STRUCTURAL PHASE TRANSITIONS IN THIN CONVECTION AT DEPENDENCE OF VISCOSITY ON TEMPERATURE

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Conditions for a second-order phase transition in a thin liquid layer with poorly conductive boundaries under conditions of dependence of viscosity on temperature are considered. It is shown that, when the dependence is weak, the shaft system of toroidal vortices develops first, and then a field of square convective cells is being formed. If the viscosity dependence on temperature is strong, even for the system described by Proctor-Sivashnsky equations, transitions from the shaft structure to hexagonal convection cells are possible.

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INTRODUCTION

In a number of applications, such as thin clouds, convection between closely spaced heat-poor surfaces, the Proctor-Sivashinsky model [1, 2] was of great interest, which was used to describe the development of convection in a thin fluid layer with poorly conductive heat boundaries. The authors of [3] discovered stationary solutions and studied their stability.

Below we discuss the possibility of phase transitions of the second kind in a thin layer of liquid between poorly conducting walls. In contrast to the traditionally used Swift-Hohenberg equations, we use the Proctor-Sivashinsky 3D equation more appropriately to the real conditions. This problem is obviously three-dimensional in space and non-stationary, which at first sight creates significant problems. However, the Proctor-Sivashinsky model makes it possible to reduce the dimensionality of the description and focus on topological aspects, that is, the type, size and time of the development of spatial structures.

The peculiarity of the model is that it singles out one spatial scale of interaction, leaving for the evolution of the system the possibility to choose the character of symmetry. All spatial disturbances of the same size, but of different orientations, interact with each other. Nonlinearity in the system is vector. It turned out that the presence of the minima of the mode interaction potential, the absolute value of the wave number vectors of which is unchanged, determines the choice of symmetry and, accordingly, the characteristics of the spatial structure.

1. PROCTOR-SIVASHINSKY EQUATION

Assuming a thin layer of liquid (or gas), it is possible to integrate all perturbations due to convection along the height of the layer and go to the two-dimensional description [1, 2] (see, also, [4]). In two-dimensional geometry, the Proctor-Sivashinsky equation for the convection temperature field takes the form

$$\frac{\partial \varphi}{\partial T} + \nabla^4 \varphi + \nabla[(2 - \gamma_V \varphi - |\nabla \varphi)^2 |)\nabla \varphi] + a\varphi = 0 , \quad (1)$$

where the two-dimensional operator $\nabla \phi = \vec{i} \cdot \frac{\partial \phi}{\partial \zeta} + \vec{j} \cdot \frac{\partial \phi}{\partial \vartheta}$, moreover $\vec{i}$, $\vec{j}$ – unitary unit vectors orthogonal to each other in the plane (ζ, ϑ) of the media section. However, in contrast to [5, 6], where two-dimensional vortices are considered, this problem, despite the two-dimensional description, is a three-dimensional case.

It should be noted, to the fact that the quadratic nonlocal (that is, the presence of derivatives with respect to time or coordinate), the nonlinearity in the equation is present in the form of a term proportional to γ_V and is due to the dependence of the viscosity on the temperature on the height of the layer, and the nonlocal cubic is taken into account by a term proportional to $|\nabla \varphi)^2 |\nabla \varphi$.

In works (see review [4]) it was noted that in the process of development of convection in the absence of viscosity dependence on temperature ($\gamma_V = 0$) arise – toroidal vortex shaft structures, which turn out to be metastable and after a short time a second-order phase transition occurs, which leads to the appearance of square toroidal vortex structures.

If the viscosity in the system depends on the temperature, then we usually use the Swift-Hohenberg equation, in which the cubic nonlinearity is local, that is, the proportional φ^3. However, in problems of describing the convection of a fluid between weakly conducting walls, the nonlinearity is still a vector, then the question arises of the character of the processes in this case.

The solution is represented in the form of a series

$$\Phi = \varepsilon \sum_j A_j \exp(i\vec{k}_j \vec{r}) \text{ with } |\vec{k}_j| = 1 .$$

When replacing $T \cdot \varepsilon^2 = t$, for slow amplitudes A_j in the absence of noise, we get a convenient representation of the Proctor-Sivashinsky model for describing convection:

$$\dot{A}_\vartheta = A_\vartheta - 2\gamma (A_{\vartheta'} A_{\vartheta''})_\vartheta - A_1^2 \frac{2[1 + 2(\mathrm{Cos}\,\vartheta)^2]}{3} A_\vartheta , \quad (2)$$

where the interaction coefficients are determined by the relations

$$V_{jj} = 1 ,$$

$$V_{ij} = (2/3)\left(1 - 2\left(\vec{k}_i \vec{k}_j\right)^2\right) = (2/3)\left(1 + 2\cos^2 \vartheta_{ij}\right),$$

and ϑ_{ij} – angle between vectors $\vec{k}_i$ and $\vec{k}_j$. Expression (2) must be supplemented with the initial values of the amplitudes of the spectrum A_j. I.e. $A_j |_{t=0} = A_{j0}$. The dependence of the viscosity on temperature is determined by a term of the form proportional to γ, where $\vartheta_{n_0} = 2\pi / 3$ wherein $\vartheta_{n+n_0} = \vartheta_n + 2\pi / 3$ and

$\vartheta_{n+2n_0} = \vartheta_n + 4\pi/3$, accurate to $(\vartheta + \vartheta' + \vartheta'' = 2\pi s, s = 0, \pm 1, \pm 2...)$. Let us recall that $\gamma > 0$ is responsible for the convection of the gas layer, and $\gamma < 0$ – is responsible for the convection of the liquid.

2. ANALYSIS OF THE MODEL

The so-called "amorphous" state is obtained first because of averaging $<(\mathrm{Cos}\,\vartheta)^2>=1/2$, then $<V>\approx 4/3$, and at small γ the explosive growth of modes stops when $\sum_i < A_i^2 > \approx 3/4$. Then a system of shafts is formed. There appears one (in the half-cycle) mode A_1 with an amplitude of about unity. Let's see how other small amplitude modes (linear theory) are shifted by an angle ϑ. If the angle $\vartheta \neq 2\pi/3$, then the second term (2) is vanishingly small, and the increment Γ (determined from $\exp\{\Gamma \cdot t\}$) of this mode is

$$\Gamma = 1 - \frac{2[1+2(\mathrm{Cos}\,\vartheta)^2]}{3} = \frac{1-4(\mathrm{Cos}\,\vartheta)^2}{3} \qquad (3)$$

and is positive only for $-1/2 < \mathrm{Cos}\,\vartheta < 1/2$, i.e. in the surrounding area of $\pm\pi/2$, and when $\vartheta = \pm\pi/2$, $\Gamma_{Max1} = 1/3$ – the maximum increment of the growing modes, which form together with the developed shafts the square cells.

Let's consider the case when $\vartheta = 2\pi/3$. Let us find out how three-mode interactions can be competitive. Equation for modes $A_{\pm 2\pi/3}$ small amplitude shifted by $\vartheta = \pm 2\pi/3$ relative to the basic mode A_1 already formed shafts, can be written in the form

$$\begin{aligned}\dot{A}_{\pm 2\pi/3} &= A_{\pm 2\pi/3} - 2\gamma(A_{\mp 2\pi/3}A_1) - \\ &- A_1^2 \frac{2[1+2(\sqrt{3}/2)^2]}{3} A_{\pm 2\pi/3} = \\ &= -\frac{5}{3} A_1^2 A_{\pm 2\pi/3} - 2\gamma(A_{\mp 2\pi/3}A_1),\end{aligned} \qquad (4)$$

or for a system of coupled equations

$$\begin{aligned}(\frac{d}{dt} + \frac{5}{3}|A_1|^2)A_{2\pi/3} &= -2\gamma(A_{-2\pi/3}A_1), \\ (\frac{d}{dt} + \frac{5}{3}|A_1|^2)A_{-2\pi/3} &= -2\gamma(A_{2\pi/3}A_1).\end{aligned} \qquad (5)$$

The increment of such a process

$$\Gamma_6 = -2\gamma A_1 - \frac{5}{3}A_1^2, \qquad (6)$$

for the case of convection $|\gamma| > \frac{5A_1}{6}$. I.e. at $|\gamma| > \frac{5}{6}$ the process of forming hexagonal cells in the case of a well-developed structure of shafts only receives the right to life.

That is why smaller values $|\gamma|$ only formed shaft structures at the initial stage [7]. But in order to compete with the process of growth of the lateral spectrum (that is, the process of forming square cells), it is necessary that the increment (6) exceed the increment (3), that is $\Gamma_{Max1} = 1/3$. In other words, the increment $\Gamma_6 > \Gamma_{Max1}$, what is possible with $|\gamma| > 1$. It can be seen that the developed structure of hexagonal cells inhibits the development of square cells. That is, this instability regime leads to the same results as in the case considered in the Swift-Hohenberg model.

Thus, the formation of hexagonal cells, found in experiments [8] (see also [9] and Figs. 1, 2), is associated with a strong dependence of viscosity on temperature. At the periphery of the viewing area, the cell formation process has just begun, so the phase transition from the shaft system to the cell system has not yet occurred (see Fig. 1). That is, shaft structures dominate. In the center there is already a structural-phase transition to hexagonal convective cells.

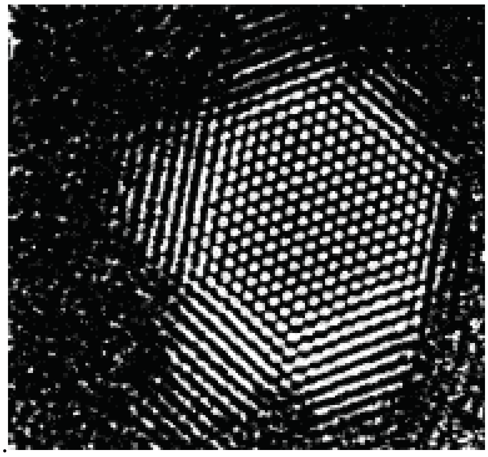

Fig. 1. Experiment. Thermal convection in a thin layer of gaseous CO_2 [8, 9]. Growth of hexagonal crystal lattice in the abovethresholdmode (figure on the right corresponds to a early period)

In Fig. 2 shows the final stage of the formation of a field of hexahedral cells – toroidal vortices.

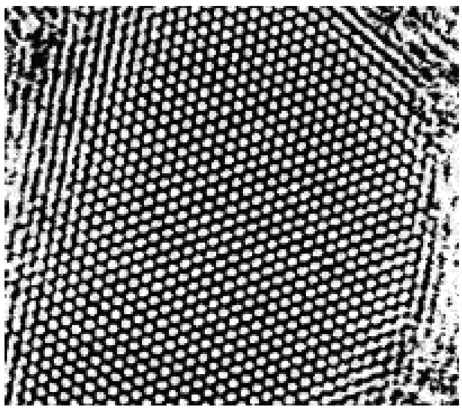

Fig. 2. Experiment. Thermal convection in a thin layer of gaseous CO_2 [8, 9]. Growth of hexagonal crystal lattice in the abovethreshold mode (figure corresponds to a later time)

CONCLUSIONS

The formation of spatial structures in the convection of thin layers of gas and liquid is similar to that for a weak viscosity dependence on temperature, one can first see the shaft eddy toroidal convection structures and then, as a result of a second-order phase transition, square convection cells, in the field of which a modulation instability can arise with the formation of large-scale poloidal vortex structures – the so-called hydrodynamic dynamo effect [11 - 12].

However, many experiments demonstrate the appearance of hexagonal cells in thin convective layers. Therefore, attempts to clarify this phenomenon led to the description in the framework of the Swift-Hohenberg model [13]. However, the structure of the nonlinearity in this model is far from real and the descriptions obtained were qualitative. Attempts to find solutions for small $|\gamma|$ also did not lead to the appearance of solutions of this type [7]. It turned out that only for much larger $|\gamma|$ there is the possibility of forming a hexagonal structure of convection, which is so widespread in nature and observed in various experiments.

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СТРУКТУРНО-ФАЗОВЫЕ ПЕРЕХОДЫ В ТОНКОЙ КОНВЕКЦИИ ПРИ ЗАВИСИМОСТИ ВЯЗКОСТИ ОТ ТЕМПЕРАТУРЫ

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Рассмотрены условия для фазового перехода второго рода в тонком слое жидкости с плохо проводящими тепло границами в условиях зависимости вязкости от температуры. Показано, что при слабой зависимости развивается сначала валиковая система тороидальных вихрей, а затем формируется поле квадратных конвективных ячеек. Если зависимость вязкости от температуры сильная, даже для системы, описываемой уравнениями Проктора-Сивашинского, возможны переходы от валиковой структуры к шестигранным конвективным ячейкам.

СТРУКТУРНО-ФАЗОВІ ПЕРЕХОДИ У ТОНКІЙ КОНВЕКЦІЇ ПРИ ЗАЛЕЖНОСТІ В'ЯЗКОСТІ ВІД ТЕМПЕРАТУРИ

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Розглянуто умови для фазового переходу другого роду в тонкому шарі рідини з границями, що погано проводять тепло, в умовах залежності в'язкості від температури. Показано, що при слабкій залежності розвивається спочатку валікова система тороїдальних вихорів, а потім формується поле квадратних конвективних осередків. Якщо залежність в'язкості від температури сильна, навіть для системи, що описується рівнянням Проктора-Сівашинського, можливі переходи від валікової структури до шестигранних конвективних осередків.