

ON PULSATING RADIATION IN WEAKLY INVERTED MEDIA

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A change in the character of maser generation in a two-level system is found when the initial population inversion exceeds some threshold value equal to the square root of the total number of atoms. Above this threshold, the number of photons begins to grow exponentially with time and the pulse with short leading edge and broadened trailing edge is generated. In this work, we attempt to explain the nature of this threshold. Coherent pulse duration, estimated by its half-width, increases significantly with increasing inversion, if all other parameters are fixed and the absorption is neglected. The inclusion of the energy loss of photons leads to the fact that the duration of coherent pulse is almost constant with increasing inversion, at least well away from the threshold. If there is a mechanism for restoring population inversion, a pulsating regime for generating stimulated emission becomes possible. The integral radiation intensity at this can be increased several times. This approach can be used to analyze cosmic radiation, which could help explain the great diversity of pulsating radiation sources in space.

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INTRODUCTION

Description of physical phenomena based on the systems of partial differential equations, derived from the observations and experimental facts, often conceals from an investigator some essential features, especially in those cases, when the researchers do not expect to find anomalies and qualitative changes in the dynamics of systems in given range of variables and parameters. Namely such a case of unusual behavior of a two-level quantum system was found in attempting to separate a induced component from the total radiation flow.

In the beginning of the past century, A. Einstein has proposed the model of two-level system, which has demonstrated the possibility of generation of both spontaneous and induced (stimulated) emission when the initial population inversion is sufficiently large [1]. Usually, the term spontaneous emission denotes the emission of oscillator (or other emitter) which not forced by external field of the same frequency. As for other influences on the characteristics of the spontaneous emission, there is nothing to say definitely. Although the dynamics of spontaneous processes usually shows a steady recurrence and invariance, there is evident [2] that the characteristics of the spontaneous processes can vary with change of environment. By induced or simulated emission is usually meant the emission produced because of an external field action on the emitting source at the radiation frequency.

There were difficulties in the quantum description with interpretation of the stimulated emission as coherent, where in contrast to the classical case it was impossible to say anything about the phases of the fields emitted by individual atoms and molecules. However, C. Townes believed that "... the energy delivered by the molecular systems has the same field distribution and frequency as the stimulating radiation and hence a constant (possibly zero) phase difference" [3].

If we assume, relying upon the results of the studies of fluctuation correlations in the laser radiation [4], that a stimulated emission has a high proportion of the coherent component, one can find a threshold of coherent radiation at a certain critical value of population inversion [5]. The specific feature of this threshold is that it

follows from the condition that the initial value of the population inversion is equal to the square root of the total number of states. On the other hand, the change in the nature of the process near the threshold is evident, even without making any other assumptions. Above this threshold, the number of photons begins to grow exponentially with time. Herewith, below the threshold there no exponential growth.

It is known that at low levels of spontaneous component and far above the maser generation threshold the number of photons grows exponentially and the radiation is largely a coherent [6, 7]. The meaningful indicator of the collective character of stimulated emission is the so-called photon degeneracy, which is defined as the average photon number contained in a single mode of optical field (see, for example [8]). For the incoherent light, this parameter does not exceed unity, but for even the simplest He-Ne maser it reaches the value of 10^{12} as was shown in the early works (see [6]).

It is of interest to go further and analyze the consequences of consideration of the spontaneous emission as a random process (at least, in a homogeneous medium) and induced process as a coherent process. It is clear that the separation of total radiation into two categories: the stimulated – coherent and spontaneous – random or incoherent will be idealized simplification. However, such separation may explain, at least qualitatively, the nature of the radiation emitted by two-level quantum system near to exposed threshold.

Another indirect proof of the existence of such a threshold is the following observation. The intensity of the spontaneous emission, which is non-synchronized (randomly distributed) over oscillators phases is known to be proportional to their number. The intensity of the coherent stimulated emission is proportional in turn to the square of the number of oscillators. It is easy to see that the exposed threshold corresponds to the case when the intensity of spontaneous and stimulated coherent radiation becomes equal.

In [5, 8] we have shown that under these conditions the pulse of induced radiation with a characteristic profile is formed when the initial population inversion slightly exceeds the threshold. The leading edge of the pulse due to the exponential growth of the field is very

sharp due to the exponential growth of the field, and the trailing edge is rather broadened. Further overriding of the threshold, that is growing of the initial population inversion, results in the ratio of the trailing edge duration to the leading edge duration becomes greater. At large times the incoherent radiation dominates.

Because very small value of the initial population inversion can provide generation of pulses of induced radiation, it is of interest to determine the shape of these pulses for different values of the initial population inversion levels and when the field energy absorption should be taken into account. These pulses can be easily detected in experiments. In addition, after experimental validation of this model, it will be possible to use these approaches for analysis of the cosmic radiation that might help explain such abundance of coherent radiation sources in space.

In this paper, we study the characteristics of the pulses of induced (practically coherent) radiation as a function of the initial inversion and absorption level in the system. The dynamics of the emission process in the simplified model is compared with the dynamics of change in the number of quanta in the traditional model, where the separation into spontaneous and induced components is not carried out.

If there is exist a recovery mechanism for the population inversion, the pulsating mode of stimulated emission generation becomes possible. The integral radiation intensity at this may be increased several times. This approach can be used for analysis of the cosmic radiation that might help explain a great variety of pulsating radiation sources in space. In present work, we investigate the characteristics of the periodic pulse generation depending on the initial inversion, the pumping level and the absorption rate.

1. THE DESCRIPTION OF TWO-LEVEL SYSTEM

Following to A. Einstein [1], a two-level system with transition frequency $\varepsilon_2 - \varepsilon_1 = \hbar\omega$ can be described by following set of equations:

$$\partial n_2 / \partial t = -(u_{21} + w_{21} \cdot N_k) \cdot n_2 + w_{12} \cdot N_k \cdot n_1, \quad (1)$$

$$\partial n_1 / \partial t = -w_{12} \cdot N_k \cdot n_1 + (u_{21} + w_{21} \cdot N_k) \cdot n_2,$$

where the sum of level populations $n_1 + n_2 = N$ remains constant, $u_{21}n_2$ is the rate of change in the number density of atoms due to spontaneous emission. The rates of change in level population due to stimulated emission and absorption are $w_{21}N_k n_2$ and $w_{12}N_k n_1$ correspondingly. The number of quanta N_k on the transition frequency ω is governed by the equation

$$\frac{\partial N_k}{\partial t} = (u_{21} + w_{21} \cdot N_k) \cdot n_2 - (w_{12} \cdot N_k) \cdot n_1. \quad (2)$$

Note that the relationship between the coefficients u_{21} and w_{21} can be represented as follows:

$$\frac{u_{21}}{w_{21}} = g = \frac{A_{21}}{\hbar\omega \cdot B_{21}} = \frac{2\omega^2}{\pi c^3},$$

where A_{21} and B_{21} – corresponding Einstein coefficients. For yellow light numerical value g equally 0.25, for violet light at the edge of the visible spectrum 0.6.

The losses of energy in active media are caused mainly by radiation outcome from a resonator. These radiative losses can be calculated by imposing the correct boundary conditions on the field. Thus, they can be estimated in rather common form with the following parameter:

$$\delta = \frac{\iiint_s \frac{\partial \omega}{\partial k} \frac{1}{4\pi} \vec{E} \times \vec{H} ds}{\iiint_v \frac{\partial[\omega \varepsilon(\omega, \vec{k})]}{\partial \omega}} \times \times \frac{1}{8\pi} (|\vec{E}|^2 + |\vec{H}|^2) dv, \quad (3)$$

i.e. as the ratio of the energy flow passing through the resonator mirrors to the total field energy within resonator. It is important, that the characteristic size of the resonator L should be much less than the characteristic time of field variation $\tau \sim |\vec{E}|^2 (\partial |\vec{E}|^2 / \partial t)^{-1}$ multiplied by the group velocity of oscillations $|\partial \omega / \partial \vec{k}|$. In this case the radiative losses through the mirrors can be replaces by distributed losses within the resonator volume. The threshold of instability leading to exponential growth of induced emission in this case is defined by condition $\mu_0 > \mu_{TH1}$ (see, for example [6], where

$$\mu_{TH1} = \delta / w_{21}. \quad (4)$$

Equations (1)-(2) can be rewritten in the form

$$\partial n_2 / \partial \tau = -g \cdot n_2 - \mu \cdot N_k, \quad (5)$$

$$\partial \mu / \partial \tau = -2g \cdot n_2 - 2\mu \cdot N_k, \quad (6)$$

$$\partial N_k / \partial \tau = g \cdot n_2 + \mu \cdot N_k, \quad (7)$$

where $\tau = w_{21} \cdot t$, $g^{-1} \cdot u_{21} = w_{21} = w_{12}$. Since the purpose of this work is to find the threshold of the initial population inversion, which starts the exponential growth of the number of emitted quanta, we will restrict our consideration by the case $\mu = n_2 - n_1 \ll n_1, n_2$. It follows from Eqs. (5) - (7) that

$$N_k = N_{k0} + (\mu_0 - \mu) / 2 \approx (\mu_0 - \mu) / 2,$$

and at large times

$$g \cdot n_{2st} \approx g \cdot N / 2 = -\mu_{st} \cdot (\mu_0 - \mu_{st}) / 2,$$

where $\mu_0 = \mu(\tau = 0)$, $N_{k0} = N_k(\tau = 0)$. Hence, we find the stationary value of the inversion

$$\mu_{st} = (\mu_0 / 2) - \sqrt{(\mu_0 / 2)^2 + g \cdot N}. \quad (8)$$

Two cases are of interest. When the initial population inversion is sufficiently large $(\mu_0 / 2)^2 \gg g \cdot N$, it rapidly decreases to its steady-state value $\mu \rightarrow \mu_{st1} = -(g \cdot N / \mu_0)$ with $|\mu_{st1}| \ll \mu_0$. The number of quanta at this growths exponentially and asymptotically tends to a stationary level $N_k \rightarrow N_{kst1} = \mu_0 / 2$. It is obviously that in this case the stimulated emission dominates (the second terms in r.h.s. of Eqs. (5) - (7)).

The second case of interest corresponds to relatively small initial inversion $(\mu_0 / 2)^2 \ll g \cdot N$. Here, μ tends to its stationary value $\mu \rightarrow \mu_{st} = -(g \cdot N)^{1/2}$, where $|\mu_{st}| > \mu_0$, and the number of quanta reaches the limit $N_k \rightarrow N_{kst2} = (g \cdot N)^{1/2}$.

If the spontaneous emission only dominated (the first terms on the r.h.s. of Eqs. (5) - (7)), the characteristic time to reach the steady-state number of photons will be of the order of $\Delta \tau \sim \tau_m = \mu_0 / gN > \mu_0^{-1}$ in the first

case and $\Delta\tau \sim 1/\sqrt{g \cdot N} < \mu_0^{-1}$ in the second case, where μ_0^{-1} is the characteristic time of exponential growth of the number of photons in the first case. This means that the exponential growth of the number of photons in the second case is suppressed and the role of the second terms in r.h.s. of Eqs. (5) - (6) comes to stabilize the number of particles and the inversion level due to the absorption process.

Thus, it is clear that the scenario of the process changes, if the initial value of the inversion μ_0 is more or less than a threshold value [5]:

$$\mu_{TH2} = 2(g \cdot N)^{1/2}. \quad (9)$$

2. THE CONDITIONS OF NEAR-THRESHOLD PERIODIC RADIATION

When taking into account the collision mechanisms create an inversion of equation (1) can be written in the form

$$\partial n_2 / \partial \tau = -g \cdot n_2 - \mu \cdot N_k + \frac{v_{col}}{u_{21}} n_1; \quad (10)$$

$$\partial n_1 / \partial \tau = +g \cdot n_2 + \mu \cdot N_k - \frac{v_{col}}{u_{21}} n_1; \quad (11)$$

$$\partial \mu / \partial \tau = (v - u_{21}) n_1 - 2\mu - 2\mu \cdot N_k, \quad (12)$$

where v_{col} - effective collision frequency, which translate the particle from the lower energy level to the top. Under equilibrium conditions, it is obvious $v_{col} \propto u_{21}$ and $(v_{col} - u_{21}) n_1 = \mu_0^2 I_0$.

Near the threshold (9), or rather when the condition $n_2 \approx n_1$, the steady state of the spontaneous emission is determined by the value $\delta \cdot N_k^{(incoh)} \approx g \cdot N / 2$, but the energy flow of the stimulated emission is equal to $\delta \cdot N_k^{(coh)}$. In order to describe the behavior of a two-level system in presence of the radiation losses and collisions account [9]

$$\partial M / \partial T = I_0 - 2M - 2M \cdot N_c, \quad (13)$$

$$\partial N_{inc} / \partial T = N_0 / 2 - \theta \cdot N_{inc}, \quad (14)$$

$$\partial N_c / \partial T = M \cdot N_c - \theta \cdot N_c, \quad (15)$$

where $N_{inc} = N_k^{(incoh)} / \mu_0$, $N_c = N_k^{(coh)} / \mu_0$, $M = \mu / \mu_0$, $T = w_{21} \cdot \mu_0 \cdot t = \mu_0 \cdot \tau$, $I_0 = I / \mu_0^2$, and the only convenient for analysis free element is $N_0 = N / \mu_0^2$. It is rational to choose $\mu_0 = \delta / w_{21}$ [9]. That is, in this case both the threshold (4) and (9) $\mu_{TH2} \approx \mu_{TH1} = \delta / w_{21}$ are close.

In this case, the relaxation oscillations appear in the system resulting in a stationary state $N_{cst} \approx -1 + I_0 / 2\theta$, $M_{st} = \theta$. The total radiation flow outside the system in assumed terms is equal to

$$\theta \cdot N_{cst} + \theta \cdot N_{incst} = (I_0 - 2\theta) / 2 + N_0 / 2 \geq N_0 / 2.$$

Note, that in presence of an external mechanism, which provides an exceedance of the inversion over its stationary value $M_{st} = \theta$, the Eq. (13) can be supplemented by the driving term

$$\partial M / \partial T = \Gamma M - 2M \cdot N_c + 2I_0. \quad (16)$$

As will be shown [10], only positive definite value Γ leads to undamped periodic oscillations and radiation inversion. One of the few physical causes such positive definiteness Γ is a convection

$$-v \cdot \partial M / \partial X \approx v \frac{M}{L} = \Gamma M > 0,$$

where $v \cdot grad M = -M \cdot L^{-1}$, convective transport is determined by the speed of inversion v of the denser areas corresponding to the coordinate scale.

The Eqs. (14) - (16) are similar to so-called Stutz-DeMars equations [11], which describe the relaxation oscillations in a two-level media in the presence of the pump and energy losses. The only difference in equation (16) is the first term in r.h.s. that provides the maintenance of the population inversion. Namely this term changes the characteristics of pulse generation from relaxation to periodic.

The Eqs. (14) - (16) have a solution in a form of periodical sequence of coherent pulses against a background of the mean radiation flow, if $v_{col} = u_{21}$:

$$\theta \cdot N_{cst} + \theta \cdot N_{incst} = (\tilde{\Gamma} \theta + N_0) / 2, \quad (17)$$

where $\tilde{\Gamma} = \Gamma - 2 > 0$, $N_{cst} \approx (I_0 + \tilde{\Gamma} \theta) / 2\theta$, and $\tilde{\Gamma} > I_0 / \theta$.

Pulse repetition rate is $\sqrt{\theta \cdot \tilde{\Gamma}}$. The integral radiation intensity on the pulse peak can exceed the background value in several times.

It should be noted that the radiation losses of the field energy θ in open systems is defined as the ratio of the energy flux from the object to the energy in its volume, and therefore this parameter decreases with increase of the radius of the system R as c / R , where c is the speed of light. This means that an increase in size R reduces the losses θ , which in turn, provides a higher intensity of the stimulated emission. That is, at the same parameters of the system, the larger objects should generate more intense pulses but with less repetition rate.

3. STRUCTURE OF RADIATION SOURCES

The equilibrium state of the gas consisting of active atoms excited by collisions with free electrons and relaxed due to spontaneous emission can be describes by the relation $I_0 \equiv v_{col} \cdot n_1 - n_2 \cdot u_{21} = 0$. The layer with a higher temperature satisfies the condition $I_0 > 0$, that means that the collisional excitation of active atoms by free electrons of the heated medium is quite significant. Analysis of the solutions of the Eqs. (14) - (16) shows that in this layer the integral radiation of these excited atoms does not have an oscillatory character (its dynamics is monotonic) [10, 11]. In the upper cold layers of the source, in contrast, the efficiency of excitation of active atoms due to collisions of atoms with the free electrons decreases and the condition $I_0 < 0$ is met, and there is no generation of the induced radiation. However, in the upper layers, both spontaneous and induced radiation can be scattered to such an extent that the integral spectrum becomes a blackbody radiation spectrum.

The appearance of dark lines in the spectrum of the source can be easily explained, by the presence of the third level in the quantum system, from which the ener-

gy is transferred to the second (radiative energy level) by the use of nonradiative or other transition processes. As this takes place, $v_{col}n_1 \approx u_{32}n_3$. Therefore, such a high level of absorption of radiation with a frequency corresponding to the excitation of this nonradiative level can be observed.

The pulse generation mode is possible only in the presence of convection ensuring the delivery of inverted atoms of the active substance out of dense underlying layers, i.e. $-v\partial\mu/\partial x \approx v\mu/l = \tau \cdot \mu$. The convection in these cases should be rather intensive $(v/l \cdot u_{21}) > \mu_{TH2}$.

In addition, hotter sources have a greater activity of convective currents, i.e. Γ increases with increasing temperature of the surface layers. Therefore, the frequency of the intensity oscillations of small hot sources is higher, but for large and cold sources, it is lower.

CONCLUSIONS

The threshold of coherent emission generation, discussed in this paper, corresponds to the case when the intensity of spontaneous and stimulated coherent radiation becomes equal. The stimulated emission in this case can be considered as completely coherent or as a set of narrow wave packets of coherent radiation. When the initial population inversion crosses the threshold (9), the process of generation undergoes qualitative changes. The excess of the threshold (9) leads to an exponential growth in the number of quanta. If we make the assumption that the stimulated emission is mainly coherent, the nature of this threshold can be explained as follows: generation of coherent radiation begins only after crossing of this threshold. In this work, we have tried to develop a qualitative model of this process.

If we fix all parameters except the inversion, the duration of the coherent pulse estimated by its half-width significantly increases with increasing initial inversion in the absence of absorption [9]. For relatively small inversion levels $\sqrt{g \cdot N} \ll \mu_0 \ll g \cdot N$ the coherent emission is always presents as a rather short pulse with duration of $\tau \propto (\mu_0 / gN)$. At large times $\tau > (\mu_0 / gN)$ the incoherent radiation dominates. If there is a mechanism for supporting the inversion, such as convection from optically dense layers into less dense one, transparent to the radiation, the sign of the first term of the equation (16) is positive definite. Then we may receive periodic pulses of stimulated emission.

The intensity of the field of these pulses is comparable and may exceed the intensity of the integrated spontaneous emission, because even at very small inversion

$$\mu = (n_2 - n_1) \ll g \cdot N = g \cdot (n_1 + n_2) < \mu^2 = (n_2 - n_1)^2$$

the intensity of the spontaneous emission is proportional to $g \cdot N / 2$ near the threshold (9), the intensity of the involuntary largely coherent radiation is proportional to

the $\mu^2 = (n_2 - n_1)^2 \geq g \cdot N / 2$. That is, such a mechanism may be responsible for the periodic change in the luminosity of stars with a rarefied atmosphere, which should be strong convective currents [12].

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ПЕРИОДИЧЕСКОЕ ИЗЛУЧЕНИЕ В СЛАБОИНВЕРТИРОВАННЫХ СРЕДАХ

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Рассмотрено изменение характера генерации лазера в двухуровневой системе, когда начальная инверсия населенности превышает некоторое пороговое значение, равное квадратному корню из общего числа состояний. Выше этого порога генерируется импульс с коротким передним фронтом и расширенным задним фронтом. Длительность когерентного импульса, оцененная по его полуширине, значительно возрастает с увеличением инверсии, если все остальные параметры фиксированы и поглощение пренебрегается. Включение потери энергии фотонов приводит к тому, что длительность когерентного импульса почти постоянна с увеличением инверсии, по крайней мере, далеко от порога. Если существует механизм восстановления инверсии населенностей, становится возможным пульсирующий режим генерации вынужденного излучения. Интегральная интенсивность излучения при этом может быть увеличена в несколько раз. Этот подход может быть использован для анализа космического излучения, которое могло бы помочь объяснить большое разнообразие пульсирующих источников излучения в космосе.

ПЕРІОДИЧНЕ ВИПРОМІНЮВАННЯ В СЛАБКОІНВЕРТОВАНИХ СЕРЕДОВИЩАХ

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Розглянуто зміну характеру генерації в дворівневій системі, коли початкова інверсія населеності перевищує порогове значення, яке дорівнює квадратному кореню із загальної кількості станів. Вище цього порога генерується імпульс з коротким переднім фронтом і розширеним заднім фронтом. Тривалість когерентного імпульсу, оцінююча за його напівшириною, значно зростає зі збільшенням інверсії, якщо всі інші параметри фіксовані і поглинання нехтується. Включення втрат енергії фотонів призводить до того, що тривалість когерентного імпульсу майже постійна зі збільшенням інверсії, принаймні, далеко від порогу. Якщо існує механізм відновлення інверсії заселеності, стає можливим пульсуючий режим генерації вимушеного випромінювання. Інтегральна інтенсивність випромінювання при цьому може бути збільшена в декілька разів. Цей підхід може бути використаний для аналізу космічного випромінювання, яке могло б допомогти пояснити велику різноманітність пульсуючих джерел випромінювання в космосі.