

# MODELLING OF THE ELECTROMAGNETIC SURFACE WAVES PROPAGATION ON THE INTERFACE BETWEEN THE LEFT-HANDED METAMATERIAL AND THE DISSIPATIVE DIELECTRIC

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The properties of surface electromagnetic waves propagating along a planar structure consisting of an ideal dielectric, a dielectric layer with large losses and a high dielectric constant, and a left-handed metamaterial with "amplification" are studied. All media were assumed isotropic. Dispersion relations for the eigenmodes of such a waveguide structure are obtained. The possibility of full compensation of the energy losses of surface waves is demonstrated.

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## INTRODUCTION

There are a lot of experimental and theoretical studies of the metamaterials that have been already done and currently are carried out in the world laboratories. These metamaterials possess many unique physical properties that are not found in ordinary natural materials, such as: simultaneously negative value of permittivity and permeability, reverse Doppler and Cherenkov effects, etc. So, it is expected that the existence of such unusual properties will open the possibilities for creation of new devices [1, 2]. Previous studies were mainly focused on the variety of planar waveguide structures that contain the left-handed metamaterials without losses [3, 4].

In our recent paper [5] it was demonstrated the possibility of full loss compensation for the surface electromagnetic wave at the boundary between the dielectric with losses and the isotropic left handed material with the frequency-independent gain. Previously, it was mainly studied the metamaterials with dielectric claddings with rather low permittivity and without a considerable losses.

In the present work it was studied the planar waveguiding structure in which the left-handed metamaterial with gain [1] is cladding by the finite thickness layer with high-permittivity dissipative dielectric material. It is assumed the existence of the gain in the LHM to compensate the wave energy losses [1, 2] and is not discussed mechanisms of such gain in this paper.

## 1. TASK SETTINGS

The considered electromagnetic wave propagates along the planar waveguide structure that consists of ideal dielectric ( $\epsilon_1 = 4.5, \mu_1 = 1$ ), the rather thin layer of thickness  $d$  with high-permittivity lossy dielectric medium ( $\epsilon_2, \mu_2$ ) and the left-handed metamaterial with gain. All media were assumed to be isotropic. The left-handed material is characterized by the effective permittivity  $\epsilon(\omega)$  and permeability  $\mu(\omega)$  that depend on the wave frequency and commonly expressed with the help of experimentally obtained relations [3]:

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega - i\nu)}, \quad (1)$$

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega(\omega - i\gamma) - \omega_0^2}, \quad (2)$$

here  $\omega_p$  is effective plasma frequency,  $\omega_0$  is the characteristic frequency of LHM. In further study it was considered the LHM with  $\omega_p / 2\pi = 10$  GHz and  $\omega_0 / 2\pi = 4$  GHz and parameter  $F = 0.56$  [3]. The model parameters  $\nu, \gamma$  simulate the "electric" and "magnetic" gain of electromagnetic waves in the metamaterial, respectively.

Such proportion of the parameters leads to the existence of the frequency region where  $\epsilon(\omega) < 0$  and  $\mu(\omega) < 0$  simultaneously.

It was assumed that the layer of high-permittivity dissipative dielectric is non-magnetic ( $\mu_2 = 1$ ), has thickness  $d$  and possesses the constant dielectric permittivity that equals to  $\epsilon_2 = \epsilon_2' + \epsilon_2'' \sim 35 + i \cdot 30$ . Such values of task parameters were chosen due to the attempt of modelling of the electromagnetic wave propagation along the biological sample.

Let us consider the electromagnetic wave that propagates along this structure. It was assumed that the wave disturbances exponentially tend to zero far away from both boundaries. The dependence of the wave components on time  $t$  and coordinates  $x$  and  $z$  is expressed the following form:

$$E, H \propto E(x), H(x) \exp[i(k_3 z - \omega t)], \quad (3)$$

here  $z$  lies at the separation plane, and  $x$  is the coordinate rectangular to the wave propagation direction and  $k_3 = \text{Re}(k_3) + i \cdot \text{Im}(k_3)$ .

In the considered case it is possible to split the system of Maxwell equations on two sub-systems. One of them describes the waves of  $H$ -type and another – waves of  $E$ -type.

The wave of  $E$ -type in the given structure possesses the dispersion relation of the following form:

$$e^{h_1 d} \{(\varepsilon(\omega) h_1 + \varepsilon_1 \kappa)(\varepsilon_2 h_1 + \varepsilon_1 h_2)\} - e^{-h_1 d} \{(\varepsilon(\omega) h_1 - \varepsilon_1 \kappa)(\varepsilon_2 h_1 - \varepsilon_1 h_2)\} = 0, \quad (4)$$

here  $h_{1,2} = \sqrt{k_{1,2}^2 - \varepsilon_{1,2} \mu_{1,2} k^2}$ ,  $\kappa = \sqrt{k_3^2 - \varepsilon(\omega) \mu(\omega) k^2}$  - are the transverse wave vectors of appropriate exponents,  $k = \omega/c$ , where  $c$  is the speed of light in vacuum.

Similarly the wave of  $H$ -type possesses the dispersion relation of the similar form:

$$e^{h_1 d} \{(\mu(\omega) h_1 + \mu_1 \kappa)(\mu_2 h_1 + \mu_1 h_2)\} - e^{-h_1 d} \{(\mu(\omega) h_1 - \mu_1 \kappa)(\mu_2 h_1 - \mu_1 h_2)\} = 0. \quad (5)$$

## 2. MAIN RESULTS

The results of numerical solution of the dispersion equations for  $E$ - and  $H$ -waves for the given set of parameters are shown at Fig. 1. It were found three solutions of two dispersion equations (4), (5): two solutions for  $E$ -wave and one solution for  $H$ -wave.

Curves  $EWave1$ ,  $EWave2$  correspond to the waves of  $E$ - type and the curve  $HWave1$  corresponds to wave of  $H$ - type. For the chosen set of parameters the domain where the metamaterial demonstrates left-handed properties (i.e. simultaneously  $\varepsilon(\omega) < 0$  and  $\mu(\omega) < 0$ ) lies in the region where  $1 < \Omega = \omega/\omega_0 < 1.5$ . The inclined straight lines corresponds to  $\zeta = ck_3/\omega_0 = \Omega\sqrt{\varepsilon_1}$  and  $\zeta = \Omega\sqrt{\text{Re}(\varepsilon_2)}$ . The curve  $\zeta = \Omega\sqrt{\varepsilon(\omega)\mu(\omega)}$  corresponds to the condition  $\kappa(\zeta, \Omega) = 0$  and separates region, where electromagnetic wave of the surface type can exist.

The considered modes are both forward (both phase and group velocities are positive) and backward (phase velocity is positive and group velocity is negative) in the correspondent  $\zeta$ - region.

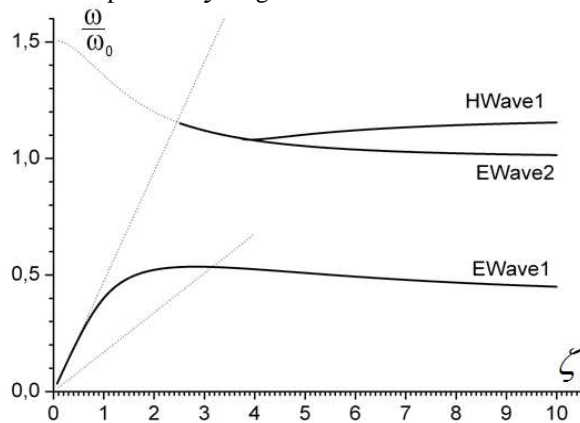


Fig. 1. The dependence of the frequency  $\Omega = \omega/\omega_0$  on the wavenumber  $\xi = k_3 c/\omega_0$  for the metamaterial gain values  $\gamma = \nu = 0$  and for dielectric layer:  $\varepsilon_2 = 35$ ;  $\Delta = \omega_0 d/c = 0.1$

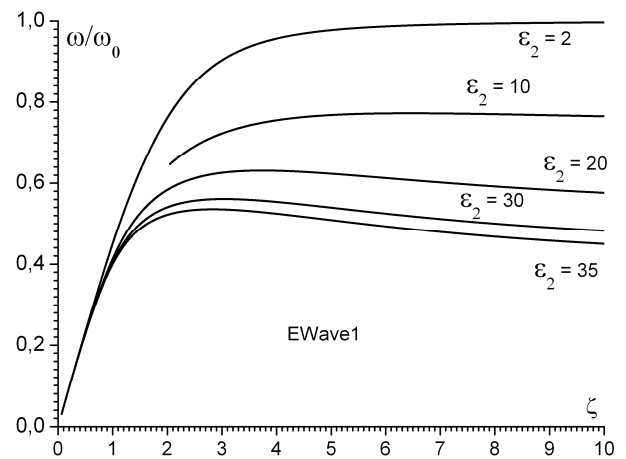


Fig. 2. The dependences of the of the  $EWave1$  frequency on the wavenumber for different value of real part of high-permittivity dissipative dielectric layer  $\varepsilon_2$

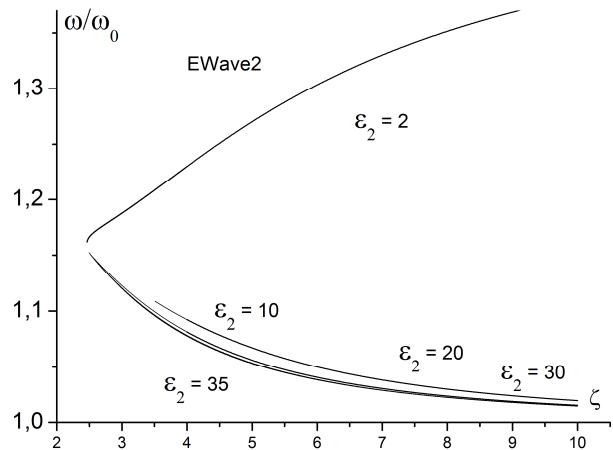


Fig. 3. The dependences of the of the  $EWave2$  frequency on the wavenumber for different value of real part of high-permittivity dissipative dielectric layer  $\varepsilon_2$

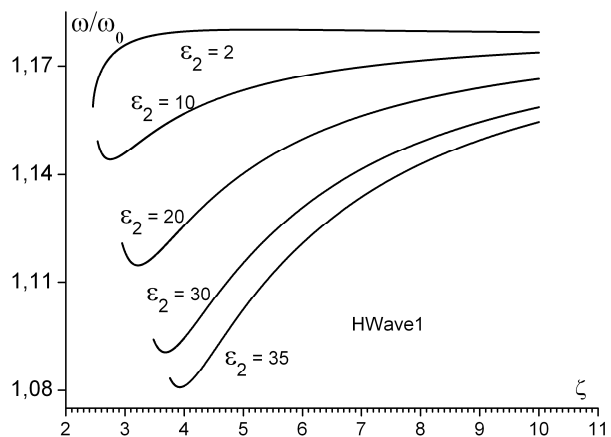


Fig. 4. The dependences of the of the  $HWave1$  frequency on the wavenumber for different value of real part of high-permittivity dissipative dielectric layer  $\varepsilon_2$

Figs. 2-4 present the modification of the dispersion curves with the changing of  $\varepsilon_2$  value for all three modes. For each mode the obtained modification is essential. If the value of  $\varepsilon_2$  is increased, then the eigenfrequency of the considered surface modes is decreased.

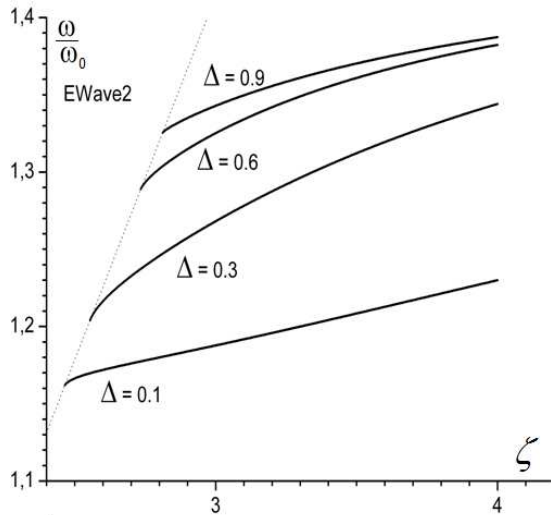


Fig. 5. The dependence of the of the EWave2 frequency on the wavenumber for different value of thickness of high-permittivity dissipative dielectric layer

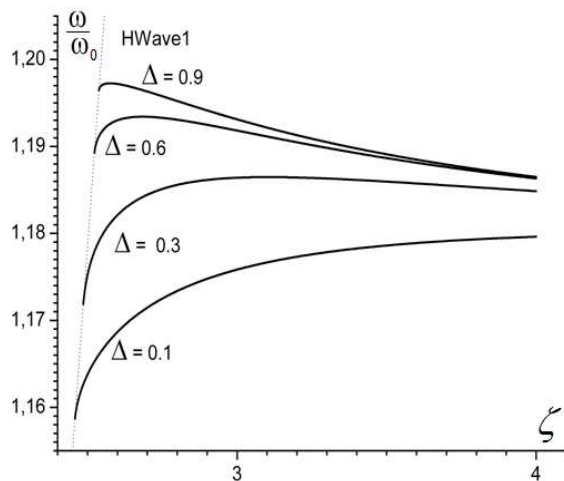


Fig. 6. The dependence of the imaginary part of HWave1 frequency on the wavenumber for different value of thickness of high-permittivity dissipative dielectric layer

Figs. 5, 6 present the modification of the dispersion curves with the changing of the thickness of high-permittivity dissipative dielectric layer. It was obtained that the thickness of dissipative layer strongly influences on the surface wave dispersion properties.

The dependencies of the imaginary part of the wave number  $\zeta''$  versus on the values of the “electric” and the “magnetic” gain of the left-handed material are presented on the Figs. 7, 8. The condition  $\zeta'' = 0$  implies a full compensation of losses in the high-permittivity dis-

sipative dielectric layer. The upper graphs on Figs. 7,8 show the slight deviation of the eigenfrequencies for the correspondent modes under “switching-on” of the such gains.

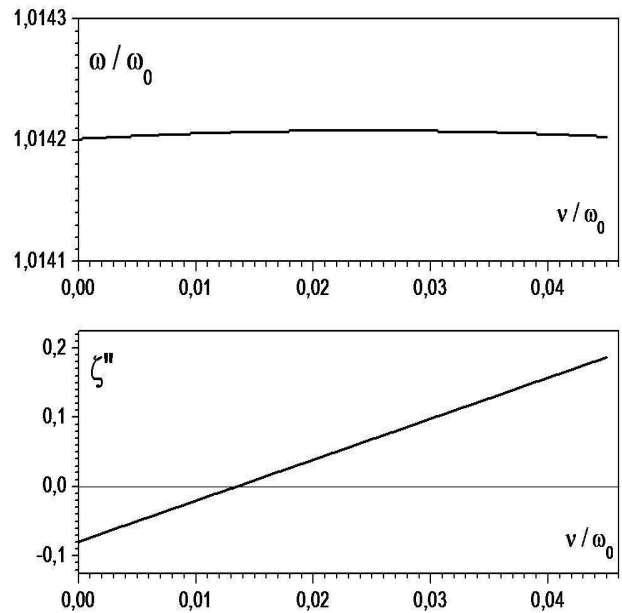


Fig. 7. The dependence of the wave frequency and the imaginary part of wavenumber of EWave2 on the values on “electric” gain  $\nu$  for  $\varepsilon_2=35+i30$ ;  $\gamma=10^{-3}$  and  $\zeta=10.0$

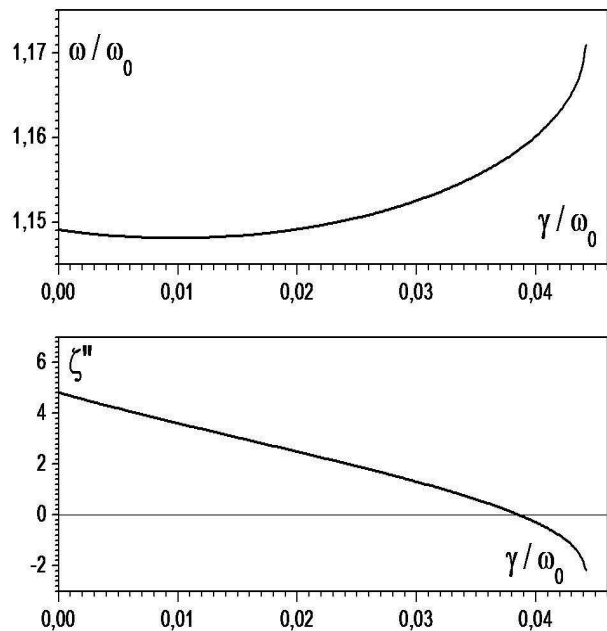


Fig. 8. The dependence of the wave frequency and the imaginary part of wavenumber of HWave1 on the values of “magnetic” gain  $\gamma$  for  $\varepsilon_2=35+i30$ ;  $\nu=10^{-3}$  and  $\zeta=10.0$

It was obtained, that rather low gain values are needed for the full compensation of the electromagnetic surface waves energy losses in the dissipative dielectric.

Thus, such full compensation of the EWave2 mode energy losses takes place if  $\nu \approx 0.013$ , and for the HWave1 mode such effect was observed at  $\gamma \approx 0.037$ .

Naturally, for another set of problem parameters the full wave energy compensation will take place at another gain values.

## CONCLUSIONS

It was obtained that surface electromagnetic waves that propagate along the interface between the ordinary dissipative dielectric with strong losses and the isotropic metamaterial are strongly damped. It was shown the possibility of full loss compensation for the surface electromagnetic wave that propagates in such waveguide structure along the interface between the high-permittivity dielectric with strong losses and the isotropic left handed material with gain. However, it is necessary to choose the appropriate values of metamaterial gain for each mode considered here. The required gain values for such compensation are seemed to be quite reachable.

The results obtained in this work can be useful for the various practical applications of metamaterials in science, medicine and technology.

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## МОДЕЛИРОВАНИЕ РАСПРОСТРАНЕНИЯ ЭЛЕКТРОМАГНИТНЫХ ПОВЕРХНОСТНЫХ ВОЛН НА ГРАНИЦЕ МЕЖДУ ЛЕВОСТОРОННИМ МЕТАМАТЕРИАЛОМ И ДИССИПАТИВНЫМ ДИЭЛЕКТРИКОМ

*В.К. Галайдыч, Н.А. Азаренков, В.П. Олефир, А.Е. Споров*

Изучены свойства поверхностных электромагнитных волн, распространяющихся вдоль плоской структуры, состоящей из идеального диэлектрика, слоя диэлектрика с большими потерями и большой диэлектрической проницаемостью и левостороннего метаматериала с «усилением». Все среды считались изотропными. Получены дисперсионные зависимости для собственных мод такой волноводной структуры. Продемонстрирована возможность полной компенсации потерь энергии поверхностных волн.

## МОДЕЛЮВАННЯ ПОШИРЕННЯ ЕЛЕКТРОМАГНІТНИХ ПОВЕРХНЕВИХ ХВИЛЬ НА МЕЖІ МІЖ ЛІВОСТОРОННІМ МЕТАМАТЕРІАЛОМ ТА ДИСИПАТИВНИМ ДІЕЛЕКТРИКОМ

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Вивчено властивості поверхневих електромагнітних хвиль, що поширюються уздовж пласкої структури, що складається з ідеального діелектрика, шару діелектрика з великими втратами та великою діелектричною проникливістю і лівостороннього метаматеріалу з «підсиленням». Усі середовища вважались ізотропними. Отримано дисперсійні залежності для власних мод такої хвильоводної структури. Продемонстровано можливість повної компенсації втрат енергії поверхневих хвиль.