A DISPERSION EQUATION OF THE CYLINDRICAL IDEAL WALL VACUUM CAVITY SINUSOIDALLY CORRUGATED IN AZIMUTHAL DIRECTION. PART II. INVESTIGATION OF THE DISPERSION EQUATION

A.V. Maksimenko[1], V.I. Tkachenko[1,2], I.V. Tkachenko[1]
[1]National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine;
[2]V.N. Karazin Kharkiv National University, Kharkov, Ukraine
E-mail: tkachenko@kipt.kharkov.ua

Dispersion equation of a cylindrical cavity with an ideally conducting outer wall has been investigated, whose radius is described by a sinusoidal-periodic dependence on the azimuth angle. Numerically and analytically it is shown that in the neighborhood of intersection points of neighboring harmonics here appear non-transmission bands in which there are no oscillations of the cavity. The dependence of the width of the unblocking band on the corrugation depth is determined. It is shown that for an arbitrary choice of the corrugation depth and the number of corrugations of the cavity in its frequency spectrum, it is possible that there are no natural frequencies that were originally assumed to be working.

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INTRODUCTION

Due to the wide range of applications of corrugated resonance systems, a correct analysis of the dispersion properties of such systems is relevant. In this paper we investigate the dispersion equation obtained in [1]. The non-transmission bands are investigated, and the dispersion curves are compared with the corresponding dispersion curves, which are obtained in the traditional way.

THE INTERACTION OF TWO ADJACENT HARMONICS OF THE AZIMUTHALLY CORRUGATED CAVITY

NUMERICAL SOLUTION OF THE DISPERSION EQUATION OF THE AZIMUTHALLY CORRUGATED CAVITY

To describe the interaction of two neighboring harmonics, it is necessary to specify the point or points of their intersection, since harmonics can interact with each other only when intercepted [2].

In the analytical form, this is difficult to do. Therefore, we solve the problem of the presence of intersection points using numerical methods. For this purpose, similar to the numerical calculations carried out in [3], the dispersion equation (14) in [1] was solved in order to determine the dependence of the corrugation depth of the azimuthally corrugated cavity with the number of corrugations $M = 2m$ from cutoff frequency x_0.

In Fig. 1,a,b the dispersion dependences that determine the depth of corrugation of the azimuthally corrugated cavity with the number of corrugations $M = 2m = 4$ (a) and $M = 2m = 6$ (b) from cutoff frequency x_0.

As follows from Fig. 1 results of numerical calculations, at the points of intersection of the first mirror harmonics 2'(3') with second harmonics 6(9) intervals are formed along the depth of the corrugation, in which there are no real values of the cut-off frequency. In the figure, these regions are marked by ovals 1 and 2.

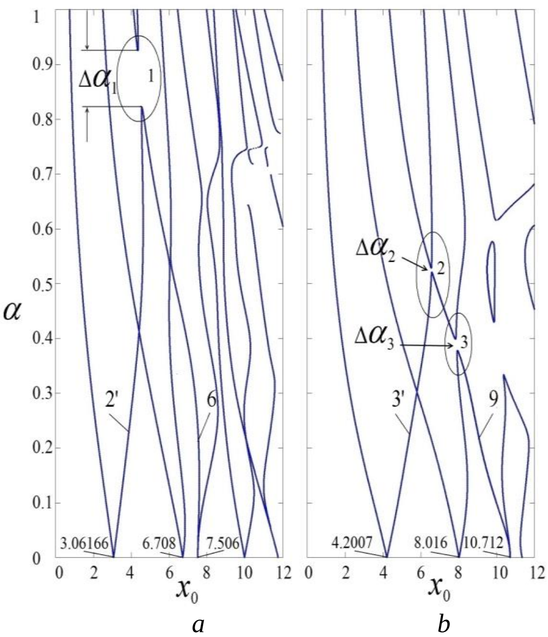

Fig. 1. Dependence of depth of ripple α from cutoff frequency x_0 for mirror reflections of the first harmonic $l_0 = 1$ (curves 2' and 3') and second harmonic $l_0 = 2$ (curves 6 and 9) azimuthally corrugated cavity with a number of corrugations $M = 4$ (a) and $M = 6$ (b)

The membership of the curves 2'(3') and 6(9) first $(l_0 = 1)$ and second $(l_0 = 2)$ harmonics are respectively determined from the values of the cutoff frequencies in the absence of corrugation (when $\alpha = 0$). The values of these cut-off frequencies are shown in Fig. 1 in the form of numbers above the axis of abscissas. It can be seen from them that the curves 2'(3') first and curves 6(9) of the second harmonic are determined by the first zeros ($n = 1$ in (26) in [1]) the derivative of the Bessel function $dJ_{m(2l_0-1)}(x_0)/dx$.

However, as follows from the figure, the second harmonic also interacts with the first-harmonic harmonics $(l_0 = 1)$, formed by the second zeros ($n = 2$ in (26) in [1]) the derivative of the Bessel function $dJ_{m(2l_0-1)}(x_0)/dx$ (oval 3).

From the above numerical calculations it follows that for the given cutoff frequencies x_0 there are intervals along the depth of ripple $(\Delta\alpha)_i$, $i=1;2;3$, where the cutoff frequencies of the corrugated cavity are absent. This means that in these intervals there are no eigen oscillations of the cavity.

On the basis of the foregoing, it can be concluded that for an arbitrary choice of the corrugation depth and the number of corrugations of the cavity in its frequency spectrum, the absence of those natural frequencies that were originally assumed by workers.

ANALYTICAL STUDY OF THE DISPERSION EQUATION OF THE AZIMUTHALLY CORRUGATED CAVITY

On the basis of the resulting convergent infinite products, we investigate the dispersion properties of a corrugated cavity.

The convergent infinite product (22) in [1] implies the convergence of not only the first harmonic (26) in [1] ($\left|C_{l_0}^m\right|=1$ with $l_0=1$ and $m=2,3,4,5,6$), but also the convergence of the product of two neighboring harmonics, for example, $\left|C_{l_0}^m\right|\cdot\left|C_{l_0+1}^m\right|=B_{l_0}^m<\infty$, where $B_{l_0}^m$ is the finite number.

For an analytical description of the interaction of two neighboring harmonics, it is necessary to specify the point of their intersection. Harmonics can interact with each other only when suppressed [2].

Let us describe in general form the interaction of neighboring harmonics l_0 and l_0+1 at an arbitrary value m.

We specify the coordinates of the points of intersection of neighboring harmonics in the form:

$$\alpha=\alpha^*_{m(2l_0-1)},\ x_0=x^*_{m(2l_0-1)}.$$

A numerical comparison of the ones is shown in Fig. 3 in [1] for the first harmonics of TE modes of the corrugated cavity and curves 2'(3') and 6(9) in Fig. 1 that shows that $\left|C_{l_0}^m\right|\cdot\left|C_{l_0+1}^m\right|=\left|C_{l_0+1}^m\right|=B_{l_0}^m>1$. Inequality $B_{l_0}^m>1$ is a consequence of the large slope of the curves 6(9).

On the basis of the foregoing, we form a bounded positive definite form of the form:

$$Q_{m(2l_0-1)}(\alpha)=\left(1+C_{m(2l_0-1)}\right)\left(1+C_{m(2l_0+1)}\right)<\infty, \quad (1)$$

where

$$C_{m(2l_0-1)}=\pm\frac{\alpha}{f_{m(2l_0-1)}(x_0)},\ C_{m(2l_0+1)}=\pm\frac{\alpha}{B_{l_0}^m f_{m(2l_0+1)}(x_0)},$$

$Q_{m(2l_0-1)}(\alpha)>0$ is the bounded positive definite function.

The form $Q_{m(2l_0-1)}(\alpha)$ must be set so that far from the point of intersection the harmonics are not influenced, and in the vicinity of the point of intersection they are connected in accordance with the requirements of mode coupling in corrugated systems [4].

In accordance with (22) in [1], the form (1) must depend on the square of the depth of the ripple α, since in this case the change in sign α does not change the dispersion of waves.

As a function $Q_{m(2l_0-1)}(\alpha)$, satisfying the requirements of the mode coupling in corrugated systems, we choose a model function of the form:

$$Q_{m(2l_0-1)}(\alpha)=p^2_{m(2l_0-1)}\alpha^{2N_{m(2l_0-1)}}\exp\left(-\frac{\left(\alpha-\alpha^*_{m(2l_0-1)}\right)^2}{\left(\alpha^*_{m(2l_0-1)}\right)^2}\right). \quad (2)$$

It follows from (2) that in the neighborhood of the intersection point of the dispersion curves (for $\left|\alpha-\alpha^*_{m(2l_0-1)}\right|\ll\alpha^*_{m(2l_0-1)}$) limited positive form $Q_{m(2l_0-1)}(\alpha)$ is described by expression $Q_{m(2l_0-1)}(\alpha)=p^2_{m(2l_0-1)}\alpha^{2N_{m(2l_0-1)}}\ll 1$, where $p_{m(2l_0-1)}$ is a number depending on the number of intersecting harmonics l_0, $N_{m(2l_0-1)}=0,1,2,3\ldots$ are the positive integers.

Far from the intersection point of the dispersion curves of the corrugated cavity (for $\left|\alpha-\alpha^*_{m(2l_0-1)}\right|\gg\alpha^*_{m(2l_0-1)}$) the mutual influence of the harmonics is not significant, because function $Q_{m(2l_0-1)}(\alpha)$ is quite small: $Q_{m(2l_0-1)}(\alpha)\ll p^2_{m(2l_0-1)}\alpha^{2N_{m(2l_0-1)}}$. In this case, Eq. (1) describes two independent harmonics: l_0 and l_0+1.

Thus, from the assumptions made above on the dependence $Q_{m(2l_0-1)}(\alpha)$ from corrugation depth α, it follows that near the point of intersection of the harmonics their frequency spectra turn out to be connected.

To describe the connection between the harmonics, we can perform the following substitutions in (1) (for negative signs of the second terms of the factors in the right-hand side of (1)), the following substitutions:

$$\alpha=\alpha^*_{m(2l_0-1)}+\tilde{y}_{m(2l_0-1)},\ x_0=x^*_{m(2l_0-1)}+\tilde{x}_{m(2l_0-1)},$$

where we consider $\left|\tilde{y}_{m(2l_0-1)}\right|\ll\alpha^*_{m(2l_0-1)}$, $\left|\tilde{x}_{m(2l_0-1)}\right|\ll x^*_{m(2l_0-1)}$. As a result of such substitutions, we obtain an equation describing the coupling of harmonics (henceforth everywhere the wavy sign of $\tilde{y}_{m(2l_0-1)}$ and $\tilde{x}_{m(2l_0-1)}$ omit):

$$\left(y_{m(2l_0-1)}-\left|k_{m(2l_0-1)}\right|x_{m(2l_0-1)}\right)\left(y_{m(2l_0-1)}+B_{l_0}^m\left|k_{m(2l_0+1)}\right|x_{m(2l_0-1)}\right)=$$
$$=p^2_{m(2l_0-1)}\alpha^{2N_{m(2l_0-1)}}B_{l_0}^m\cdot\left(\alpha^*_{m(2l_0-1)}\right)^2, \quad (3)$$

where $k_{m(2l_0-1)}=\left.\frac{d}{dx}\alpha(x)\right|_{x^*_{m(2l_0-1)}}$, and the depth of ripple $\alpha(x)$ is determined by the expressions (29), (32) from [1] in the appropriate ranges of the argument variation.

In equation (3), the parameters $N_{m(2l_0-1)}$ and $p_{m(2l_0-1)}$ are uncertain.

Therefore, to analyze (3), we set these parameters.

Parameter $N_{m(2l_0-1)}$ we define, for example, assuming $N_{m(2l_0-1)}=K-1$, where the number $K=1,2,3,\ldots$ determines the mismatch between a given harmonic and a higher harmonic l_0+K, which intersects with the given l_0.

Thus, for example, $m=2$ and $l_0=1$ for neighboring harmonics the exponent K is equal to one $(K=1)$. This corresponds to the intersection of the first mirror harmonic $m(2l_0-1)=2'$ with harmonics $m(2(l_0+K)-1)=6$.

If the K is equal to two $(K=2)$, this corresponds to the intersection of the first harmonic $m(2l_0-1)=2'$ with a harmonic located one from the first: $m(2(l_0+K)-1)=10$.

If the indicator is equal to three $(K=3)$, this corresponds to the intersection of the first harmonic $m(2l_0-1)=2'$ with a harmonic located two from the first: $m(2(l_0+K)-1)=14$ etc.

It should be noted that the proposed definition of the parameter $N_{m(2l_0-1)}$ follows from the property of the interaction of spatial harmonics in corrugated waveguide systems [5, 2].

Parameter $p_{m(2l_0-1)}$ we determine from the geometric characteristics of the intersecting harmonics.

From Fig. 1 it follows that with growth m for neighboring harmonics $(K=1)$ the coordinates of their intersection points $x^*_{m(2l_0-1)}$ with an increase in the number of harmonics l_0 shifted from zero the derivative of the Bessel function $\gamma'_{m(2l_0+1),1}$ to the lower zero $\gamma'_{m(2l_0-1),1}$ so that the conditions:

$$\left|x_{m(2l_0+1)}\right| = \left|x^*_{m(2l_0-1)} - \gamma'_{m(2l_0-1),1}\right| = p_0 << 1,$$

$$\left|y_{m(2l_0-1)}\right| << \left|k_{m(2l_0-1)}\right| x_{m(2l_0-1)}, B^m_{l_0}\left|k_{m(2l_0+1)}\right| x_{m(2l_0-1)}, \quad (4)$$

where p_0 is the maximum approach along the abscissa axis of the derivative of the Bessel function $\gamma'_{m(2l_0-1),1}$ with a point of intersection of harmonics.

From the last condition (4) it can be shown that for neighboring harmonics, when the parameter $N_{l_0m}=0$, it is always possible to find conditions under which

$$p^2_{m(2l_0-1)} = p_0^2\left|k_{m(2l_0-1)}\right|\cdot\left|k_{m(2l_0+1)}\right|.$$

Thus, summarizing the foregoing, the equation of coupling of neighboring harmonics can be represented in the form:

$$\left(y_{m(2l_0-1)} - \left|k_{l_0m}\right| x_{m(2l_0-1)}\right)\left(y_{m(2l_0-1)} + B^m_{l_0}\left|k_{m(2l_0+1)}\right| x_{m(2l_0-1)}\right) =$$

$$= p_0^2\left|k_{m(2l_0-1)}\right|\cdot\left|k_{m(2l_0+1)}\right| B^m_{l_0}\cdot\left(\alpha^*_{m(2l_0-1)}\right)^2. \quad (5)$$

We consider the particular case of intersection of neighboring (order of intersection $K=1$) harmonics $l_0=1$ and $l_0+1=2$ azimuthally corrugated cavity with $m=2$. Equation (5) in this case describes the non-transmission bandwidth, i.e. corrugation depth interval α, where there are no real cut-off frequencies x_0.

Fig. 2 shows the graph of the hyperbola arising in the neighborhood of the intersection point of the curves $m=2'$ and $m=6$ with coordinates $\alpha^*_2=0.878$ and $x^*_2=4.366$. Parameters B_1^2 and p_0 were estimated by numerical calculations (see Fig. 1,a), and were chosen equal: $B_1^2=5.7$; $p_0=0.2$.

Numerical calculations show that the width of the non-transmission bandwidth in the considered case of the order of

$$(\Delta\alpha)_1 = p_0\sqrt{B_1^2|k_2||k_6|}\alpha^*_2 = 0.2\sqrt{5.7|k_2||k_6|}\alpha^*_2 \approx 0.$$

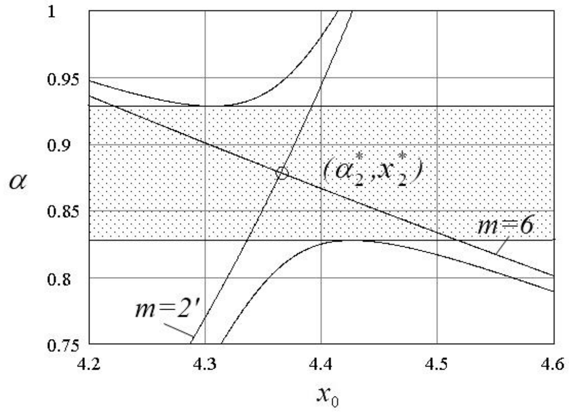

Fig. 2. The unblocking band (the shaded area) in the vicinity of the intersection point of the dispersion curves $m=2'$ and $m=6$ with coordinates $\alpha^*_2=0.878$ and $x^*_2=4.366$

This value corresponds quantitatively to the numerical method.

Thus, in this section it is shown that in a corrugated cavity for a given m in the neighborhood of the points of intersection of the harmonics $(2l_0+1)$ with mirror reflection of harmonics $(2l_0-1)$ the non-transmission bands appear. In these bands there is no oscillation of the cavity. For the order of intersection K the width of the unblocking band depends on the depth of the corrugation in proportion $\alpha^{K-1}\left(\alpha^*_{l_0m}\right)$, where $\alpha^*_{l_0m}$ is the ordinate of the point of intersection of harmonics $2l_0-1$ and $2(l_0+K)-1$.

The above-described appearance of non-transmission bands at the intersection of harmonics $2l_0+1$ with mirror reflection of harmonics $2l_0-1$ follows from direct numerical calculations of the dispersion equation (14) in [1].

CONCLUSIONS

The dispersion equation of an ideally conducting cylindrical vacuum cavity with sinusoidal corrugated boundaries in the azimuth direction was investigated. The dispersion of the first harmonics of a corrugated cavity is studied for different, even number of corrugations. Analytical dependencies are quantitatively consistent with the experimental data.

The interaction of two neighboring harmonics of the azimuthally corrugated cavity is studied. It is shown that for a given m, in the neighborhood of the points of intersection of the harmonic l_0+1 with mirror reflection of harmonics l_0 the non-transmission bands appear. In these bands there is no oscillation of the cavity. The width of the unblocking band depends on the depth of the corrugation as $\alpha^{K-1}\left(\alpha^*_{l_0m}\right)$, where $\alpha^*_{l_0m}$ is the ordinate of the point of intersection of harmonics l_0 and l_0+K, $K=1,2,3,\ldots$ is the mismatch of harmonics. The proposed analytical description of oscillatory processes in an ideally conducting cylindrical vacuum cavity with sinusoidal corrugated boundaries in the azimuth direction is confirmed by numerical calculations.

It is shown that for an arbitrary choice of the corrugation depth and the number of corrugations of the cavity in its frequency spectrum, it is possible that there are no eigenfrequency that were originally assumed to be working.

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ДИСПЕРСИОННОЕ УРАВНЕНИЕ ЦИЛИНДРИЧЕСКОГО ВАКУУМНОГО РЕЗОНАТОРА С ИДЕАЛЬНЫМИ ГОФРИРОВАННЫМИ В АЗИМУТАЛЬНОМ НАПРАВЛЕНИИ СТЕНКАМИ. ЧАСТЬ II. ИССЛЕДОВАНИЕ ДИСПЕРСИОННОГО УРАВНЕНИЯ

А.В. Максименко, В.И. Ткаченко, И.В. Ткаченко

Исследовано дисперсионное уравнение цилиндрического резонатора с идеально проводящими стенками, радиус которого описывается синусоидально-периодической зависимостью относительно азимутального угла. Численно и аналитически показано, что в окрестности точек пересечения соседних гармоник появляются полосы непропускания, в которых колебания резонатора отсутствуют. Определена зависимость ширины полосы непропускания от глубины гофрировки. Показано, что при произвольном выборе глубины гофрировки и количества гофров резонатора в его частотном спектре возможно отсутствие тех собственных частот, которые первоначально предполагались рабочими.

ДИСПЕРСІЙНЕ РІВНЯННЯ ЦИЛІНДРИЧНОГО ВАКУУМНОГО РЕЗОНАТОРА З ІДЕАЛЬНИМИ ГОФРОВАНИМИ В АЗИМУТАЛЬНОМУ НАПРЯМКУ СТІНКАМИ. ЧАСТИНА II. ДОСЛІДЖЕННЯ ДИСПЕРСІЙНОГО РІВНЯННЯ

А.В. Максименко, В.І. Ткаченко, І.В. Ткаченко

Досліджено дисперсійне рівняння циліндричного резонатора з ідеально провідними стінками, радіус якого описується синусоїдально-періодичною залежністю відносно азимутального кута. Чисельно та аналітично показано, що поблизу точок перетину сусідніх гармонік з'являються смуги непропускання, в яких коливання резонатора відсутні. Визначено залежність ширини смуги непропускання від глибини гофрування. Показано, що при довільному виборі глибини гофрування і кількості гофрів резонатора в його частотному спектрі можлива відсутність тих власних частот, які спочатку передбачалися робочими.