VELOCITY FIELD BASED METHOD FOR DATA PROCESSING IN RADIONUCLIDE STUDIES

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In this paper, we consider a method of construction of a velocity field based on an optimization approach. A general formulation of the problem is given. It is shown how, under the appropriate assumption, it reduces to known particular case. Two-dimensional case of the velocity field construction is considered in detail under the gradient constancy assumption of the density of the radiopharmaceutical distribution. The problem is reduced to solving a sparse system of large dimension, and the convergence of the iterative algorithm to the solution is considered. This method can be used in the radionuclide data processing.

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INTRODUCTION

BASIC CONCEPTS

The solution of various inverse problems has always been of great practical interest. In [1], the problem of determining velocity field from a given density of the distribution of charged particles was considered. In this paper, we propose to use this approach for the radionuclide data processing, and to search for the velocity field from the known radiopharmaceutical distribution density [2].

Radionuclide studies are performed using gamma cameras and gamma tomographs [3]. Radionuclide methods are one of the most modern methods of functional diagnostics of diseases of the cardiovascular system, nephrology system, hepatobiliary systems etc. [2, 4, 5]. They require using of mathematical methods for data processing and analysis.

PROBLEM STATEMENT

Let us consider following system of differential equations

$$\dot{x} = f\,, \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \sum_{i=1}^{3} \frac{\partial \rho}{\partial x_i} f_i + \rho\, div f = 0. \tag{2}$$

In accordance with the statement of the problem in article [1], we assume that the transport of the radiopharmaceutical is described by equation (1) and the distribution density of the radiopharmaceutical satisfies the Liouville's equation (2). Here t is time, $x = (x_1, x_2, x_3)^T$ – the spatial coordinate's vector, $\rho = \rho(t,x) = \rho(t, x_1, x_2, x_3)$ – radiopharmaceutical distribution density, $f = f(t,x) = (f_1, f_2, f_3)^T$ is the velocity field and the superscript T denotes transposition of vector.

We suppose that given function $\rho(t,x)$ satisfies the equation (2). The problem is to restore the velocity field of the system (1), i.e. to find function $f(t,x)$. In common case, this is ill-posed problem [6]. So in [1] the regularization method is used and the corresponding variational problem is investigated.

Further we will denote

$$x_1 = x,\ x_2 = y,\ x_3 = z\,,$$
$$f_1 = u,\ f_2 = v,\ f_3 = w.$$

Following suggested approach [1] let us fix some moment t and formulate the problem of determination function $f(t,x)$ as a minimization problem. We introduce the functional

$$J(u,v,w) = \int_M \left(\varphi^2 + \alpha^2 \psi^2\right) dxdydz\,, \tag{3}$$

where $\varphi^2 = (\rho_t + \rho_x u + \rho_y v + \rho_z w + \rho(u_x + v_y + w_z))^2$,

$$\psi^2 = u_x^2 + u_y^2 + u_z^2 + v_x^2 + v_y^2 + v_z^2 + w_x^2 + w_y^2 + w_z^2\,,$$

α^2 is a regularization parameter, M is a nonzero measure region in R^3, ρ_t, ρ_x, ρ_y are notations for partial derivatives of first order in t, x, y respectively.

Thus, the problem of finding velocity field is considered as a functional minimization problem (3) [1, 7].

If we put in the equation (2) $div f = 0$, we get a case of the so-called optical flow, when the density of the indicator remains constant along the trajectories of the system (1) [8, 9].

Further we consider two-dimensional case

$$x_1 = x,\ x_2 = y,\quad f = (u,v)^T.$$

The Euler-Lagrange equations for the integral functional (3) were written in [1] for the general three-dimensional case. For the two-dimensional case, taking into account $div f = 0$, we obtain the well-known equations for finding the velocity field [8]:

$$\begin{aligned} -\alpha^2 \Delta u + \rho_x^2 u + \rho_x \rho_y v &= -\rho_t \rho_x\,, \\ -\alpha^2 \Delta v + \rho_y^2 v + \rho_x \rho_y u &= -\rho_t \rho_y\,, \end{aligned} \tag{4}$$

here $\Delta -$ Laplace operator.

As a result of this approach, the problem for determining functions u, v reduces to solving the system (4) under the appropriate boundary conditions.

In this paper we will consider in more detail the case of the gradient constancy assumption, i.e. when the gradient of the distribution density along the trajectories of the system (1) remains constant:

$$\begin{aligned} \rho_{xx} u + \rho_{xy} v &= -\rho_{xt}\,, \\ \rho_{yx} u + \rho_{yy} v &= -\rho_{yt}\,, \end{aligned} \tag{5}$$

here $\rho_{xx}, \rho_{xt}, \rho_{xy}, \rho_{yy}, \rho_{yt}$ are notations for partial derivatives of the second order.

In this case, the integral functional (3) will be considered for the two-dimensional case and we put

$$\varphi^2 = (\rho_{xt} + \rho_{xx} u + \rho_{xy} v)^2 + (\rho_{yt} + \rho_{yx} u + \rho_{yy} v)^2.$$

The Euler-Lagrange equations will have following form

$$\begin{aligned} &-\alpha^2\Delta u+(\rho_{xx}^2+\rho_{yx}^2)u+\rho_{xy}(\rho_{xx}+\rho_{yy})v= \\ &-\rho_{xx}\rho_{xt}-\rho_{yx}\rho_{yt}, \\ &-\alpha^2\Delta v+(\rho_{xy}^2+\rho_{yy}^2)v+\rho_{xy}(\rho_{xx}+\rho_{yy})u= \\ &-\rho_{xy}\rho_{xt}-\rho_{yy}\rho_{yt}. \end{aligned} \quad (6)$$

To find the velocity field of the system (1), it is necessary to solve the system (6) under the appropriate boundary conditions.

DATA PROCESSING

CONSTRUCTION OF THE SPARSE SYSTEM OF SPECIAL FORM

We consider the system (6), where $(x,y)\in M$ and functions *u, v* are defined at the boundary of the region *M*.

The dynamic data acquisition mode allows observing radiopharmaceutical density distribution in the studied system as a function of time [2]. As a result we obtain radiopharmaceutical density distribution as a function of time and spatial coordinates $\rho=\rho(t,x,y)$, $t\in[0,T]$, $(x,y)\in M$. Taking into account the discretization with respect to time and spatial coordinates we get the sequence of matrices.

We denote the density distribution of the radiopharmaceutical at the point located at the intersection of *i*-th line, *j*-th column, and *k*-th moment of time as $\rho(i,j)$, $i,j=0,\ldots,N+1$. The solution of system (6) can be considered at the nodes of a square grid with a step equal to the one pixel change in the distance along any axis. In the grid point (i,j) the approximation to the solution of system (6) can be written as $u(i,j), v(i,j)$. Laplacians in (6) then can be changed with finite differences and partial derivatives of the second order can be calculated using the density values in the neighboring grid points according to the chosen scheme. So we obtain linear system of equations

$$\begin{cases} -\alpha^2(u(i-1,j)+u(i+1,j)+u(i,j-1)+ \\ u(i,j+1))+(4\alpha^2+\rho_{xx}^2(i,j)+\rho_{yx}^2(i,j))u(i,j)+ \\ (\rho_{xx}(i,j)+\rho_{yy}(i,j))\rho_{xy}(i,j)v(i,j)= \\ -\rho_{xx}(i,j)\rho_{xt}(i,j)-\rho_{yx}(i,j)\rho_{yt}(i,j), \\ -\alpha^2(v(i-1,j)+v(i+1,j)+v(i,j-1)+ \\ v(i,j+1))+(4\alpha^2+\rho_{xy}^2(i,j)+\rho_{yy}^2(i,j))v(i,j)+ \\ (\rho_{xx}(i,j)+\rho_{yy}(i,j))\rho_{xy}(i,j)u(i,j)= \\ -\rho_{xy}(i,j)\rho_{xt}(i,j)-\rho_{yy}(i,j)\rho_{yt}(i,j). \end{cases} \quad (7)$$

$$i,j=1,\ldots,N.$$

According to our assumption that the functions u,v are given on the boundary of the region, only $2N^2$ at the interior points of the grid are unknown in equations (7). Thus, we obtain a linear system of $2N^2$ equations, the solution of which gives an approximation to the solution of the system (6) at grid points. Further we will denote

$$u(i,j)=u_{ij}, v(i,j)=v_{ij}.$$

As a result of the discretization of the system of partial differential equations (6), a linear system of difference equations (7) was obtained, and then we consider its solution. Let introduce the following notation

$$z=(z_1,z_2,z_3,\ldots,z_n)^T, \text{ where } z_s=\begin{pmatrix}u_s\\v_s\end{pmatrix},$$

$$(u_1,u_2,\ldots u_n)^T=$$

$$(u_{11},\ldots u_{1N},u_{21},\ldots u_{2N},\ldots u_{N1},\ldots u_{NN})^T,$$

$$(v_1,v_2,\ldots,v_n)^T=$$

$$(v_{11},\ldots,v_{1N},v_{21},\ldots,v_{2N},\ldots,v_{N1},\ldots,v_{NN})^T.$$

The system (7) can be written in the following form

$$Hz=q. \quad (8)$$

The matrix H and right-hand side and vector of unknowns are portioned as follows

$$\begin{pmatrix} H_{11} & H_{12} & \cdots & H_{1n} \\ H_{21} & H_{22} & \cdots & H_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ H_{n1} & \cdots & H_{n,n-1} & H_{nn} \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix}=\begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{pmatrix}, \quad (9)$$

the partitioning of q and z into subvectors q_i and z_i of size 2 are identical and compatible with the partitioning of H. Here $H-$ block matrix of size $n\times n$, $n=N^2$, with square blocks of second order

$$H_{ss}=\begin{pmatrix} a_{ss} & b_{ss} \\ b_{ss} & c_{ss} \end{pmatrix},\ a_{ss}=4\alpha^2+\rho_{xx}^2(i,j)+\rho_{yx}^2(i,j),$$

$$b_{ss}=\rho_{xy}(i,j)(\rho_{xx}(i,j)+\rho_{yy}(i,j)),$$

$$c_{ss}=4\alpha^2+\rho_{yy}^2(i,j)+\rho_{yx}^2(i,j),\ i,j=\overline{1,N},$$

$$H_{sr}=\begin{pmatrix} a_{sr} & 0 \\ 0 & c_{sr} \end{pmatrix},\ s\neq r,\ a_{sr}=c_{sr},$$ and nonzero elements are equal to $-\alpha^2$; $q_s=\begin{pmatrix}d_s\\e_s\end{pmatrix}$, $s,r=\overline{1,N^2}$,

$$(d_1,d_2,\ldots,d_n)^T=$$

$$(d_{11},\ldots,d_{1N},d_{21},\ldots,d_{2N},\ldots,d_{N1},\ldots,d_{NN})^T,$$

here $d_{ij}=-\rho_{xx}(i,j)\rho_{xt}(i,j)-\rho_{yx}(i,j)\rho_{yt}(i,j),\ i,j=\overline{1,N}$.

$$(e_1,e_2,\ldots,e_n)^T=$$

$$(e_{11},\ldots e_{1N},e_{21},\ldots e_{2N},\ldots,e_{N1},\ldots e_{NN})^T,$$

here $e_{ij}=-\rho_{xy}(i,j)\rho_{xt}(i,j)-\rho_{yy}(i,j)\rho_{yt}(i,j),\ i,j=\overline{1,N}$.

Fig. 1 shows the matrix *H* scheme for the mesh 6×6.

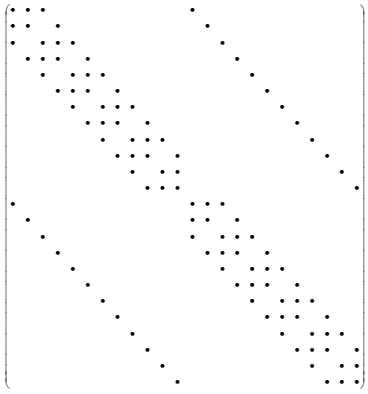

Fig. 1. Pattern of matrix associated with the 6×6 mesh

System (8) is large sparse linear system, and we will solve it by iterative block Gauss-Seidel method [10].

BLOCK GAUSS-SEIDEL METHOD. THE CONVERGENCE OF METHOD

Let us represent matrix H in following way

$$H = D - E - F. \qquad (10)$$

Matrix $D-$ block diagonal, E and $F-$ lower triangular and upper triangular block matrices, respectively.

$$D = \begin{pmatrix} H_{11} & 0 & \cdots & 0 \\ 0 & H_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & H_{nn} \end{pmatrix},$$

$$E + F = \begin{pmatrix} 0 & -H_{12} & \cdots & -H_{1n} \\ -H_{21} & 0 & \cdots & -H_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -H_{n1} & \cdots & -H_{n,n-1} & 0 \end{pmatrix},$$

note, that zeros here are for zero blocks of second order.

Block Gauss-Seidel method for solving system (8), corresponding to introduced partition (9), has the following form

$$H_{ii}\, z_i^{k+1} = -\sum_{j<i}^{n} H_{ij} z_j^{k+1} - \sum_{j>i}^{n} H_{ij} z_j^{k} + q_i, \qquad (11)$$

$$i = \overline{1,n},\ k = 0,1,\ldots,$$

or taking into account (10) and using matrix notations

$$(D - E) z^{k+1} = F z^{k} + q, \quad k = 0,1,2,\ldots. \qquad (12)$$

For the convergence of the method it is sufficient that the conditions obtained in [11] be satisfied:

$$a)\ a_{ss} c_{ss} - b_{ss}^2 > 0, \quad a_{ss} > 0, \quad c_{ss} > 0,$$

$$s = \overline{1,n},$$

$$b)\ \frac{a_{ss} + c_{ss}}{2} \geq \sum_{\substack{r=1 \\ r \neq s}}^{n} |a_{sr}| + \sqrt{\left(\frac{a_{ss} - c_{ss}}{2}\right)^2 + b_{ss}^2}, \qquad (13)$$

$$s = \overline{1,n} \quad and \quad for \quad some \quad s$$

$$\frac{a_{ss} + c_{ss}}{2} > \sum_{\substack{r=1 \\ r \neq i}}^{n} |a_{sr}| + \sqrt{\left(\frac{a_{ss} - c_{ss}}{2}\right)^2 + b_{ss}^2},$$

$$c)\text{"}block\text{"}\ irreducible\ condition.$$

It is easy to verify that conditions (13) are satisfied for system (8), and thus the Gauss-Seidel method converges to a unique solution of system (8) for any initial approximation.

METHOD APPLICATIONS FOR DATA PROCESSING IN RADIONUCLIDE STUDIES

This method can be used to process radionuclide images. Important stages of processing radionuclide studies are motion correction [12 - 14], contour construction and analysis of the regions of interest (ROI). This method can be used to solve these problems. Figs. 2 - 5 give examples of the results of constructing the velocity field for radionuclide images. In Fig. 2, we see two images obtained in the study of the human hepatobiliary system in which there is a shift of the studied organ. In the region indicated by the dotted line, the velocity field has the form shown in Fig. 3.

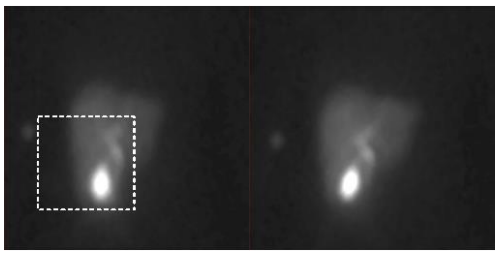

Fig. 2. Two radionuclide images of hepatobiliary system

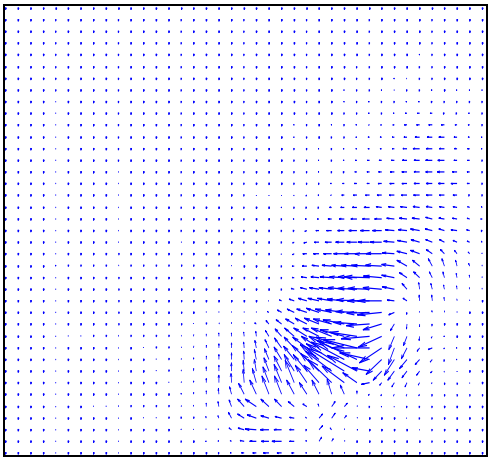

Fig. 3. Velocity field

In Fig. 4, we see the images of region of interest at the moment of time k and $k+1$, these are fragments of images of cardiac radionuclide research. In Fig. 5 the velocity field constructed in this region is represented.

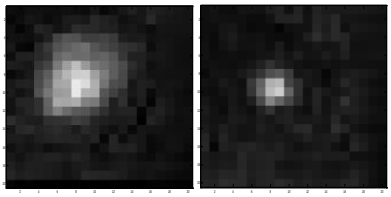

Fig. 4. Images of region of interest (ROI)

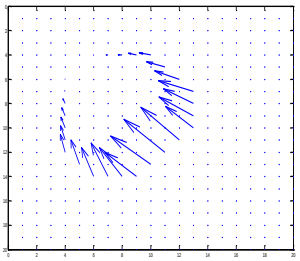

Fig. 5. Velocity field

The data illustrate the possibility of using this method for the motion detecting and its subsequent correction, and also for constructing the contours of the investigated objects on radionuclide images.

CONCLUSIONS

The method of constructing the velocity field proposed in the article can be used in solving the problems of data processing of radionuclide studies. The variants of application given in the article can be used for the analysis of radionuclide images, on their basis it is possible to build algorithms of construction of contours of the investigated areas of interest, and also to carry out

correction of motion for dynamic radionuclide researches.

This method can also be used in other problems, for example, in the study of dynamics of charged particles.

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МЕТОД ПОЛЯ СКОРОСТЕЙ ДЛЯ ОБРАБОТКИ ДАННЫХ РАДИОНУКЛИДНЫХ ИССЛЕДОВАНИЙ

Е.Д. Котина, Д.А. Овсянников

Радионуклидные методы являются одними из современных методов функциональной диагностики различных органов и систем организма человека, которые требуют использования математических методов обработки и анализа данных, полученных в ходе исследования. Поэтому развитие современных методов обработки радионуклидных изображений является актуальной задачей. В статье рассматривается метод для обработки радионуклидных изображений, основанный на построении поля скоростей. Данный метод может применяться для коррекции движения, построения контуров, анализа радионуклидных изображений, также он может использоваться для анализа и формирования динамики заряженных частиц.

МЕТОД ПОЛЯ ШВИДКОСТЕЙ ДЛЯ ОБРОБКИ ДАНИХ РАДІОНУКЛІДНИХ ДОСЛІДЖЕНЬ

О.Д. Котіна, Д.А. Овсянников

Радіонуклідні методи є одними з сучасних методів функціональної діагностики різних органів і систем організму людини, які вимагають використання математичних методів обробки та аналізу даних, отриманих у ході дослідження. Тому розвиток сучасних методів обробки радіонуклідних зображень є актуальним завданням. У статті розглядається метод для обробки радіонуклідних зображень, заснований на побудові поля швидкостей. Даний метод може застосовуватися для корекції руху, побудови контурів, аналізу радіонуклідних зображень, також він може використовуватися для аналізу та формування динаміки заряджених частинок.