

SIMULATION OF REGIMES IN THE PULSE-FORMING LINES BY OPERATOR METHOD

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In this paper the application of the Laplace transform to the analysis of circuits with distributed parameters is considered. As an example pulse-forming lines based on distributed parameters lines are analyzed. Such devices are widely applied in electronics, high-voltage technology and high-current charged particle accelerators. In the above examples, both of classic methods for solving partial differential equations: Fourier method and d'Alembert method (method of travelling waves), and method based on Laplace transform of the periodic function. The detailed solution of the problem of formation of voltage pulses using circuits with distributed parameters is carried out. The repeated Laplace transform is shown to allow to pass from partial differential equations to algebraic equations and to simplify the solution of the problem. The time diagrams of the processes of charge and discharge of the line for various loads are given.

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INTRODUCTION

Processes in electrical devices are typically described by differential equations or systems of differential equations with respect to some unknown functions typically the currents in the branches of the circuit or voltage on the elements. Various methods [1, 2] apply to their solutions. In radio- and electrical-engineering analysis is performed using an electric scheme of the device, so in engineering practice methods have the greatest appeal, formalizing the task, provided the least departure from the scheme. Such methods include the Laplace transform, underlying operational method of analysis of radio- and electric systems and devices [3 - 7].

The main advantage of the operator method is the ability to transition from the n -th order differential equation with respect to the unknown function $f(t)$ to an algebraic equation for the complex variable function $F(p)$. Such transform proposed Heaviside, is performing by the replacing of the differential operator d/dt on formally operator p , which in this case is a complex number, and on which trivial algebraic transformations possible.

A bijective correspondence (in terms of Laplace transform) of the original and the image $f(t) \leftrightarrow F(p)$ allows us to find the solution of the problem by the reverse transition from the image to the original using known methods. The conditions which function $f(t)$ should satisfy to be performed as the original of the image $F(p)$, as a rule take place in radio and electrical devices, so check in this compliance may not carry out [8].

It is crucial that the originals may be functions that describe the processes in circuits with distributed parameters and is a function of both time and coordinates. These circuits include the transmission and formation lines, in the analysis of which the unknowns are usually instantaneous values of voltage $u(t, x)$ and current $i(t, x)$, in the various sections of line x . Although the simulation of such circuits yet requires the solution of differential equations in partial derivatives, the operator method nonetheless performs as a reliable and demonstrative tool of the analysis.

1. MODEL OF THE DISTRIBUTED PARAMETERS CIRCUIT

Consider the two-wire line in which the current i flows in the same direction as the positive direction of the x axis (Fig. 1). Any arbitrarily taken an infinitesimal piece of the line Δx can be represented by an equivalent circuit consisting of lumped elements infinitesimal ΔL , ΔR , ΔC and ΔG which characterize the resistance and inductance of the wire piece and the capacitance between the wires and the leakage conductance insulation of the considered piece.

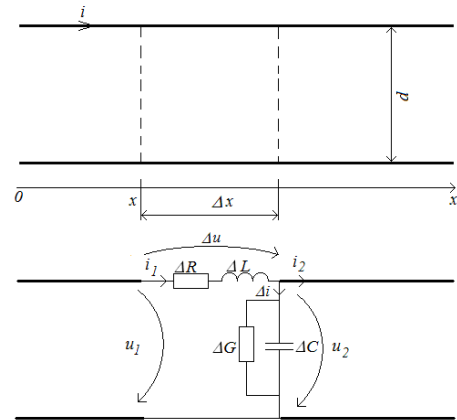


Fig. 1. Two-wire transmission line and equivalent scheme of the elementary piece

On the basis of the Kirchoff equations for the equivalent scheme of line's piece Δx results

$$\frac{u(x) - u(x + \Delta x)}{\Delta x} = \frac{\Delta L}{\Delta x} \frac{\partial i}{\partial t} + \frac{\Delta R}{\Delta x} i, \quad (1a)$$

$$\frac{i(x) - i(x + \Delta x)}{\Delta x} = \frac{\Delta C}{\Delta x} \frac{\partial u}{\partial t} + \frac{\Delta G}{\Delta x} u, \quad (1b)$$

and then proceed to the limit at $\Delta x \rightarrow 0$ and take into account the definition of a derivative with respect to the left-hand side of each equation (1) [9]. As a result, we obtain a system of equations of a transmission line

$$-\partial u / \partial x = L \partial i / \partial t + Ri, \quad (2a)$$

$$-\partial i / \partial x = C \partial u / \partial t + Gu, \quad (2b)$$

where linear parameters introduced

$$R = \lim_{\Delta x \rightarrow 0} \frac{\Delta R}{\Delta x} = \frac{dR}{dx}, \quad L = \lim_{\Delta x \rightarrow 0} \frac{\Delta L}{\Delta x} = \frac{dL}{dx},$$

$$C = \lim_{\Delta x \rightarrow 0} \frac{\Delta C}{\Delta x} = \frac{dC}{dx}, \quad G = \lim_{\Delta x \rightarrow 0} \frac{\Delta G}{\Delta x} = \frac{dG}{dx}.$$

2. BASIS OF THE OPERATOR METHOD

To get the operator model of a transmission line let's apply to the system of equations (2) Laplace transform [4, 5]. Voltage and current (2) are a function of two variables $u(t, x)$, $i(t, x)$, whereas in the Laplace transform the integration is made by one variable. Taking into account that the domain of the originals definition by the variable t is a semi-infinite $(0, \infty)$ or finite intervals $(0, t_1)$, (t_1, t_2) , we calculate the integral of the Laplace transform by the variable t . Then for the left-hand part (2a), we obtain

$$\int_0^{\infty} \frac{\partial u}{\partial x} e^{-pt} dt = \frac{d}{dx} \int_0^{\infty} u e^{-pt} dt = \frac{dU}{dx}. \quad (3)$$

It is considered here that the variables x and t are independent, so changing the order of integration and differentiation is rightly. In addition, the partial derivative is replaced by the total one, because the variable p is regarded as a parameter [6].

Further, for the right side (2a), using the integration by parts, we get

$$\int_0^{\infty} (L \partial i / \partial t + Ri) e^{-pt} dt = -Li(0) + pI + RI. \quad (4)$$

In the formulas (3) and (4) by the symbols U and I the transformed Laplace function of voltage and current in the line are marked, which in the operational method are called as the images. Thus we have $u(t, x) \Leftrightarrow U(p)$, $i(t, x) \Leftrightarrow I(p)$. To simplify entries function further the arguments will be omitted.

Having done a similar transformation to the equation (2b), we obtain as a result the following relations

$$-dU/dx = (pL + R)I - Li(0), \quad (5a)$$

$$-dI/dx = (pC + G)U - Cu(0). \quad (5b)$$

We note at once the Laplace transform important property: the initial conditions of the task are taken into account automatically, resulting last term in (5). For the lossless line $R=0$, $G=0$ and zero initial conditions the equations (5) are simplified

$$-dU/dx = pLI, \quad (6a)$$

$$-dI/dx = pCU. \quad (6b)$$

Differentiating (6a) on the x and substituting (6b) in this expression we obtain:

$$d^2U/dx^2 - \gamma^2U = 0. \quad (7a)$$

The same transform may be performed with equation (6b) that gives as the result the expression for the current

$$d^2I/dx^2 - \gamma^2I = 0, \quad (7b)$$

where the operational wave number

$$\gamma = p\sqrt{LC}. \quad (8)$$

As is known [4, 10], the general solution of a homogeneous linear differential equation II order form (7) is given by

$$U = A \operatorname{ch} \gamma x + B \operatorname{sh} \gamma x, \quad (9)$$

and for the current

$$I = -1/Z_0 (A \operatorname{sh} \gamma x - B \operatorname{ch} \gamma x). \quad (10)$$

Here, the designation for the operational characteristic (wave) impedance has been introduced

$$Z_0 = \sqrt{L/C}. \quad (11)$$

3. CALCULATION EXAMPLES

Here consider some examples of calculation voltage and current distributions in line at different regimes of loading by means of operational method.

3.1. THE LINE CHARGING FROM DC VOLTAGE SOURCE

A segment of a line opened at the end of length l at time $t = 0$ is connected to a DC voltage source U_0 . The sequence of the solution of the problem is set forth in [4]. The initial energy supply in the line is absent, therefore, we have a problem with zero initial conditions: $u(0, x) = 0$, $i(0, x) = 0$.

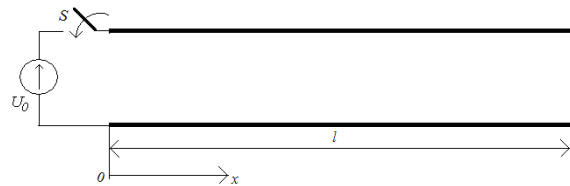


Fig. 2. Scheme of the line charging from DC voltage source

The boundary condition at the beginning of the line, where the DC voltage source connected (Fig. 2.), is written as $u(t, 0) = U_0$. The second boundary condition for the right end of the line: $u(t, l) = U_0$. These conditions are valid for any time $t > 0$, therefore we can write in the operator form:

$$U|_{x=0} = U_0/p, \quad I|_{x=l} = 0.$$

Then, setting $x = 0$ in (9), on the basis of the first boundary condition we obtain

$$A = U_0/p.$$

Substituting $x = l$ in (10), on the basis of the second boundary condition,

$$A \operatorname{sh} \gamma l - B \operatorname{ch} \gamma l = 0,$$

from which we find the second integration constant

$$B = \frac{U_0 \operatorname{sh} \gamma l}{p \operatorname{ch} \gamma l}.$$

Now substitute the integration constants calculated in (9), (10), and obtain images of voltage and current

$$U = \frac{U_0}{p} \frac{\operatorname{ch} \gamma (l-x)}{\operatorname{ch} \gamma l}, \quad (12a)$$

$$I = \frac{U_0}{p} \frac{\operatorname{sh} \gamma (l-x)}{Z_0 \operatorname{ch} \gamma l}. \quad (12b)$$

As is known, to calculate the original, expressed by the ratio of the functions F_1 and F_2 , one can use the decomposition theorem in the form [6]:

$$f(t) = \frac{F_1(0)}{F_2(0)} + \sum_{n=1}^N \frac{F_1(p_n)}{p_n F_2'(p_n)} e^{p_n t}, \quad (13)$$

where the summation is over all the roots of the denominator

$$p_n = \frac{\gamma_n}{\sqrt{LC}} = j \frac{2n+1}{l\sqrt{LC}} \frac{\pi}{2}$$

Denoting by: $\tau = 2x\sqrt{LC}$, $q = T/\tau$,
 $\omega_1 = 2\pi/4l\sqrt{LC}$,

$A_n = U_0 \sin\left[(2n+1)\frac{\pi}{2} \frac{x}{l}\right] / \left[(2n+1)\frac{\pi}{2} \frac{x}{l}\right]$ the final solution obtains in form

$$u(t,x) = U_0 + \sum_{n=1}^{\infty} A_n [\cos(n\omega_1 t + \pi) + \cos(n\omega_1 t + n\pi)],$$

that corresponds to Fourier expansion formulas for the two sequences of rectangular pulses [11].

According to d'Alembert method the general solution of equations (7) is written in the form

$$U = C_1 e^{\gamma x} + C_2 e^{-\gamma x},$$

$$I = -\frac{1}{z_0} (C_1 e^{\gamma x} - C_2 e^{-\gamma x}),$$

where C_1 and C_2 again denote the integration constants, which are related to the constants A and B by the relations $C_1 = (A+B)/2$, $C_2 = (A-B)/2$.

System of equations for calculating the integration constants looks like as

$$C_1 + C_2 = U_0/p$$

$$C_1 e^{\gamma l} + C_2 e^{-\gamma l} = 0$$

solving which, we get an image of the voltage

$$U = \frac{U_0}{p} \frac{e^{-\gamma x} + e^{-\gamma(2l-x)}}{1 + e^{-2\gamma l}} =$$

$$= \frac{U_0}{p} \frac{e^{-\gamma x} + e^{-\gamma(2l-x)} - e^{-\gamma(2l+x)} - e^{-\gamma(4l-x)}}{1 - e^{-4\gamma l}}. \quad (14)$$

Next, note that if $\text{Re}(p) > 0$, then $|e^{-4\gamma l}| < 1$. As a result the modulus of the second term in the denominator (14) is also less than unity, and we can construct the series

$$\frac{1}{1 - e^{-4\gamma l}} = \sum_{n=0}^{\infty} e^{-4n\gamma l},$$

the substitution of which in (14) leads to

$$U = \frac{U_0}{p} \left[e^{-\gamma x} + e^{-\gamma(2l-x)} - e^{-\gamma(2l+x)} - e^{-\gamma(4l-x)} \right] \sum_{n=0}^{\infty} e^{-4n\gamma l}.$$

Rewrite the series in expanded form

$$U = \frac{U_0}{p} \left(e^{-p\frac{x}{V}} + e^{-p\frac{2l-x}{V}} - e^{-p\frac{2l+x}{V}} - e^{-p\frac{4l-x}{V}} + \dots \right). \quad (15)$$

Passing to the originals of the terms of the resulting expression, we find

$$u(t,x) = \frac{U_0}{p} \left[1\left(t - \frac{x}{V}\right) + 1\left(t - \frac{2l-x}{V}\right) - 1\left(t - \frac{2l+x}{V}\right) - 1\left(t - \frac{4l-x}{V}\right) + \dots \right], \quad (16)$$

where $1(\cdot)$ – Heaviside function.

It is seen that the inverse Laplace transform of expression (16), taking into account the delay theorem of the original, gives a voltage in the form of a superposi-

tion of step waves shifted relative to each other by an amount that is a multiple of $4\gamma l$ and propagating towards each other: from the beginning of the line and from the load. Thus, the first term determines a wave of height U_0 arising at the time l/V and propagating towards positive x . The second term corresponds to a wave of height U_0 arising at the moment $(2l-x)/V$ at the end of the line, and propagating in the direction of decreasing x , etc. The imposition of waves propagating in opposite directions forms an interference pattern in the line (Fig. 3).

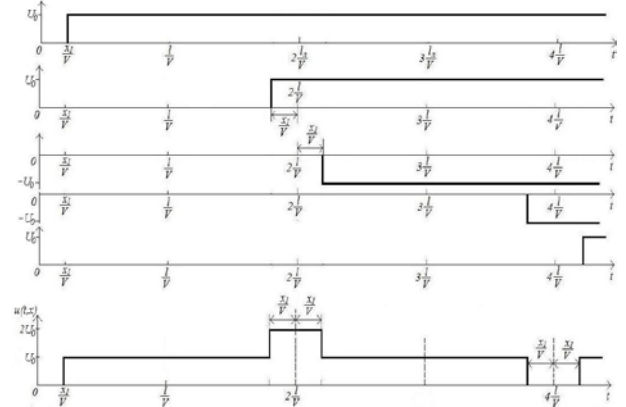


Fig. 3. Travelling waves and formation of the transfer characteristic of the open transmitting line

3.2. DISCHARGE LINE ON SHORT CIRCUIT

An important case for the practice is to use the line as a storage element [12, 13] Consider the case of the short circuit lossless line pre-charged to voltage U_0 (Fig. 4).

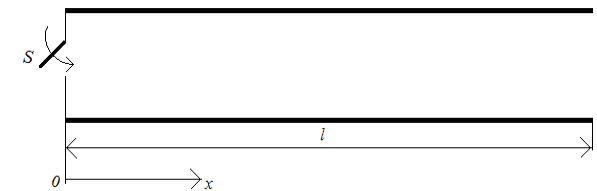


Fig. 4. Line discharge on short circuit

Initial conditions

$$u(0,x) = U_0, \quad i(0,x) = 0 \quad \text{at } 0 < x < l$$

using the formula (1) are provided in operator form

$$U = U_0/p, \quad I = 0.$$

Similarly boundary conditions

$$u(t,0) = 0, \quad i(t,l) = 0 \quad \text{at } t > 0$$

have a such operational form

$$U|_{x=0} = 0, \quad I|_{x=l} = 0.$$

Now again we apply the Laplace transform to the system of telegrapher's equations for the lossless line (6) and obtain

$$-dU/dx = pLI, \quad (17a)$$

$$-dI/dx = pCU - CU_0. \quad (17b)$$

Here, the second term on the right side (17b) implies that the line was charged to a voltage U_0 . Its appearance is due to the use of the Laplace transform to the right side of the equation (2b) on the condition $G = 0$. Along the way, once again we see that in the operational method the initial conditions are accounted automatically. Differentiating (17a) in x and substituting in the re-

sulting expression in place of the current's derivative the expression (17b), we arrive at the inhomogeneous equation for voltage in the form of

$$d^2U/dx^2 - \gamma^2U = -pLCU_0. \quad (18)$$

Solution of the (18) is written as:

$$U = C_1e^{\gamma x} + C_2e^{-\gamma x} + U_0/p. \quad (19a)$$

The image of the current is

$$I = -\frac{1}{pL} \frac{dU}{dx} = -\frac{1}{Z_0} (C_1e^{\gamma x} - C_2e^{-\gamma x}). \quad (19b)$$

In order to calculate the integration constants, we substitute the boundary conditions in (19) and obtain a system of equations

$$C_1 + C_2 = -U_0/p \quad (20a)$$

$$C_1e^{\gamma l} - C_2e^{-\gamma l} = 0. \quad (20b)$$

To solve it the Cramer's method is used

$$\tilde{N}_1 = \frac{\begin{vmatrix} -\frac{U_0}{p} & 1 \\ 0 & -e^{-\gamma l} \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ e^{\gamma l} & -e^{-\gamma l} \end{vmatrix}} = -\frac{U_0}{p} \frac{e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}}, \quad (21a)$$

$$\tilde{N}_2 = \frac{\begin{vmatrix} 1 & -\frac{U_0}{p} \\ e^{\gamma l} & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ e^{\gamma l} & -e^{-\gamma l} \end{vmatrix}} = -\frac{U_0}{p} \frac{e^{\gamma l}}{e^{\gamma l} + e^{-\gamma l}}. \quad (21b)$$

Substituting the integration constants in (19a) and taking into account the equality $\gamma = p\sqrt{LC} = p/V$, we obtain

$$U = \frac{U_0}{p} \left[1 - \left(e^{-\frac{p}{V}x} + e^{-\gamma \frac{2l-x}{V}} - e^{-\gamma \frac{2l+x}{V}} - e^{-\frac{4l-x}{V}} + e^{-\frac{4l+x}{V}} + e^{-\frac{6l-x}{V}} - e^{-\frac{6l+x}{V}} - e^{-\frac{8l-x}{V}} + \dots \right) \right], \quad (22)$$

whose original

$$u(t, x) = U_0 \left[1(t) - 1\left(t - \frac{x}{V}\right) - 1\left(t - \frac{2l-x}{V}\right) + 1\left(t - \frac{2l+x}{V}\right) + 1\left(t - \frac{4l-x}{V}\right) - 1\left(t - \frac{4l+x}{V}\right) - 1\left(t - \frac{6l-x}{V}\right) + 1\left(t - \frac{6l+x}{V}\right) + 1\left(t - \frac{8l-x}{V}\right) - \dots \right].$$

The corresponding time diagram of the voltage is shown in Fig. 5. As can be seen, the voltage in the line represents an infinite sequence of rectangular pulses of opposite polarity with a height U_0 and duration of $2x/V$, which is formed as a superposition of step positive and negative polarity functions shifted in time.

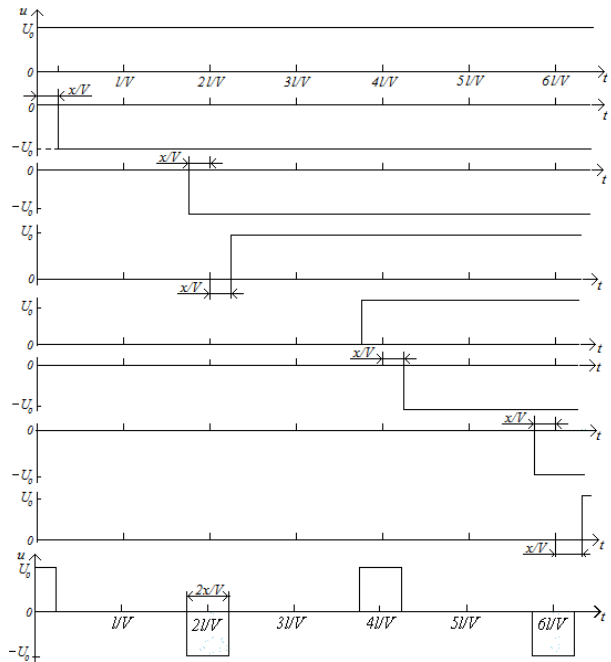


Fig. 5. Formation of voltage in arbitrary cross-section of the sort-circuited line

3.3. LINE DISCHARGING ON THE MATCHED LOAD

Let us now consider the case in which a line charged to a voltage U_0 is discharged through a resistance equal to the wave resistance $R_l = Z_0$ (Fig. 6).

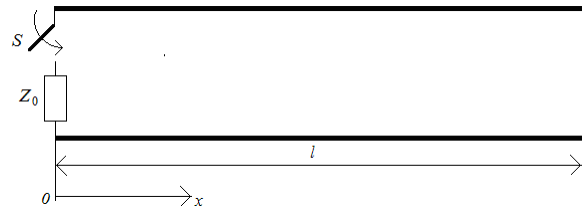


Fig. 6. Scheme of line discharging on the load

Obviously, this task differs from the previous one only by boundary conditions, which now look like this:

$$u(t, 0) = Z_0 i(t, 0), \quad i(t, l) = 0 \text{ at } t > 0$$

or in operator form

$$U|_{x=0} = Z_0 I|_{x=0}, \quad I|_{x=l} = 0.$$

Substitute these conditions in (19) and obtain

$$C_1 + C_2 + U_0/p = C_1 - C_2, \\ C_1e^{\gamma l} - C_2e^{-\gamma l} = 0,$$

from which we find the constants of integration

$$C_2 = -U_0/2p, \quad C_1 = -U_0/2p e^{-2\gamma l}.$$

Then the images of voltage and current respectively look like

$$U = \frac{U_0}{p} \left[1 - \frac{1}{2} e^{-\gamma x} - \frac{1}{2} e^{-\gamma(2l-x)} \right], \quad (23a)$$

$$I = \frac{U_0}{2pZ_0} \left[e^{-\gamma(2l-x)} - e^{-\gamma x} \right]. \quad (23b)$$

Let's turn to the originals of the received expressions:

$$u(t) = U_0 \left[1(t) - \frac{1}{2} 1\left(t - \frac{x}{V}\right) - \frac{1}{2} 1\left(t - \frac{2l-x}{V}\right) \right], \quad (24a)$$

$$i(t) = \frac{U_0}{2Z_0} \left[1\left(t - \frac{l-x}{V}\right) - 1\left(t - \frac{l+x}{V}\right) \right]. \quad (24b)$$

It is important for practice to study the voltage and current directly on the matched load. Substituting $x = 0$ in (24), we obtain the corresponding expressions for the instantaneous values

$$u(t, 0) = \frac{U_0}{2} \left[1(t) - 1\left(t - \frac{l}{V}\right) \right], \quad (25a)$$

$$i(t, 0) = \frac{U_0}{2Z_0} \left[1(t) - 1\left(t - 2\frac{l}{V}\right) \right]. \quad (25b)$$

We are convinced that the regime of discharge of the line for a coordinated load ensures the formation of a rectangular voltage pulse with height $U_0/2$ and duration $2l/V$ (Fig. 7). Therefore, the corresponding electrical circuit is called the forming line. Forming lines are used as storage elements in modulators of powerful radio transmitters, in accelerators of charged particles.

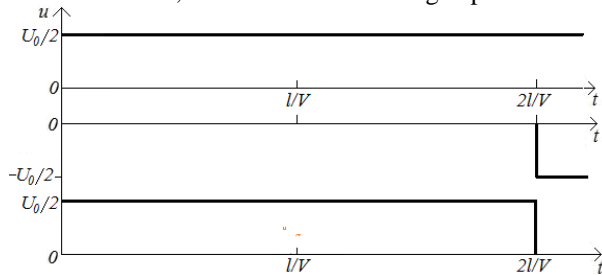


Fig. 7. Voltage pulse on the load of the matched line segment $x=0$

4. REPEATED LAPLACE TRANSFORM

We will consider a case now, when to the line, charged to tension U_0 , runs down through resistance equal to the wave $R_i = Z_0$. We will consider the charge of line without losses long l from the source of permanent tension $e(t) = U_0$ through resistance of R_i (Fig. 8).

Such mode arises up in the case when internal resistance of source of charge tension it is impossible to ignore. Including of tension will be realized by means of the ideal key of S . We will count resistance of R_i to active, that, generally, does not limit applicability of method of calculation.

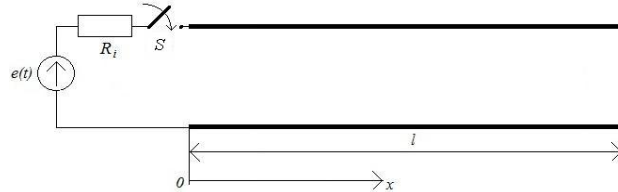


Fig. 8. Charging line scheme

We will take the system of telegraph equations in a statement form (6) and again will apply transformation of Laplace to her. Multiplying right and left parts of equations (8) on e^{-sx} and integrating after x limits from 0 to ∞ , we will get

$$s\bar{U} + pL\bar{I} = U(0), \quad (26a)$$

$$pC\bar{U} + s\bar{I} = I(0). \quad (26b)$$

We will consider thus, that functions $U(x, p)$ an $I(x, p)$ satisfy to the corresponding terms, i.e. are originals. Here \bar{U} and \bar{I} are the functions got as a result of the repeated application of transformation of La-

place to the images U and I , s – complex variable of Laplace transform. Right parts of the system of equalizations (26) reflect property of transformation of Laplace, consisting in the automatic account of border terms, again. The decision of the system (26) gives expression for tension

$$\bar{U} = \frac{sU(0) - pLI(0)}{s^2 - \gamma^2}. \quad (27)$$

Here connection of tension of source of charge of line and tension on an entrance is expressed by statement correlation

$$U(0) = \frac{U_0}{p} \frac{Z_{in}}{Z_{in} + R_i},$$

where Z_{in} is entrance statement resistance of line, equal in this case

$$Z_{in} = Z_0 \frac{1 + e^{-2\gamma l}}{1 - e^{-2\gamma l}}, \quad (28)$$

U_0/p it is an image of tension of source.

We will put (28) in (27) and we will get after transformations

$$\bar{U} = \frac{U_0}{p} \frac{Z_0}{Z_0 + R_i} \frac{1}{1 - k_{in} e^{-2\gamma l}} \left(\frac{1}{s + \gamma} + \frac{e^{-2\gamma l}}{s - \gamma} \right), \quad (29)$$

where $k_{in} = \frac{R_i - Z_0}{R_i + Z_0}$ is a reflectivity on the entrance of line.

We will consider a case, when $R_i > Z_0$, that gives $k_{in} > 0$.

The inverse Laplace transform of expression (29) allows one to obtain an image of the voltage $U(p, x)$ in the form

$$U = \frac{U_0}{p} \frac{Z_0}{Z_0 + R_i} \frac{1}{1 - k_{in} e^{-2\gamma l}} \left[e^{-\gamma x} + e^{-\gamma(2l-x)} \right]. \quad (30)$$

The terms in brackets (30) represent the incident and reflected waves, respectively. Performing a second inverse Laplace transform of (30), we find the expression for the instantaneous voltage in an arbitrary section of the line x in the form of traveling waves

$$u(t, x) = \frac{U_0 Z_0}{Z_0 + R_i} \left[1\left(t - \frac{x}{V}\right) + 1\left(t - \frac{2l-x}{V}\right) + k_{in} 1\left(t - \frac{2l+x}{V}\right) + k_{in} 1\left(t - \frac{4l-x}{V}\right) + k_{in}^2 1\left(t - \frac{4l+x}{V}\right) + k_{in}^2 1\left(t - \frac{6l-x}{V}\right) + \dots \right]. \quad (31)$$

Here it is taken into account that the factor $1/(1 - k_{in} e^{-2\gamma l})$ in (29) is a sign of the periodic original.

The time diagram of the voltage (Fig. 9) shows that the line charge occurs as a result of successive passes of the incident and reflected waves, the initial height $U_0 Z_0 / (Z_0 + R_i)$ of which decreases in proportion to the value k_{in}^n , where $n = 0, 1, 2, \dots$. The envelope of the charging voltage is described by a function $U_0 (1 - e^{-t/\tau})$ where the time constant $\tau = R_i C l$ depends on the source resistance R_i and capacitance of the $C_1 = C l$ line. The

maximum charge voltage is U_0 . In a time of 3τ , the line will be charged to a voltage of $0.95U_0$, which requires the passage of the incident and reflected waves.

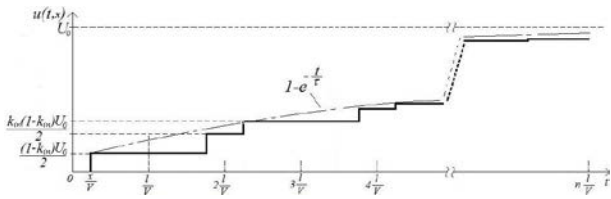


Fig. 9. Line charging from DC voltage source

For $R_i < Z_0$ a quantity $k_{in} < 0$, and (31) reduces to the form

$$U = \frac{U_0}{p} \frac{Z_0}{Z_0 + R_i} \frac{1}{1 - k_{in}^2} \left[e^{-\gamma x} + e^{-\gamma(2l-x)} + k_{in} e^{-\gamma(2l+x)} + k_{in} e^{-\gamma(4l-x)} \right], \quad (32)$$

and in the formula for the instantaneous value of the voltage (31) it is necessary to take into account the negative value k_{in} .

If $R_i = Z_0$, then in (31) there remain only two terms, hence, the line charge is carried out during two passes of the wave along the line, i.e. the charge time is $t_c = 2l\sqrt{LC}$.

Note that, if $U_0 = 1$, then (31) is a transient response $h(t, x)$.

Then the line voltage when the input is acted upon with an arbitrary time dependence of the voltage can be calculated using the Duhamel integral.

Such a result allows performing calculations, for example, in the case of using a non-ideal commutator S.

Substituting in (30) instead U_0/p of the coefficient 1, we obtain the transfer function of the line

$$K(p) = \frac{Z_0}{Z_0 + R_i} \frac{1}{1 - k_{in}^2} \left[e^{-\gamma x} + e^{-\gamma(2l-x)} \right] \quad (33)$$

the original of which is an impulse response $g(t)$ [1].

Then the line voltage for an arbitrary exciting function at the input can be calculated using the overlap integral [1, 2]. So, for example, with an exponential voltage pulse at the input $u_1(t) = U_0(1 - e^{-\alpha t})$, we get:

$$u(t, x) = \frac{U_0 Z_0}{Z_0 + R_i} \left\{ \left[1 - e^{-\alpha \left(t - \frac{x}{V} \right)} \right] 1 \left(t - \frac{x}{V} \right) + \left[1 - e^{-\alpha \left(t - \frac{2l-x}{V} \right)} \right] 1 \left(t - \frac{2l-x}{V} \right) + k_{in} \left[1 - e^{-\alpha \left(t - \frac{2l+x}{V} \right)} \right] 1 \left(t - \frac{2l+x}{V} \right) + k_{in} \left[1 - e^{-\alpha \left(t - \frac{4l-x}{V} \right)} \right] 1 \left(t - \frac{4l-x}{V} \right) + \dots \right\}.$$

5. DUAL FORMING LINE (BLUMLEIN LINE)

The scheme of the double forming line is two segments of a line of equal length with the wave resistance Z_0 included in series. In the rupture of the lower, "earth" conductor of the line, a load resistance of $R = 2Z_0$ is included (Fig. 10).

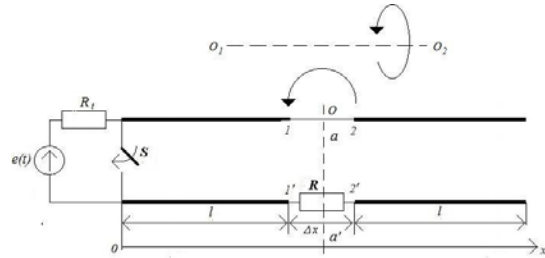


Fig. 10. Dual forming line

DFLs have found application in the technology of high-current accelerators of relativistic electron beams, as elements of powerful modulators of radio transmitters, as well as generators of high-voltage pulses of nanosecond duration [12, 14]. In Fig. 11 is a diagram of the DFL of a coaxial structure. The coaxial line can be obtained from a two-wire line, which is shown in Fig. 11, if the latter is first folded by turning with the center at the point O . As a result, the upper electrodes of the lines coincide, and the lower ones form two parallel equal-sized segments. All three segments are generators of three coaxial cylinders, which can be constructed if the generated linear construction is rotated around the axis O_1O_2 . Since the wave resistance of the coaxial line is proportional to the logarithm of the ratio of the electrode diameters [5], in order to ensure the equality of the wave impedances of each of the lines to a value Z_0 , the diameters should be in the ratio $d_2 = \sqrt{d_1 d_3}$.

Coaxial DFL is charged from a high-voltage source E by feeding it to an intermediate electrode. In practice, such a source can serve as a generator of Marx [15]. Then, after the activation of the key S1, the process of forming a voltage pulse on the load takes place. As a load, a segment of a coaxial line appears in this case, formed by the electrodes d2 and d3, with a wave resistance of $2Z_0$.

The generated voltage pulse $\tau_p = l\sqrt{LC}$ reaches the key S2, after which it is applied to the cathode of the vacuum diode of the accelerator. As a result of field emission, an electron beam is formed in the vacuum tube. The inductance of the throttle L_{dr} provides an AC decoupling of the internal and external electrodes during the duration of the pulse on the load. The high-voltage insulator Ins separates the water part of the accelerator from the vacuum one.

As a dielectric filling the interelectrode space, distilled water is used. So, for example, to form a pulse with a duration of $\tau_p = 50$ ns, the line length should equal $l = c\tau_p / \sqrt{\epsilon_{H_2O}} = 1.6$ m, where c – is the speed of light in a vacuum. At the same time, the shortening of the electrical length of the line is provided by slowing the propagation velocity of the wave in the line due to the high value of the dielectric constant of water: $\epsilon_{H_2O} \approx 80$. The required electrical strength of water insulation is achieved by distillation, as well as deionization of water.

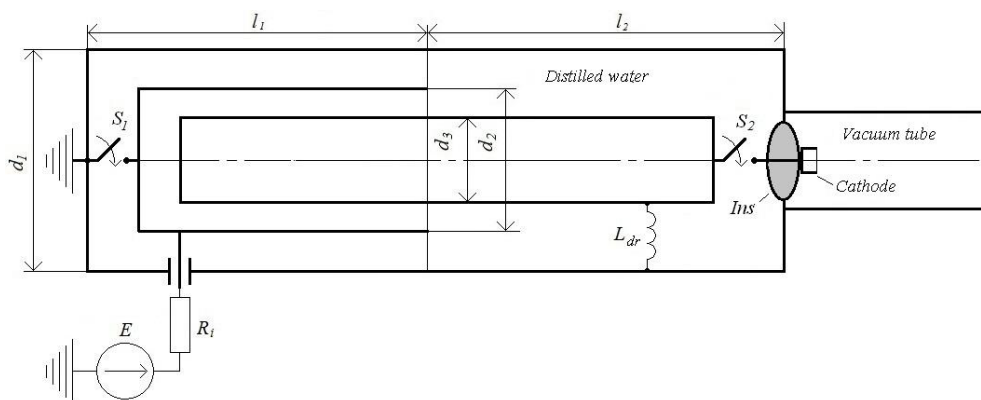


Fig. 11. Coaxial dual-forming line in the high-current electron accelerator

The equivalent circuit for the calculation is shown in Fig. 12. The expression for the instantaneous voltage value on the load of the DFL follows from the Kirchhoff law.

$$u_l(t) = u_{22'}(t, 0) - u_{11'}(t, l), \quad (34)$$

where $u_{22'}(t, 0)$ and $u_{11'}(t, l)$ are the stresses in the corresponding sections of the line. We calculate each of these quantities.

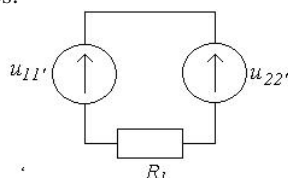


Fig. 12. Equivalent scheme of the dual-forming line

In the initial state, both lines are charged to voltage U_0 , i.e. initial conditions: $u(x, 0) = U_0$. Boundary conditions for the first line

$$u(t, 0) = 0, \quad u(t, l) = 3Z_0 i(t, l) \text{ for } t > 0$$

or in the operator form

$$U|_{x=0} = 0, \quad U|_{x=l} = 3Z_0 I|_{x=l}.$$

This implies a system of equations for calculating the integration constants

$$C_1 + C_2 + U_0/p = 0,$$

$$C_1 e^{\gamma l} + C_2 e^{-\gamma l} = -3(C_1 e^{\gamma l} - C_2 e^{-\gamma l}).$$

Omitting the intermediate calculations, we write the expression for the voltage in section 11'

$$U = \frac{U_0}{p} \left(1 - \frac{3}{2} e^{-\gamma l} + \frac{1}{2} e^{-3\gamma l} \right). \quad (35)$$

Here the first term is a stress wave propagating in the load direction from the short-circuited beginning of the line, the second term represents a wave reflected from the end of the line and propagating to its beginning, the third term is a secondary reflection from the line beginning. In each case, the reflection coefficient is determined by the impedance value in the corresponding section of the line. At the beginning of the line at $x = 0$, we get $k = -1$, at the end of the line $k = 1/2$.

Expand the series and use the delay theorem of the original, write the expression for the instantaneous voltage at the end of the line for $x=l$

$$u_{11'} = U_0 \left[1(t) - \frac{3}{2} 1\left(t - \frac{l}{V}\right) + \frac{1}{2} 1\left(t - \frac{3l}{V}\right) \right]. \quad (36)$$

Let us pass to the analysis of the second line. Input of the second line is forced by the wave refracted from ISSN 1562-6016. BAHT. 2018. №4(116)

the first line into the second one. Refraction coefficient here is equal

$$q = 1 - k = 1/2$$

so, input voltage in cross section 22' at $t=l/V$ equals $-U_0/2$. Final expression for voltage $u_{22'}$.

$$u_{22'} = U_0 \left[1(t) - \frac{1}{2} 1\left(t - \frac{l}{V}\right) + \frac{1}{2} 1\left(t - \frac{3l}{V}\right) \right]. \quad (37)$$

Resulted pulse voltage on the line load is

$$u_l = U_0 \left[1\left(t - \frac{l}{V}\right) - 1\left(t - \frac{3l}{V}\right) \right].$$

Corresponding time diagram is shown in Fig. 13.

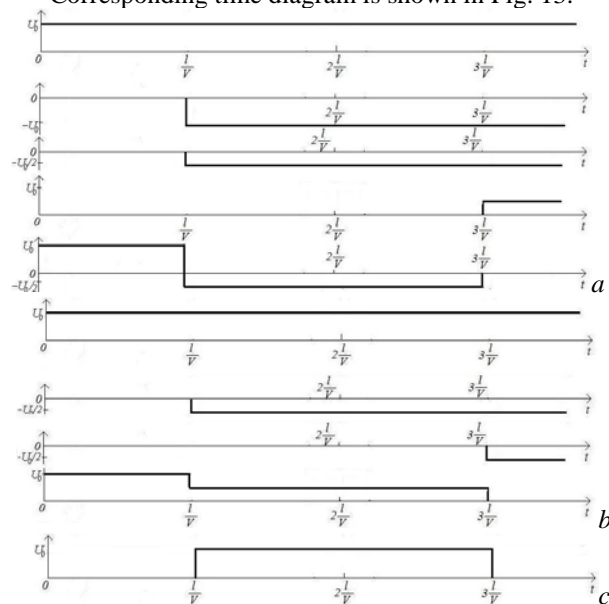


Fig. 13. Forming output pulse of the Blumlein line
a) first line output; b) second line input;
c) voltage on the load

The result can be verified by calculating the energy balance in the system. Thus, the initial energy reserve was

$$W_1 = Cl U_0^2.$$

The only dissipative element in the system is the load resistance. The energy released in it in the form of Joule heat is

$$W_2 = \frac{1}{2Z_0} \int_0^\infty u_l^2 dt = \frac{2U_0^2 l}{2Z_0 V} = CU_0^2 l.$$

As you can see, the results obtained coincide.

CONCLUSIONS

1. It is shown that the Laplace transform makes it possible to simplify considerably the problem of analyzing processes in forming lines. It is noted that the Laplace transform of the telegraph equations of the line allows us to take into account the initial conditions of the problem automatically.

2. Examples of calculation of various line regimes that can be used in practice are given. The solution of the line equations by the classical method allows one to obtain an expression for the desired functions in the form of a Fourier series. Thus, as a result, we obtain a spectrum of voltage or current in the line, which can be very useful for further frequency analysis.

3. The method of traveling waves makes it possible to obtain a clear picture of the distribution of voltage and current in the line. It is shown that wandering waves arise as a result of the reflection of voltage and current waves in a line from the inhomogeneities of its structure.

Processes in the line are a manifestation of the interference pattern of wandering waves. We also note that the method of wandering waves most adequately reflects the picture of processes in the transmission lines of electric energy in the regime of unintentional impulse overloads or the influence of external factors (for example, lightning discharge) [15, 16]. Examples are given of calculating the time characteristics of a line and calculating the voltage in the case of an arbitrary source function at the input.

4. It is shown that by the repeated Laplace transform of the operators from telegrapher equations one can go over to algebraic equations, which greatly simplifies the calculation. The boundary conditions are automatically taken into account.

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МОДЕЛИРОВАНИЕ РЕЖИМОВ В ИМПУЛЬСНЫХ ФОРМИРУЮЩИХ ЛИНИЯХ ОПЕРАТОРНЫМ МЕТОДОМ

В.И. Чумаков, Ю.Ф. Лонин, О.И. Харченко

Рассмотрено применение преобразования Лапласа для анализа цепей с распределенными параметрами. В качестве примера анализируются импульсные формирующие линии. Такие устройства широко применяются в электронике, высоковольтной технике и сильноточных ускорителях заряженных частиц. В приведенных примерах рассматриваются как классические методы решения дифференциальных уравнений в частных производных: метод Фурье и метод Даламбера (метод блуждающих волн), так и метод, основанный на преобразовании Лапласа периодической функции. Проведено детальное решение задачи формирования импульсов напряжения с использованием схем с распределенными параметрами. Показано, что повторное преобразование Лапласа позволяет перейти от уравнений в частных производных к алгебраическим уравнениям и упростить решение проблемы. Приводятся временные диаграммы процессов заряда и разряда линии для различных нагрузок.

МОДЕЛЮВАННЯ РЕЖИМІВ В ІМПУЛЬСНИХ ФОРМУЮЧИХ ЛІНІЯХ ОПЕРАТОРНИМ МЕТОДОМ

В.І. Чумаков, Ю.Ф. Лонін, О.І. Харченко

Розглянуто застосування перетворення Лапласа для аналізу ланцюгів з розподіленими параметрами. Як приклад аналізуються імпульсні формуючі лінії. Такі пристрої широко застосовуються в електроніці, високовольтній техніці і сильноточових прискорювачах заряджених частинок. У наведених прикладах розглядаються як класичні методи розв'язання диференціальних рівнянь у часткових похідних: метод Фур'є і метод Даламбера (метод блукаючих хвиль), так і метод, заснований на перетворенні Лапласа періодичної функції. Проведено детальне розв'язання задачі формування імпульсів напруги з використанням схем з розподіленими параметрами. Показано, що повторне перетворення Лапласа дозволяє перейти від рівнянь у приватних похідних до алгебраїчних рівнянь і спростити розв'язання задачі. Наводяться часові діаграми процесів заряду і розряду лінії для різних навантажень.