SUPERRADIANCE OF STATIONARY OSCILLATORS

V.M. Kuklin, D.N. Litvinov, A.E. Sporov
V.N. Karazin Kharkiv National University, Kharkov, Ukraine
E-mail: v.m.kuklin@karazin.ua

The mathematical model of superradiance of the system of nonlinear oscillators that are restricted in space is considered. Each oscillator generates its own field. The interaction of the oscillators occurs between each other and through the integral radiation field. The effects of nonlinearity of oscillators due to relativistic effects are taken into account. The features of the synchronization process of oscillators are discussed. A comparison of the superradiance efficiency of a system of interacting oscillators and a dissipative generation regime in a similar system is discussed, where the oscillators interacts only with the field of the excited wave.

PACS: 05.45.Xt, 52.40.Mj

INTRODUCTION

Dicke has found in his work [1] that taking into account the interaction of two-level atoms in a small volume leads to an acceleration of the relaxation processes that remove their excitation and synchronization of radiation. This effect was called superradiance. In the classical case, one can also take into account the interaction of radiating particles, because each of them interacts with others through its field. In the absence of an external field, such an interaction can lead to a certain phase (and, in some cases, spatial) synchronization of oscillators and the appearance of a coherent component in the integral radiation of the system [2].

In the work [3], the nature of the radiation of Langmuir oscillations both by extensive and by comparable with the wavelength of non-relativistic electron beams of has been considered. The latter case corresponds to the superradiation of a cluster beam and qualitatively corresponds to the dissipative generation regime. In this case, the process of the synchronizing of oscillators, creating the conditions for the formation of coherent radiation, was determined by their spatial grouping.

However, for such particles whose centers of oscillations are stationary the spatial grouping is impossible and the phase synchronization is hampered [4 - 7]. But this type of nonlinear oscillators can be synchronized with the help of an external small amplitude field [5 - 7].

In general, the scenarios of particle dynamics in fields can be different and can depend on a variety of reasons (see, for example, the model [8 - 13]), but in this case the only phase synchronization of oscillators can be realized.

In this article, the main attention is drawn to the relationship between descriptions in different physical realizations of oscillators whose centers are fixed. It is made a comparison between the superradiance efficiency of a system of interacting oscillators and the dissipative generation regime in a similar system where the oscillators interact only with the field of the excited wave.

1. THE SYSTEM OF OSCILLATORS WITH THE FIXED CENTERS

Let us consider the processes of electromagnetic waves generation by a system of oscillators with the fixed centers. Let consider the case when the wave frequency and the oscillators frequency coincides and equal to ω. The wave vector of the oscillations is $\vec{k} = (0,0,k)$, the wave components are $\vec{E} = (E,0,0)$, $\vec{B} = (0,E,0)$, where $E = |E| \cdot \exp\{-i\omega t + ikz + i\varphi\}$. The oscillators are placed along the OZ axis with quantity of N per wave length $2\pi / k$. The oscillator's mass is m, the charge is equal to $-e$ and oscillator's frequency coincides with wave frequency ω. The oscillator's initial amplitude is equal to a_0. Let assume that the oscillator moves only along the axis OX. In this case, the influence of the magnetic field of the wave on the oscillator dynamics can be neglected [5].

For the extended systems, or for the case of small group velocity of the excited oscillations, it becomes possible to store the field energy in the active zone, even at a finite level of radiation losses. Further, we will assume that the system is rather extended, and the group velocity of the wave is small, for example, because of the effective permittivity ($c_{eff} = v_g = k_0 c^2 / \omega_0 \varepsilon_0$) or near the boundary frequency of the waveguide, as, for example, realized in gyrotrons. Also, the reflection from the boundaries of the system of oscillators is neglected. In this case the effective decrement of absorption is $\delta_D = 2c_{eff} / b$ and $\theta = \delta / \gamma_1 = 2c_{eff} / b \cdot \gamma_1$. In general, it is necessary to take into account electromagnetic waves of induced radiation, which propagate in both directions. As for the spontaneous radiation of each particle, and of the oscillator system, it is always oriented in both directions. The equations of the oscillators motion can take the form:

$$\frac{d}{d\tau_1} A_j = -\frac{1}{\theta N} \sum_{s=1}^{N} A_s \cdot [\mathrm{Cos}\{\psi_j + 2\pi(Z_j - Z_s) + \psi_s\} \cdot U_+(Z_j - Z_s) - \\ -\mathrm{Cos}\{\psi_j - 2\pi(Z_j - Z_s) + \psi_s\} \cdot U_+(Z_s - Z_j)] - \mathrm{E}_0 \cdot \mathrm{Cos}\{2\pi Z_j\}, \quad (1)$$

$$A_j [\frac{d}{d\tau_1} \psi_j - \Delta_i] = \\ = -\frac{1}{\theta N} \sum_{s=1}^{N} A_s \cdot [\mathrm{Sin}\{\psi_j + 2\pi(Z_j - Z_s) + \psi_s\} \cdot U_+(Z_j - Z_s) - \\ -\mathrm{Sin}\{\psi_j - 2\pi(Z_j - Z_s) + \psi_s\} \cdot U_+(Z_s - Z_j)] - \mathrm{E}_0 \cdot \mathrm{Sin}\{2\pi Z_j\}. \quad (2)$$

The right-hand sides of equations (1) and (2) contain the normalized values of the amplitude of the total field of the spontaneous radiation of its neighbors and the external synchronizing field acting on the oscillator.

In the case of excitation of the same oscillators, but whose spontaneous radiation is neglected and only their interaction with the field of integral waves $\mathrm{E}_{\pm}$, is taken into account we can write the following equations:

$$\frac{\partial}{\partial \tau_1} E_{\pm} + \theta \cdot E_{\pm} = \frac{1}{N}\sum_{j=1}^{N} A_j \cdot \exp\{\psi_j \mp 2\pi Z_j - \varphi_{\pm}\}, \quad (3)$$

$$E_{\pm}\frac{\partial \varphi}{\partial \tau_1} = \frac{1}{N}\sum_{j=1}^{N} A_j \cdot \exp\{\psi_j \mp 2\pi Z_j - \varphi_{\pm}\}, \quad (4)$$

with the equation of motion:

$$\frac{d}{d\tau_1} A_j = \quad (5)$$
$$-[\mathrm{E}_{+} \cdot \mathrm{Cos}\{\varphi_{+} + 2\pi Z_j - \psi_j\} + \mathrm{E}_{-} \cdot \mathrm{Cos}\{\varphi_{-} - 2\pi Z_j - \psi_j\}],$$

$$A_j[\frac{d}{d\tau_1}\psi_j - \Delta_i] = \quad (6)$$
$$= -[\mathrm{E}_{+} \cdot \mathrm{Sin}\{\varphi_{+} + 2\pi Z_j - \psi_j\} + \mathrm{E}_{-} \cdot \mathrm{Sin}\{\varphi_{-} - 2\pi Z_j - \psi_j\}],$$

where we have used the following variables: $\mathrm{E} = eE / \omega m \gamma a_0$, $\tau = \gamma t$, $A_i = |x_i| / a_0$, and $\gamma^2 = \pi e^2 n_0 / m$ is the maximum increment of the system (3)-(6) in the absence of losses, $kz_i = Z_i \subset (0, 2\pi)$. In the non-relativistic case $\Delta_i = 0$, and taking the relativism into account leads to the non-linearity of the oscillators $\Delta_i = \alpha \cdot (A_i^2 - A_{i0}^2)$, $\alpha = 3\omega(k \cdot a_0)^2 / 4$.

It was obtained that for the waves of different polarization (when the particles move only along the direction of the electric wave field vector), the same equations can be used. Let us define the conditional operator $U_{+}(z - z_s) = 1$ when $z > z_s$, and $U_{+}(z - z_s) = 0$ when $z \le z_s$, the density of the particles in a unit section per unit length $n_0 = M / b$, where M is the total number of particles on the length of the bunch, and each model particle contains M / N real particles, $\pi e^2 M / 2mc = \gamma^2 / \delta = \gamma_D^2$ is the square of the increment (inverse characteristic time of development) of the process in both cases (i.e., superradiance and dissipative instability regimes for the same $\theta = \delta / \gamma >> 1$ values), and $\delta = 2c_{eff} / b$ is the effective decrement of radiation losses in the system.

It was earlier noted in [6] that for some θ values the characteristic times of the development of the superradiance process and the dissipative instability process of the system of oscillators coincide. Moreover the maximum possible radiation intensity levels of a spatially confined particle beam were coincided.

It is also interest to determine the efficiency of the superradiance regime of a system of oscillators and to compare it with the regime of dissipative instability considered earlier in [7].

2. THE EFFICIENCY OF THE SUPERRADIANCE REGIME

For the case when the number of oscillators N = 10 000, the average integral field $|E|$ of a system of oscillators with a random distribution of the initial phase is in 100 times smaller than the maximum possible value of the field in the case when all the oscillators are synchronized in phase. That is, the spontaneous and absolutely coherent induced radiation satisfies the relation $10^{-2} - 1$. For the squares of the amplitudes $|E|^2$, this relation takes the form $10^{-4} - 1$.

In the absence of a external synchronizing field with amplitude $E_0 = 0$, one can observe a significant scatter of the maximum values of the radiation field at the ends of the system whose size is equal to the wavelength of the radiation $\Delta Z = 1$. For rather great nonlinearity of the oscillators $\alpha = 1$, the squares of the maximum amplitudes $|E|^2$ become to be the largest and change for random distributions of the initial phases in the range from 0.04 to 0.08. The increasing of the system size reduces the spreading of the maximum values for any nonlinearity values, but it also reduces the maximum possible radiation amplitude at the ends of the system.

The influence of an external field that only in five times greater than the average spontaneous level $E_0 = 0.05$ leads to a sharp decrease of the spreading of the maximum values of the field at the boundaries of the system. So, for the size of the system $\Delta Z = 1$ for different values of nonlinearity, the values $|E|^2$ are shown in Fig. 1.

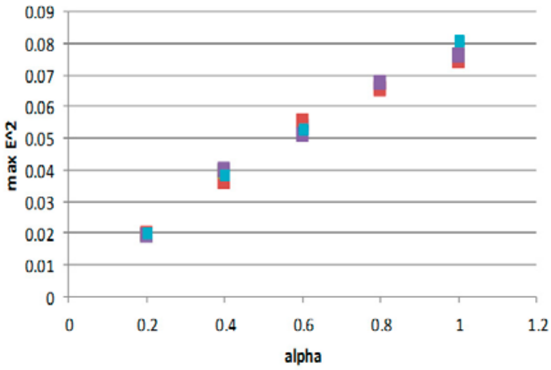

Fig. 1. The values of $|E|^2$ depending of the level on nonlinearity α

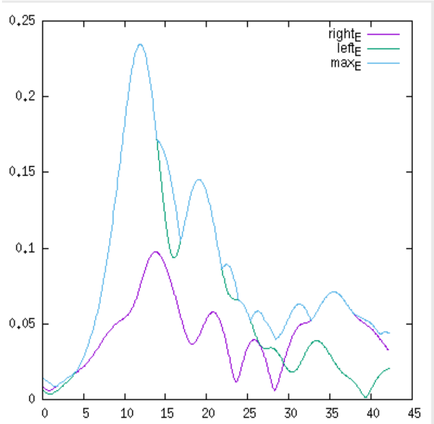

Fig. 2. The dependence of the field to the right of the system, to the left of the system and the maximum within the system upon the time, $\Delta Z = 1, \alpha = 1$

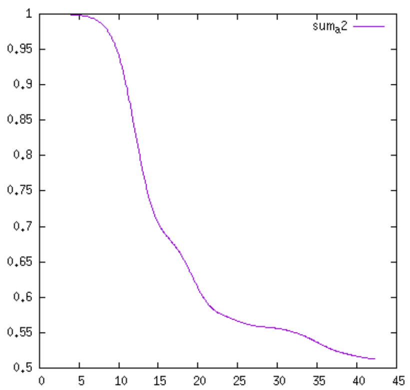

Fig. 3. The normalized total energy of the particles as a function of time, $\Delta Z = 1, \alpha = 1$

Thus (see Figs. 2-3), it can be concluded that the external field has a stabilizing effect on the generation pattern. The greatest value of the field in the opposite side of the system according to the direction of propagation of the external wave $|E| \approx 0.23...0.25$ and $|E|^2 \approx 0.08$, that is, the degree of coherence is of 8%.

In the paper [7] the behavior of the dissipative generation regime of the system (3) - (6) is analyzed. It is shown that for values $\theta \approx 10$ the maximum of the energy radiation from the system is reached (that is, the value $\theta \cdot |E|^2$ is maximal) and at the same time $|E|^2$ reaches the same values as in the case of superradiation. That is, the degree of coherence of the radiation of the oscillator system is comparable (Figs. 4, 5).

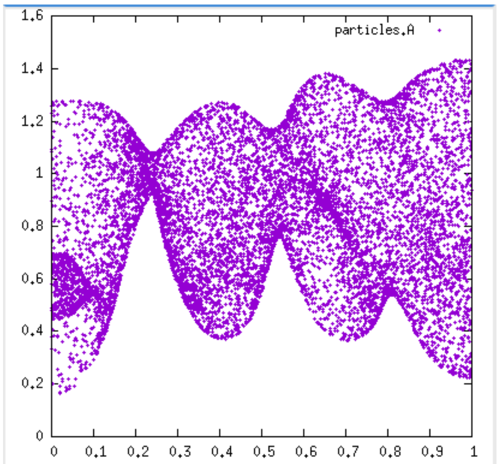

Fig. 4. The amplitude of the oscillators at different points along Z at the moment of reaching the maximum of the field, $\Delta Z = 1, \alpha = 1$

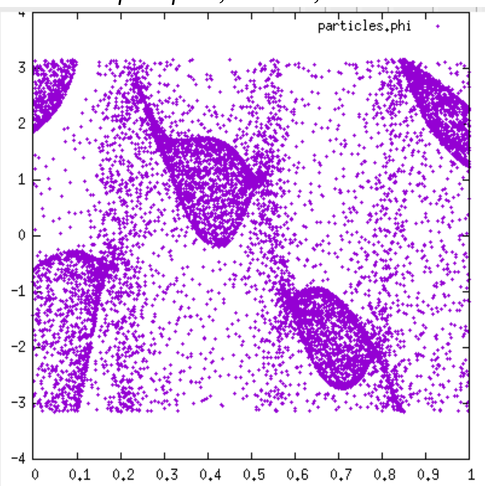

Fig. 5. The phase of the oscillators at different points along Z (to the right) at the moment of reaching the maximum of the field, $\Delta Z = 1, \alpha = 1$

CONCLUSIONS

In the case of radiation from a short beam – bunch of electrons moving in plasma the superradiance regime is realized. At that a wake is formed behind the bunch [2, 3]. The beam particles interact with each other due to the fields radiated by them. This circumstance made it possible to consider this process as superradiance [2]. One can consider the generation of radiation by beam particles (whose density and velocity are similar to the case discussed above) in a system of the same dimensions. While the radiation energy extracts from the ends of such short system, the so-called dissipative regime of beam instability is realized. Usually in this case the particles spontaneous radiation is neglected, thereby excluding the direct interaction of the particles with each other. Only the interaction of particles with the fields of the wave (or of the wave packet) is taken into account. Nevertheless, the increments of the processes are the same and the achievable amplitudes of the generated fields are of the same order. It is possible to estimate the degree of coherence of the oscillators in relation to the achievable field intensity in the system to the maximum possible intensity of ideally phased radiation sources. In this research, this value reaches, as a rule, 25%.

In this paper, we discuss an analogous problem of elucidating the radiation efficiency of a short system of nonlinear oscillators. Here, is also possible to realize the regime of the interaction of the oscillators with each other due to the intrinsic (spontaneous) fields of their radiation. This corresponds to the superradiance regime and the generation regime at the same level of radiation absorption due to the limited system. This regime is the dissipative regime that was discussed in [7]. In this case, spontaneous radiation is neglected and the interaction of the oscillators occurs only with the wave fields, generally, propagating in both directions. It is also shown here that the increments of processes at the same level of radiation absorption are the same, and the maximum possible amplitudes are of the same order. The degree of coherence of the oscillators that can be estimated from the ratio of the achievable field intensity in the system to the maximum possible intensity of perfectly-phased radiation sources, reaches 8% here. It is important to note that, for better repeatability of the results, an external synchronizing wave should be used (for example, in the same way as suggested in [14]), the intensity of which exceeds the integrated spectral noise of unphased oscillators. In the case discussed above, the noise intensity was about 0.01% and the intensity of the external field is 0.25% with respect to the maximum possible intensity of perfectly-phased sources.

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Article received 04.06.2018

СВЕРХИЗЛУЧЕНИЕ СТАЦИОНАРНЫХ ОСЦИЛЛЯТОРОВ

В.М. Куклин, Д.Н. Литвинов, А.Е. Споров

Рассмотрена математическая модель сверхизлучения системы нелинейных пространственно ограниченных осцилляторов. Каждый осциллятор генерирует собственное поле. Взаимодействие осцилляторов происходит между собой и за счет интегрального поля излучения. Учитываются эффекты нелинейности осцилляторов за счет релятивистских эффектов. Обсуждаются особенности процесса синхронизации осцилляторов. Обсуждается сравнение эффективности сверхизлучения системы взаимодействующих осцилляторов и диссипативного режима генерации в подобной системе, где осцилляторы взаимодействуют только с полем возбужденной волны.

НАДВИПРОМІНЮВАННЯ СТАЦІОНАРНИХ ОСЦИЛЯТОРІВ

В.М. Куклін, Д.М. Литвинов, О.Є. Споров

Розглянуто математичну модель надвипромінювання системи нелінійних осциляторів, обмежених у просторі. Кожен осцилятор генерує власне поле. Взаємодія осциляторів відбувається між собою та за рахунок інтегрального поля випромінювання. Враховуються ефекти нелінійності осциляторів за рахунок релятивістських ефектів. Обговорюються особливості процесу синхронізації осциляторів. Обговорюється порівняння ефективності надвипромінювання системи взаємодіючих осциляторів і дисипативного режиму генерації в подібній системі, де осцилятори взаємодіють тільки з полем збудженої хвилі.