is increased to 30. In this case criterion (2) is fulfilled and SIM calculations appear to be fairly accurate. As a result, SIM and SHM predict similar attenuation for the $TE_{8,3}$ mode of the circular corrugated waveguide made of cooper, if the number of corrugations satisfies inequality $N > 30$ (see Fig. 6,b).

CONCLUSIONS

Electromagnetic properties of a cylindrical gyrotron cavity with longitudinal wall corrugations have been studied on the basis of the approximate SIM and the rigorous SHM approaches. The good convergence of the SHM calculations with respect to number of spatial harmonics has been demonstrated for the mode eigenvalue and eigenfields. The results of SHM have been used to evaluate attenuation of TE modes due to finite conductivity of the cavity material. For this purpose, the perturbation approach has been extended to circular waveguide with corrugated wall. It has been shown that SIM may yield inadequate results, even though the number of corrugations N satisfies the well-known criterion of SIM validity. To expand SIM validity, this number must be increased. For the $TE_{8,3}$ mode, as an example, the required value of N has been determined. It has been demonstrated that for such number of corrugations SHM and SIM agree closely in eigenvalues, eigenfields and attenuation of the $TE_{8,3}$ mode.

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СОБСТВЕННЫЕ ЗНАЧЕНИЯ И СОБСТВЕННЫЕ ПОЛЯ ГОФРИРОВАННОГО РЕЗОНАТОРА ГИРОТРОНА С ПРОВОДЯЩИМИ СТЕНКАМИ

Т.И. Ткачева, В.И. Щербинин, В.И. Ткаченко

На основе приближенной поверхностной импедансной модели (SIM) и строгого метода пространственных гармоник (SHM) проведен электромагнитный анализ цилиндрического резонатора гиротрона с продольными гофрами. Показана хорошая сходимость метода SHM с увеличением числа пространственных гармоник. Теория возмущений обобщена на случай гофрированного резонатора гиротрона с конечной проводимостью стенок. С ее помощью исследовано затухание TE-мод резонатора в результате омических потерь в стенках. В качестве примера, для моды $TE_{8,3}$ определено количество гофров, обеспечивающее достаточную точность SIM. Для такого количества гофров продемонстрировано хорошее согласие расчетов SIM и SHM для собственного значения моды, ее собственных полей и затухания.

ВЛАСНІ ЗНАЧЕННЯ ТА ВЛАСНІ ПОЛЯ ГОФРОВАНОГО РЕЗОНАТОРА ГІРОТРОНА З ПРОВІДНИМИ СТІНКАМИ

Т.І. Ткачова, В.І. Щербінін, В.І. Ткаченко

На основі наближеної поверхневої імпедансної моделі (SIM) та строгого методу просторових гармонік (SHM) проведено електромагнітний аналіз циліндричного резонатора гіротрону з поздовжніми гофрами. Показано добру збіжність методу SHM зі збільшенням числа просторових гармонік. Теорію збурень узагальнено на випадок гофрованого резонатора гіротрону з кінцевою провідністю стінок. З її допомогою досліджено згасання TE-мод резонатора в результаті омічних втрат у стінках. Як приклад, для моди $TE_{8,3}$ визначено кількість гофрів, що забезпечує достатню точність SIM. Для такої кількості гофрів продемонстровано добру згоду розрахунків SIM та SHM для власного значення моди, її власних полів та загасання.