FORMATION OF CAVITIES IN THE IONOSPHERIC PLASMA DUE TO LOCALIZED ELECTROSTATIC TURBULENCE

D.V. Chibisov
V.N. Karazin Kharkiv National University, Department of Physics, Kharkov, Ukraine
E-mail: dmitriychibisov@karazin.ua

The possibility of formation of lower hybrid cavities in plasma of the terrestrial ionosphere and magnetosphere due to the interaction of plasma with inhomogeneous axially symmetric electrostatic turbulence is investigated. Heating, turbulent diffusion, as well as the drift of ions due to the radial inhomogeneity of electrostatic turbulence is considered. The equation determining the evolution of the plasma distribution function as a result of these effects is obtained. Stationary solution of this equation is found.

PACS: 52.35.Ra, 52.35.Mw

INTRODUCTION

Lower hybrid cavities (LHC) are the phenomenon that is often observed by satellites and sounding rockets near the auroral zone of the topside ionosphere and magnetosphere of the Earth [1 - 6]. LHC are the localized cylindrically symmetric regions in ionospheric and magnetospheric plasma, elongated along the geomagnetic field which are characterized by an increased level of electrostatic lower hybrid oscillations in comparison with the background plasma, increased ion temperature as well as depletion of the plasma density. LHC have dimensions across the magnetic field of a few ion Larmor radius and the dimensions along the magnetic field, considerably exceeding their transverse ones. As an explanation of this phenomenon several authors have proposed that LHC are the result of modulation instability and self-similar collapse [7 - 10]. However, it was shown in [11] that the theoretical conclusions do not correspond to the statistical properties of most of the observed cavities.

In this paper we propose a mechanism for the formation of cavities in plasma, due to the effect of the interaction of charged particles with the electrostatic turbulence of plasma. In the Earth's ionosphere upflowing and downflowing beams of charged particles are observed. It is known that the passage of charged particle beams through plasma leads to various instabilities and to the turbulent state of plasma. In turn, the turbulence of uniform plasma is the cause of the change in its distribution function, that is, the heating of plasma. However, in addition to heating, electrostatic turbulence can also lead to the diffusion of plasma particles. Such motion of charged particles is similar to the motion of Brownian particles, which is caused by collisions with molecules of a liquid. Charged plasma particles (ions or electrons) "collide" with chaotic pulsations of the electric field, which leads to diffusion motion. If the electrostatic turbulence is spatially nonuniform, then in addition to the diffusion process, the drift motion of the plasma particles induced by the ponderomotive force also occurs, which leads to transport of particles from region with an increased level of turbulence. We investigate these processes by analyzing the motion of single particle; determine the rate of change in the averaged values of the square of the particle's velocity, and also its radial coordinate. We neglect the influence of the magnetic field, assuming the frequency spectrum of the electrostatic turbulence to be sufficiently high frequency in comparison with the cyclotron frequency. Since the frequency spectrum in cavities is located in the region of the lower hybrid frequency, we will consider the motion of only ions. We use the obtained quantities in the Fokker-Planck equation and find the stationary solution of this equation that will give the final radial distribution of the plasma density.

1. HEATING AND DIFFUSION

In homogeneous plasma we consider the region with an increased electrostatic turbulence in comparison with the background plasma and having axial symmetry. We assume that the turbulent state is the result of instabilities arising when a cylindrically symmetric beam passes through plasma. The resulting turbulent pulsations of an electrostatic field lead to perturbations of the trajectories and the thermal velocities of charged plasma particles passing through this region. To determine the effect of electrostatic turbulence on the trajectory, we solve the equation of motion for the perturbed value of the velocity $\vec{v}$ of ion

$$\frac{d\vec{v}}{dt} = \frac{e}{m} F(r)\vec{E}(\vec{r},t), \tag{1}$$

where e and m are the charge and mass of ion, $\vec{E}(\vec{r},t)$ is the electric field strength of electrostatic turbulence, far from the region with a high level of turbulence, $F(r)$ is the envelope of turbulent electric field. It is assumed that $F(r)$ have a maximum at $r=0$ and $F(\infty)=1$, so $F(r)\geq 1$. Equation (1) is a stochastic differential equation with random force acting on the particle. For random velocity changes from (1) we obtain

$$\vec{v}(t) = \frac{e}{m} F(r)\int_{t_0}^{t} \vec{E}(\vec{r},t')dt', \tag{2}$$

where $t=t_0$ is the time when turbulence appeared. Find the rate of change in the mean square velocity. Multiply (1) by (2) and average this product over a large time interval

$$\left\langle \vec{v}\frac{d\vec{v}}{dt}\right\rangle = \frac{1}{2}\frac{d\left\langle \overline{v^2}\right\rangle}{dt} = \frac{e^2}{m^2}F^2(r)\int_{t_0}^{t}\left\langle \vec{E}(\vec{r},t')\vec{E}(\vec{r},t)\right\rangle dt'. \tag{3}$$

Suppose that a turbulent electric field satisfies the following conditions

$$\left\langle \vec{E}(\vec{r},t)\right\rangle = 0,\ \left\langle \vec{E}(\vec{r},t')\vec{E}(\vec{r},t)\right\rangle = \left\langle \vec{E}^2(\vec{r},t)\right\rangle \delta(t'-t), \tag{4}$$

that is the electric field at a time t is assumed to be completely uncorrelated with it at any other time (white noise). Here $\left\langle \vec{E}^2(\vec{r},t)\right\rangle$ is the average value of the square of the amplitude of electric field strength far from the region with a high level of turbulence. Substitution of (4) into (3) yield

$$\frac{d\left\langle \overline{v^2}\right\rangle}{dt} = \frac{e^2}{m^2}F^2(r)\left\langle \vec{E}^2(\vec{r},t)\right\rangle. \qquad (5)$$

The solution of (5),

$$\left\langle \overline{v^2}\right\rangle = \left\langle \overline{v_0^2}\right\rangle + \frac{e^2}{m^2}F^2(r)\left\langle \vec{E}^2(\vec{r},t)\right\rangle t, \qquad (6)$$

shows that the mean square velocity, in fact the ion temperature, in the region of an increased level of electrostatic turbulence increases linearly with time. In (6) $\left\langle \overline{v_0^2}\right\rangle$ is the initial value of the mean square of the velocity of ions before the appearance of turbulence, that is, the corresponding value for the environment plasma.

Now we find the value of the random displacement $\vec{r}(t)$ of the particle, by integrating the expression (2) twice

$$\vec{r}(t) = \int_{t_0}^{t}\delta\vec{v}dt' = \frac{e_\alpha}{m_\alpha}F(r)\int_{t_0}^{t}\int_{t_0}^{t'}\vec{E}(\vec{r},t'')dt''dt' \qquad (7)$$

and then obtain the rate of change in mean square displacement multiplying (2) by (7) and averaging over a large time interval

$$\left\langle \vec{r}(t)\frac{d\vec{r}(t)}{dt}\right\rangle = \frac{1}{2}\frac{d\left\langle \overline{\vec{r}(t)^2}\right\rangle}{dt} = \frac{e^2}{m^2}F^2(r)\times$$

$$\times\left\langle \int_{t_0}^{t}\int_{t_0}^{t'}\vec{E}(\vec{r},t'')dt''dt'\int_{t_0}^{t}\vec{E}(\vec{r},t')dt'\right\rangle. \qquad (8)$$

Write $\vec{E}(\vec{r},t)$ in the form of a Fourier integral

$$\vec{E}(\vec{r},t) = \int_{-\infty}^{\infty}\vec{E}(\vec{r},\omega)\exp(-i\omega t)d\omega. \qquad (9)$$

Substituting (9) into (8) we get

$$\frac{1}{2}\frac{d\left\langle \overline{\vec{r}^2(t)}\right\rangle}{dt} = \frac{e^2}{m^2}F^2(r)\times$$

$$\times\left\langle \int_{t_0}^{t}\int_{t_0}^{t'}\int_{-\infty}^{\infty}\vec{E}(\vec{r},\omega)\exp(-i\omega t'')d\omega dt''dt'\int_{t_0}^{t}\vec{E}(\vec{r},t')dt'\right\rangle. \qquad (10)$$

Now we integrate (10) with respect to t twice

$$\frac{1}{2}\frac{d\left\langle \overline{\vec{r}(t)^2}\right\rangle}{dt} = \frac{e^2}{m^2}F^2(r)\times$$

$$\times\left\langle \int_{-\infty}^{\infty}\frac{\vec{E}(\vec{r},\omega)}{-\omega^2}\exp(-i\omega t)d\omega\int_{t_0}^{t}\vec{E}(\vec{r},t')dt'\right\rangle. \qquad (11)$$

It is obvious that the spectrum of oscillations of the electric field is not infinite, but is determined by the processes responsible for the turbulent state. If we assume that the width of the spectrum of the electric field oscillation is $\Delta\omega$, then we can approximately consider that in the integrand $\omega \approx \Delta\omega$ and then write

$$\frac{1}{2}\frac{d\left\langle \overline{\vec{r}(t)^2}\right\rangle}{dt} = -\frac{e^2}{m^2\Delta\omega^2}F^2(r)\times$$

$$\times\left\langle \int_{-\infty}^{\infty}\vec{E}(\vec{r},\omega)\exp(-i\omega t)d\omega\int_{t_0}^{t}\vec{E}(\vec{r},t')dt'\right\rangle. \qquad (12)$$

Performing in (12) the inverse Fourier transformation we get

$$\frac{1}{2}\frac{d\left\langle \overline{\vec{r}(t)^2}\right\rangle}{dt} \approx -\frac{e^2}{m^2\Delta\omega^2}F^2(r)\int_{t_0}^{t}\left\langle \vec{E}(\vec{r},t)\vec{E}(\vec{r},t')\right\rangle dt'. \qquad (13)$$

Taking into account (4), we finally obtain

$$\frac{1}{2}\frac{d\left\langle \overline{\vec{r}(t)^2}\right\rangle}{dt} \approx -\frac{e^2}{2m^2\Delta\omega^2}F^2(r)\left\langle \vec{E}^2(\vec{r},t)\right\rangle. \qquad (14)$$

Relation (14) is the diffusion coefficient of the ion component of plasma in a turbulent electric field. We note that the diffusion coefficient in the region with an increased level of turbulence is higher than in the environment, which leads to depletion of the plasma density.

2. DRIFT MOTION OF PARTICLES

Let us now consider the drift motion of ions due to the ponderomotive force, which is induced by the radial inhomogeneity of electrostatic turbulence. Ponderomotive force affecting on the particle in the case of an inhomogeneous coherent electric field E oscillating with the frequency ω is equal

$$F_p = -\frac{e^2}{4m\omega^2}\nabla\left(E^2\right). \qquad (15)$$

Now we obtain the equation of motion of ion with the ponderomotive force in the case of a radially inhomogeneous electrostatic turbulence. We start from the equation of motion of ion along the radius

$$m\frac{d^2r}{dt^2} = eF(r)E_r(\vec{r},t), \qquad (16)$$

where $E_r(\vec{r},t)$ is the radial projection of the electric field strength. We represent the coordinate of ion as the sum of its initial radial coordinate r_0, as well as the oscillatory $\tilde{r}$ and drift $\bar{r}$ changes of this coordinate.

$$r = r_0 + \tilde{r} + \bar{r}, \qquad (17)$$

where $\bar{r}$ means the change in the mean value of the radial coordinate of ion over a long period of time, $\bar{r} = \langle r\rangle - r_0$. Assuming that $\bar{r} << \tilde{r} << r_0$ we can use Taylor expansion on the equation of motion about r_0. Substituting (17) into (16), we obtain

$$m\frac{d^2r}{dt^2} = e\left(F(r_0) + \tilde{r}\nabla F(r_0)\right)E_r(\vec{r},t). \qquad (18)$$

For the oscillating part of the trajectory, we obtain

$$m\frac{d^2\tilde{r}}{dt^2} = eF(r_0)E_r(\vec{r},t). \qquad (19)$$

Equation (19) is the radial component of equation (1) and therefore has the solution (7)

$$\tilde{r}(t) = \frac{e}{m} F(r_0) \int_{t_0}^{t}\int_{t_0}^{t'} E_r(\vec{r}, t'') dt'' dt' . \quad (20)$$

Writing $E_r(\vec{r}, t)$ in the form of a Fourier integral (9) for $\tilde{r}(t)$ we get

$$\tilde{r}(t) = \frac{e}{m} F(r_0) \int_{t_0}^{t}\int_{t_0}^{t'}\int_{-\infty}^{\infty} E_r(\vec{r}, \omega) \exp(-i\omega t'') d\omega dt'' dt' . \quad (21)$$

Integrating (21) with respect to t twice yield

$$\tilde{r}(t) = -\frac{e}{m} F(r_0) \int_{-\infty}^{\infty} \frac{E_r(\vec{r}, \omega)}{\omega^2} \exp(-i\omega t) d\omega \approx$$

$$\approx -\frac{e}{m} F(r_0) \frac{E_r(\vec{r}, t)}{\Delta\omega^2} . \quad (22)$$

Now we average (18) over a large period of time

$$m \frac{d^2\bar{r}}{dt^2} = e \nabla F(r_0) \langle \tilde{r}(t) E_r(\vec{r}, t') \rangle . \quad (23)$$

Substitute (22) into (23)

$$m \frac{d^2\bar{r}}{dt^2} = -\frac{e^2}{m\Delta\omega^2} F(r_0) \nabla F(r_0) \langle E_r(\vec{r}, t) E_r(\vec{r}, t') \rangle$$

and use the condition (4)

$$m \frac{d^2\bar{r}}{dt^2} = -\frac{e^2}{2m\Delta\omega^2} \nabla F^2(r_0) \langle E_r^2(\vec{r}, t) \rangle \delta(t' - t) . \quad (24)$$

Thus, we have obtained the equation for the drift motion of ion due to the effect of a nonuniform turbulence field. The expression on the right side of the equation (24) is a ponderomotive force which is the effect of the radial inhomogeneity of electrostatic turbulent field. This force in contrast to coherent oscillations (15) is proportional to the delta function. This is due to the fact that the affecting of this force occurs only at certain instants of time, corresponding to the moments of "collisions" of particles with random pulsations of the electrostatic field.

Integrating (24), we obtain the velocity of the drift motion of ion along the radius

$$\frac{d\bar{r}}{dt} = \int_{t_0}^{t} \frac{d^2\bar{r}}{dt^2} dt' = -\frac{e^2}{4m^2\Delta\omega^2} \nabla F^2(r_0) \langle E_r^2(\vec{r}, t) \rangle . \quad (25)$$

Since $\nabla F^2(r_0) < 0$, meaning that the level of turbulence decreases with the radius, we have that $\frac{d\bar{r}}{dt} > 0$ and thus the ion drift occurs from the center of the region with an increased level of turbulence.

3. STATIONARY DISTRIBUTION OF PLASMA DENSITY

The evolution of the distribution function $f(r,t)$ as a result of diffusion as well as the drift motion of ions is described by the Fokker-Planck equation

$$\frac{\partial f(r,t)}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(rA(r) f(r,t)) +$$

$$+ \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{B(r)}{2} \frac{\partial f(r,t)}{\partial r} \right) , \quad (26)$$

where A is the drift velocity along the radius, $B/2$ is the diffusion coefficient. Above it was obtained that diffusion coefficient is (11)

$$\frac{B}{2} = \frac{1}{2} \frac{d\left\langle \overline{\tilde{r}(t)^2} \right\rangle}{dt} \approx -\frac{e^2}{2m^2\Delta\omega^2} F^2(r) \langle \vec{E}^2(\vec{r}, t) \rangle , \quad (27)$$

and drift velocity is (24)

$$A = \frac{d\bar{r}}{dt} = -\frac{e^2}{4m^2\Delta\omega^2} \nabla F^2(r_0) \langle E_r^2(\vec{r}, t) \rangle . \quad (28)$$

Now assuming that the evolution of the distribution function after a long time is over we obtain a stationary dependence of the plasma density $n(r) = f(r)$ on the radius by setting in (26) the derivative of the distribution function with respect to time equal to zero

$$-\frac{1}{r}\frac{d}{dr}(rA(r)n(r)) + \frac{1}{r}\frac{d}{dr}\left(r \frac{B(r)}{2} \frac{dn(r)}{dr} \right) = 0 . \quad (29)$$

Integrating (29) with respect to r yield

$$-A(r)n(r) + \frac{B(r)}{2}\frac{dn(r)}{dr} = 0 . \quad (30)$$

The separation of the variables in (30) gives

$$\frac{2A(r)dr}{B(r)} = \frac{dn(r)}{n(r)} . \quad (31)$$

Substitution of (27) and (28) into (31) yield

$$\frac{dF(r)}{F(r)} \frac{\langle E_r(\vec{r}, t) \rangle}{\langle \vec{E}^2(\vec{r}, t) \rangle} = \frac{dn}{n} . \quad (32)$$

Assuming that $\langle E_r(\vec{r}, t) \rangle / \langle \vec{E}^2(\vec{r}, t) \rangle = 1/3$ and integrating (32), we obtain

$$n(r) = \frac{C}{F^{1/3}(r)} . \quad (33)$$

We choose the integration constant C using the condition $n(\infty) = n_0$, where n_0 is the plasma density far from the region with increased turbulence. To the same value the plasma density was equal in this region before the appearance of turbulence, that is, when $t < t_0$. So we get $C = n_0$ since, as we suggested, $F(\infty) = 1$. Finally, we obtain the distribution of the plasma density after a long time from the occurrence of turbulence in homogeneous plasma

$$n(r) = \frac{n_0}{F^{1/3}(r)} . \quad (34)$$

In accordance with (34), the minimum plasma density is reached in the region with the maximum level of electrostatic turbulence. Thus, in a region with an increased level of electrostatic turbulence, the plasma density cavity may be formed.

CONCLUSIONS

The up flowing and down flowing beams of charged particles in the earth's ionosphere that are observed near the auroral zone can lead to the formation of regions elongated along the geomagnetic field with an increased level of electrostatic turbulence. The turbulence of plasma leads to the heating as well as diffusion of plasma. In addition, due to the spatial in homogeneity of the turbulence, a drift motion of the plasma particles directed from the region with increased turbulence arises. This leads to a decrease in the plasma density in this region. It is established that in the steady state the plasma density and the level of turbulence are related by the condition (34) that can also be written in the form

$$n(r)F^{1/3}(r)=n_0, \quad (35)$$

where $F(r)$ is the envelope of turbulent electric field, which have a maximum at $r=0$ and $F(\infty)=1$. It was shown also that the ion temperature in the cavity exceeds the temperature of ions in the environment, which agrees with the observation data.

Thus, the appearance in the ionosphere of regions with an increased level of electrostatic turbulence, which have axial symmetry and elongated along the geomagnetic field, can lead to the formation of the plasma density cavities, such cavities having the same form as turbulence.

REFERENCES

1. J. Vago, P.M. Kintner, S.W. Chesney, et al. Transverse ion acceleration by localized hybrid waves in the topside auroral ionosphere // *J. Geophys. Res.* 1992, v. 97, № 12, p. 16935-16957.
2. K.A. Lynch, R.L. Arnoldy, J. Bonnell, P.M. Kintner. The AMICIST auroral sounding rocket – A comparison of transverse ion acceleration methods // *Geophys. Res. Lett.* 1996, v. 23, № 23, p. 3293-3296.
3. K.A. Lynch, R.L. Arnoldy, P.M. Kintner, et al. Auroral ion acceleration from lower hybrid solitary structures: A summary of sounding rocket observations // *J. Geophys. Res.* 1999, v. 104, № 12, p. 28515-28534.
4. H.L. Pecseli, K. Iranpour, O. Holter, et al. Lower-hybrid wave cavities detected by the FREJA satellite // *J. Geophys. Res*. 1996, v. 101, № 12, p. 5299-5316.
5. S.H. Kjus, H.L. Pecseli, B. Lybekk, et al. Statistics of the lower hybrid wave cavities detected by the FREJA satellite // *J. Geophys. Res*. 1998, v. 103, № 12, p. 26633-26647.
6. A. Tjulin, M. Andre, A.I. Eriksson, M. Maksimovic. Observations of lower hybrid cavities in the inner magnetosphere by the Cluster and Viking satellites // *Ann. Geophys.* 2004, v. 22, № 12, p. 2961-2972.
7. V.D. Shapiro, V.I. Shevchenko, G.I. Solov'ev, et al. Wave collapse at the lower-hybrid resonance // *Phys. Fluids B.* 1993, v. 5, № 9, p. 3148-3162.
8. P.A. Robinson. Scalings, spectra, and statistics of strong wave turbulence // *Phys. Plasmas.* 1996, v. 3, № 1, p. 192-201.
9. V. D. Shapiro. Modulational interaction of the lower-hybrid waves with a kinetic-Alfven mode // *Phys. Rev. Lett.* 1998, v. 81, № 6, p. 3415-3418.
10. P.A. Robinson. Nonlinear lower hybrid structures in auroral plasmas: Comparison of theory with observations // *Adv. Space Res.* 1999, v. 23, № 10, p. 1679-1688.
11. P.W. Schuck and G.I. Ganguli. The role of lower-hybrid-wave collapse in the auroral ionosphere // *Phys. Rev. Lett.* 2002, v. 89, № 6, p. 065002-1-065002-4.

Article received 06.06.2018

ФОРМИРОВАНИЕ ПОЛОСТЕЙ В ИОНОСФЕРНОЙ ПЛАЗМЕ ВСЛЕДСТВИЕ ЛОКАЛИЗОВАННОЙ ЭЛЕКТРОСТАТИЧЕСКОЙ ТУРБУЛЕНТНОСТИ

Д.В. Чибисов

Исследована возможность образования нижнегибридных полостей в плазме земной ионосферы и магнитосферы вследствие взаимодействия плазмы с неоднородной аксиально-симметричной электростатической турбулентностью. Рассмотрены процессы нагрева, турбулентной диффузии, а также дрейфа ионов вследствие радиальной неоднородности электростатической турбулентности. Получено уравнение, определяющее эволюцию функции распределения плазмы по радиусу в результате этих эффектов. Найдено стационарное решение этого уравнения.

ФОРМУВАННЯ ПОРОЖНИН В ІОНОСФЕРНІЙ ПЛАЗМІ ВНАСЛІДОК ЛОКАЛІЗОВАНОЇ ЕЛЕКТРОСТАТИЧНОЇ ТУРБУЛЕНТНОСТІ

Д.В. Чібісов

Досліджено можливість утворення нижньогібридних порожнин у плазмі земної іоносфери і магнітосфери внаслідок взаємодії плазми з неоднорідною аксіально-симетричною електростатичною турбулентністю. Розглянуто процеси нагріву, турбулентної дифузії, а також дрейфу іонів внаслідок радіальної неоднорідності електростатичної турбулентності. Отримано рівняння, що визначає еволюцію функції розподілу плазми вздовж радіуса в результаті цих ефектів. Знайдено стаціонарний розв'язок цього рівняння.