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Power-law low-temperature asymptotics for spatially nonhomogeneous *s*-wave superconductors

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It is shown that low temperature asymptotics of various thermodynamic and transport properties of *s*-wave superconductors can become power-law ones if wide distributions of gap values exist, originating from structure domains, charge stripes, charge-density waves or other mesoscopic nonhomogeneities. The relevant experimental data for high-T_c oxides are analyzed on the basis of the developed theory.

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1. Introduction

The controversy over the order parameter symmetry in cuprates constitutes a great challenge to investigators and is far from being resolved [1–3], contrary to what is sometimes claimed [4,5]. Really, the relevant experimental data may be divided into three main groups. The first group includes phase-sensitive methods, e.g., the phase-sensitive observations of the half-flux quantum spontaneous magnetization of the three-grain boundaries [4,6] and of the anomalous π-phase shift across the *c*-axis junction straddling a single twin [7]. These experimental results are often considered as an unequivocal evidence of the *d*-wave order parameter character. Nevertheless, as discussed in Ref.[2], the ordinary *s*-wave order parameter suppression at twin boundaries, the flux trapping there or in the corners, and meanderings of the grain boundaries on the scale of 100–1000 Å [8] can reproduce such a behavior as well. Moreover, the most recent measurements (see discussion in Ref. [9]) of the *c*-axis (perpendicular to the layers) Josephson critical current I_c between twisted bicrystals of $Bi_2Sr_2CaCu_2O_{8+y}$ ruled out the purported «naive» identification of the order parameter symmetry there with $d_{x^2-y^2}$-wave form. Namely, there was no dependence of I_c on the twist angle φ_0 , whereas in the *d*-wave case it would have been $I_c \propto \cos 2\varphi_0$. At the same time, *c*-axis tunneling between $Bi_2Sr_2CaCu_2O_{8+y}$ and Pb shows a distinct Fraunhofer pattern appropriate to s-wave order parameter, although the magnitude of the I_c is very small [10].

Two other groups are phase-insensitive. One of them probes the gap features, if any, at the Fermi surface (FS). It includes, in particular, the angle-resolved [11], tunnel [12], and point-contact [13] spectroscopies. The results obtained, making use of these methods for a number of specific hole-doped oxides, are also usually interpreted as manifestations of the *d*-wave pairing. However, this interpretation may be misleading. Namely, the emergence of the dielectric gap on the nesting FS sections due to the charge-density wave (CDW) formation may mimic the superconducting pseudogap above the critical temperature T_c and severely hamper various measurements of the superconducting gap below T_c [2,14–16]. For example, the predicted current-voltage characteristics (CVC's) asymmetricity for junctions involving CDW superconductors with *s*-type pairing [16] resemble those of the *ab*-plane tunnel CVC's for $Bi_2Sr_2CaCu_2O_{8+y}$ [17]. The same can be said about the persistence of the smeared pseudogap features in this experiment, so that the CDW's rather than the *d*-wave scenario with *V*-shaped conductance show themselves. The investigations [18] of inelastic Cooper pair tunneling for various phases of the Bi-Sr-Ca-Cu-O system clearly demonstrated the existence of the Riedel-like singularity and the subsequent steep reduction of the Josephson current inherent to Bardeen-Cooper-Schrieffer (BCS) isotropic superconductors [19], whereas the *d*-wave picture lacks such a threshold behavior [20]. The indications of the *d*-wave incon-

sistency with measured photoexcited relaxation dynamics in $YBa_2Cu_3O_{7-x}$ were also found [21]. The direct evidence of the *s*-wave pairing in $YBa_2Cu_3O_{7-x}$ and $YbBa_2Cu_3O_{7-x}$ using tunnel and point-contact measurements was also obtained in Ref. [22], where clear classical gap features were seen.

It is also claimed that the type of pairing can be, in principle, deduced as well from the phase-insensitive experiments by analyzing the properties of high-T_c oxides in the low-temperature limit. Indeed, the BCS theory leads to the following asymptotics for various thermodynamic and transport properties P_s of superconductors at temperatures T far below T_c [23]:

$$P_s^{\text{asympt}}(\Delta_0, T) = A\Delta_0^m T^l \exp\left(-\frac{\Delta_0}{T}\right). \qquad (1)$$

Here $k_B = \hbar = 1$, Δ_0 is the value of the superconducting gap at $T = 0$ and the quantities m and l are specific to the property concerned. Instead, a lot of investigations reveal power-law T-dependences [1,2,4]. Such a behavior was explained on the basis of the assumed *d*-wave symmetry of the order parameter with gap point or line nodes of the FS [1,4]. Our article is devoted just to this kind of experiments. However, in contrast to the traditional viewpoint, it is shown below that, if one takes into account the wide distribution of the superconducting order parameter magnitudes always existing in complex nonhomogeneous structures of high-T_c oxides [24–26], the same results can be explained by the conventional *s*-wave pairing.

2. Experimental low-temperature asymptotics for cuprates

To be more specific, let us consider some experimental low-T data for cuprates. In particular, for $YBa_2Cu_3O_{7-x}$ the specific heat $C_s \propto T$ [2], although the recent experiment [21] shows additional contribution $\propto T^2$ and Schottky anomalies $\propto T^{-2}$ [27], making the whole picture uncertain. At the same time, the results for the *d*-wave gap function would have been proportional to T^2 for hexagonal or T^3 for cubic lattices [28].

For $\delta\lambda_L(T) = [\lambda_L(T) - \lambda_L(0)]\,\lambda_L^{-1}(0)$, where $\lambda_L(T)$ is the constant magnetic field penetration depth, the experimental data are quite ambiguous. For nominally pure $YBa_2Cu_3O_{7-x}$ samples linear dependences on T are observed [29], whereas for Zn- and Ni-doped as well as nonhomogeneous crystals $\delta\lambda_L \propto T^2$ [29–33] in a formal accordance with the theory of *d*-wave superconductors, either dirty ones [4] or those with surface-induced Andreev bound states [33]. There are also data showing two-gap low-T asymptotics of $\delta\lambda_L(T)$ in $YBa_2Cu_3O_{7-x}$ [34]. The authors of Ref. [34] claim that this dependence is intrinsic, whereas the results of Ref. [29] are due to a non-uniform sample oxygenation. For electron-doped oxide $Nd_{1.85}Ce_{0.15}CuO_{4-\delta}$, which always reveals *s*-wave features, $\delta\lambda_L(T)$ follows the exponential law [1]. At the same time, the initial *d*-wave picture with $\delta\lambda_L(T \to 0) \propto T$ was shown to be inconsistent with the third law of thermodynamics [35]. Further modifications [36,37] (see reply in Ref. [38]) changed the electromagnetic response of the *d*-wave superconductor in such a way that the calculated in-plane dependence $\delta\lambda_{ab}(T \to 0) \propto T^2$ holds, not violating thermodynamics but destroying the apparent agreement with the experiment.

Unfortunately, it is hard to extract the electronic thermal conductivity component κ_e from the experiment due to the complex action of electrons, phonons, and impurities [39,40]. Nevertheless, the experiments indicate that $\kappa_e \propto T$ in Zn-doped $YBa_2Cu_3O_{7-x}$ [41] and below $T_c^* = 200$ mK in $Bi_2Sr_2Ca(Cu_{1-x}Ni_x)_2O_8$ [42]. The ultrasonic attenuation coefficient α_s also exhibits a power-law T^n decrease for $T << T_c$ both for $YBa_2Cu_3O_{7-x}$ [43,44] and $La_{1.8}Sr_{0.2}CuO_{4-x}$ [43] with a large scatter of the exponent n for each substance. As for the nuclear relaxation rate T_1^{-1}, it demonstrates power-law dependences with $1 \le n \le 3$ [4].

This experimental material shows that the universal dependence for any phenomenon discussed does not exist. Moreover, the agreement with the theories based on the gap function with point or line nodes is superficial. Even with additional assumptions being made, the equality between experimental and theoretical power-law exponents still cannot be ensured. It is usually considered as a basis for the adoption of the *d*-wave concept [1,4]. At the same time, while analyzing local structures observed in the nonstoichiometric superconducting and nonsuperconducting oxides [24–26,45], we drew the conclusion that there is a quite different solution to the problem. Our approach starts from the assumption of a wide distribution of order parameter Δ values in the bulk of the samples at each $T < T_c$.

3. Theory

The key idea of the theory is that not only a polycrystalline but even a single crystal superconducting oxide sample can be considered as *mesoscopically nonhomogeneous*, i.e., consisting of

domains. This domain structure is supposed to be T-independent, with each domain having the following properties:

(A) at $T = 0$ it is described by a certain superconducting order parameter Δ_0;

(B) up to the relevant critical temperature $T_{c0}(\Delta_0) = \gamma\Delta_0/\pi$, where $\gamma = 1.7810...$ is the Euler constant, it behaves as a true BCS superconductor, i.e., the temperature dependence $\Delta(T)$ of the superconducting order parameter is the Mühlschegel function $\Delta(T) = \Delta_{BCS}(\Delta_0, T)$; any property P under investigation is characterized in this interval by the function $P_s(\Delta,T)$;

(C) at $T > T_{c0}$ it changes into the normal state, and the relevant property is $P_n(T)$.

At the same time, the values of Δ_0 scatter for various domains. The current carriers move freely across domains and inside each domain acquire the respective properties. Thus, possible proximity effects resulting in the correlation of the properties of adjacent domains are neglected. The current carrier density is assumed constant all over the sample, so transient processes are excluded from consideration.

The averaging procedure considered below requires (i) the effective sample size L to be much larger than the mean size of the domains d_{mean} and (ii) the size of each domain d_i to be larger than the relevant coherence length ξ_i. The first condition is needed to regard the superconductor macroscopically homogeneous. The second one stems from the property (B) indicated above. In the opposite case, when $d_i << \xi_i$, we are led to the lattice model of superconductor with a local atomic disorder [46,47]. Such a model was applied to the description of $YBa_2Cu_3O_{7-x}$ in Ref. [48]. In essence, the domain size there is comparable to that of the elementary cell. However, in this limiting case we go beyond the scope of the BCS s-wave picture based on the long-range character of the phonon-induced interaction between electrons [23] (see discussion in Sec. 4). In contrast, the actually adopted condition $d_i > \xi_i$ is fully in line with the basic concept.

Under these conditions, we consider the current carrier liquid to involve normal, $\rho_n(T)$, and superconducting, $\rho_s(T)$, fractions with $\rho_n(T) + \rho_s(T) = 1$, and the superconducting fraction to be multicomponent. Each superconducting component corresponds to domains with a certain Δ_0. They possess the properties (A), (B), (C) mentioned above. The superconducting fraction at $T = 0$ can be described by a distribution function $f_0(\Delta_0)$ in the interval $0 \le \Delta_0 \le \Delta_o^{\max}$:

$$\rho_s(0) = \int_0^{\Delta_0^{\max}} f_0(\Delta_0)d\Delta_0 = 1 - \rho_n(0). \qquad (2)$$

The distribution is assumed wide, i.e., $f_0(\Delta_0)$ is non-zero at every point of the interval. In principle, $f_0(\Delta_0)$ can be random or not, but the former case seems more frequently occurring.

At $T \neq 0$ the superconducting components with $T_{c0} < T$, i.e., with $\Delta_0 < \Delta^*(T) = \pi T/\gamma$, lose their superconducting properties. The normal fraction of the current carriers in the sample is

$$\rho_n(T) = \rho_n(0) + \int_0^{\Delta^*(T)} f_0(\Delta_0)d\Delta_0 , \qquad (3)$$

whereas the remaining superconducting part is

$$\rho_s(T) = \int_{\Delta^*(T)}^{\Delta_0^{\max}} f_0(\Delta_0)d\Delta_0 . \qquad (4)$$

Due to the condition (B), the components, possessing at $T = 0$ order parameters within the interval $[\Delta_0, \Delta_0 + d\Delta_0]$, at $T \neq 0$ acquire order parameters within the interval $[\Delta, \Delta + d\Delta]$, where $\Delta = \Delta_{BCS}(\Delta_0, T)$. This conversion is expressed by an equation

$$f(\Delta,T)d\Delta = f_0(\Delta_0)d\Delta_0 . \qquad (5)$$

Here $f(\Delta,T)$ is a function characterizing a new distribution of components in the interval $0 < \Delta < \Delta^{\max}(T)$ where $\Delta^{\max}(T) = \Delta_{BCS}(\Delta_0^{\max}, T)$. This equation is a consequence of (i) the supposed domain structure permanence, (ii) the constant current carrier density, and (iii) the independence between superconducting components. Then, the function $\rho_s(T)$ takes the form

$$\rho_s(T) = \int_0^{\Delta^{\max}(T)} f(\Delta, T)d\Delta . \qquad (6)$$

As for any investigated property P, each component, being superconducting or not, makes its contribution to the measured (averaged) value $\langle P \rangle$:

$$\langle P(T) \rangle = P_n(T)\, \rho_n(T) + \int_o^{\Delta^{\max}(T)} P_s(\Delta, T)\, f(\Delta, T)\, d\Delta . \qquad (7)$$

This formula is valid (with restriction given above) for additive quantities, such as, e.g., the specific heat. But what about, for example, the penetration depth λ_L?. Really, in the situation when the superconducting gap changes (and in fact goes to zero) on a very short length scale, even the notion of the penetration depth becomes questionable. Moreover, since each of our elementary volumes includes an ensemble of domains with different parameters $\lambda_{L,i}$'s, the matter becomes much more entangled. Nevertheless, even in this situation one may introduce an effective penetration depth λ_L^{eff} and measure its T-dependence. Really, the measured electromagnetic response of the nonhomogeneous superconductor is the sum of individual domain responses from the sample surface layer. The quantity λ_L^{eff} is a parameter that is extracted from the essentially averaged experimental data treated as obtained for a homogeneous BCS superconductor. In the specific case of cuprates the domain sizes d_i are substantially smaller than the intrinsic penetration depths $\lambda_{L,i}$ for each domain and, therefore, the effective λ_L^{eff}. We conceive that within such a context the calculation of $\lambda_L^{\text{eff}}(T)$ as a weighted quantity is at least qualitatively reasonable.

The first term in Eq. (7) describes the contribution $\langle P(T)\rangle_n$ of the normal fraction. It is well-known and will not be considered below. The last term corresponds to the contribution $\langle P(T)\rangle_s$ of the superconducting electrons (holes). Since $f(\Delta = 0, T) = f_0[\Delta_0 = \Delta^*(T)] \neq 0$, for each T there is a nonvanishing portion of superconducting components with $\Delta \to 0$. It is their contribution that leads to the deviation of the temperature behavior $\langle P(T)\rangle_s$ from the classical one. To make our statement even more sound, we suggest that the low-T asymptotics (1) holds true for each superconducting component up to the relevant critical temperature T_{c0} , i.e.,

$$P_s(\Delta, T) = P_s^{\text{asympt}}(\Delta_0, T) \ . \qquad (8)$$

The allowance for the exact dependences may only strengthen our standpoint.

Note that $P_s^{\text{asympt}}(\Delta_0, T)$ in the framework of the BCS scheme depends on T and on Δ_0 rather than on Δ value. Accordingly, due to Eq. (5) the contribution $\langle P(T)\rangle_s$ can be rewritten as follows:

$$\langle P(T)\rangle_s = \int_{\Delta^*(T)}^{\Delta_0^{\max}} P_s^{\text{asympt}}(\Delta_0, T)\, f_0(\Delta_0)\, d\Delta_0 \ . \qquad (9)$$

The distribution function $f_0(\Delta_0)$ can be expanded into the series

$$f_0(\Delta_0) = \frac{1}{\Delta_0^{\max}} \sum_{k=k_0}^{\infty} B_k \left(\frac{\Delta_0}{\Delta_o^{\max}} \right)^k , \qquad (10)$$

where k_0 is the order of the leading expansion term. Substituting Eqs. (1) and (10) into Eq. (9) we obtain

$$\langle P(T)\rangle_s = \frac{AT^{l+m+1}}{\Delta_0^{\max}} \sum_{k=k_0}^{\infty} B_k \left(\frac{T}{\Delta_0^{\max}} \right)^k \int_{\frac{\Delta^*(T)}{T}}^{\frac{\Delta_0^{\max}}{T}} x^{m+k}\, \mathrm{e}^{-x} dx. \qquad (11)$$

Within an accuracy of the made approximations and for temperatures $T << \Delta_0^{\max}$ we may extend the upper limit of integration to infinity, so

$$\langle P(T)\rangle_s \approx AT^{l+m} \left(\frac{T}{\Delta_0^{\max}} \right) \times$$

$$\times \sum_{k=k_0}^{\infty} B_k \left(\frac{T}{\Delta_0^{\max}} \right)^k \Gamma\left[m + k + 1, \frac{\Delta^*(T)}{T} \right], \qquad (12)$$

where $\Gamma(a, x)$ is the incomplete gamma function [49]. Since $\Delta^*(T)/T = \pi/\gamma$, the apparently dominant exponential dependence of $\langle P(T)\rangle_s$ on $(-1/T)$ resulting from the second argument of $\Gamma(a, x)$ disappears altogether, whatever the particular value of k_0 .

One more important result of this formula is that in the framework of the proposed model the measured properties of the superconducting components $\langle P(T)\rangle_s$ at low temperatures are insensible to the particular profile of the distribution function $f_0(\Delta_0)$ at large Δ_0 . Hence, for $T << \Delta_0^{\max}$ a few first terms of the series (12) constitute a good approximation. Restricting ourselves to the leading k_0-term we obtain

$$\langle P(T)\rangle_s = AB_{k_0} (\Delta_0^{\max})^{\,l+m}\, \Gamma\left(m + k_0 + 1, \frac{\pi}{\gamma} \right) \left(\frac{T}{\Delta_0^{\max}} \right)^M , \qquad (13)$$

with $M = k_0 + l + m + 1$. The corrections to this expression are of the next order in $T/\Delta_0^{\max}$. This justifies the validity of substituting the upper limit of the integral in Eq. (11) by infinity. At the same time, this makes eligible the evaluating of the $\langle P(T)\rangle_s$ contribution in Eq. (7) using the low-T

asymptotics $P_s^{\text{asympt}}(\Delta_0, T)$ in the integrand instead of the exact value $P_s(\Delta, T)$. Indeed, the T-dependences of various parameters in the BCS theory are induced by the T-behavior of the gap Δ [23], e.g., the exponential multiplier in Eq. (1) originates from that in the low-T asymptotics of $\Delta(T)$. Since $\Delta(T \to T_c) \propto (T_c - T)^{1/2}$ in the BCS theory, the considered parameters have at $T \to T_c$ the power-like asymptotics as well. Thus, the use of exact functional dependences $P_s(\Delta, T)$ results not in exponential but power-law dependences $\langle P(T) \rangle_s$.

One should note that in each specific experiment only a certain lowest temperature T_{lim} is accessible, so that, according to the Eq. (9), only gap values down to $\Delta_0^{\text{lim}} = \pi T_{\text{lim}}/\gamma$ are relevant. Hence, the restriction imposed above on the distribution function $f_0(\Delta_0)$ to extend down to $\Delta_0 = 0$ may be weakened. Namely, $f_0(\Delta_0)$ should be nonzero for $\Delta_0 > \Delta_0^{\text{lim}}$. In the case when the domain ensemble possesses the minimal value $\Delta_0^{\min}$ and the lowest accessible $T_{\text{lim}} < \gamma\Delta_0^{\min}/\pi$, the value $\Delta_0^{\min}$ will manifest itself as the exponential factor $\exp(-\Delta_0^{\min}/T)$ in $\langle P(T) \rangle_s$ (cf. Ref. [23]).

Returning to Eq. (13), we see that the actual distribution function reveals itself in the final result only through the expansion parameters B_{k_0} and k_0. The most popular distribution functions [50], namely, normal Gaussian

$$f_G(\Delta_0) = \frac{1}{\Delta_0^{\max}} \left(\frac{2}{\pi \sigma^2} \right)^{1/2} \left[2\Phi\left(\frac{1}{\sigma} \right) + 1 \right]^{-1} \times$$

$$\times \exp\left[-\frac{1}{2} \left(\frac{\Delta_0 - \Delta_0^{\max}}{\sigma \Delta_0^{\max}} \right)^2 \right], \qquad (14)$$

exponential $f_E(\Delta_0) = (\alpha/\Delta_0^{\max}) \exp(-\alpha\Delta_0/\Delta_0^{\max})$, and uniform $f_U(\Delta_0)$ ones, where α and σ are dimensionless parameters and $\Phi(x)$ is the error function [49], have finite values at $\Delta_0 = 0$, so the leading term (13) in the series has the $k_0 = 0$ order of smallness. At the same time, different distribution functions have different values of coefficient B_0 . Now it is impossible to make a choice in favor of one of them. The analysis of the heat capacity measurements for various oxides [51,52] makes us to suggest that the function $f_0(\Delta_0)$ is mainly concentrated in a narrow interval near $\Delta_0 = 0$, which is beneficial for our hypothesis.

Applying the general approach to the properties concerned, taking their actual low-T expressions [23], and comparing them with Eq. (1) we come to the following dependences for the chosen case $k_0 = 0$: for the specific heat

$$\langle C_s(T) \rangle \approx B_0 \sqrt{2\pi}\, N(0)\, \Delta_0^{\max}\, \Gamma\left(\frac{7}{2}, \frac{\pi}{\gamma} \right) \left(\frac{T}{\Delta_0^{\max}} \right)^2, \qquad (15)$$

where $N(0)$ is the electron density of states at the Fermi level; for the penetration depth

$$\langle \delta\lambda_L(T) \rangle \approx B_0 \sqrt{\pi/2}\, \Gamma\left(\frac{3}{2}, \frac{\pi}{\gamma} \right) \frac{T}{\Delta_0^{\max}}\,; \qquad (16)$$

for the thermal conductivity

$$\langle \kappa_e(T) \rangle \approx B_0 \frac{2 n_e \tau_{\text{tr}}}{m_e}\, \Delta_0^{\max}\, \Gamma\left(3, \frac{\pi}{\gamma} \right) \left(\frac{T}{\Delta_0^{\max}} \right)^2, \qquad (17)$$

where n_e is the normal state electron density, τ_{tr} is the transport collision time, and m_e is the electron mass; for the ratio $\delta\alpha = \alpha_s/\alpha_n$ of the ultrasonic attenuation coefficients in superconducting, α_s , and normal, α_n , states

$$\langle \delta\alpha(T) \rangle \approx 2B_0\, \Gamma\left(1, \frac{\pi}{\gamma} \right) \frac{T}{\Delta_0^{\max}}\,. \qquad (18)$$

These results correlate well with experimental data (see Sec. 4). For other possible distribution functions with $k_0 > 0$ the preceding results will remain power-law, although with larger M. In 2D-superconductors, such as cuprates, the value $k_0 = 0$ corresponds to linear objects, i.e., lines or edges of normal regions, consisting of «nodes» ($\Delta_0 = 0$) in the real space. Point-like zeros would lead to $k_0 = 1$, so that the relevant power-law exponents would increase by one.

From the methodological point of view it is of interest to indicate an analogy between our approach dealing with the Δ-distribution in the real space and the Abrikosov's introduction [53] of the distribution function for the order parameter Δ anisotropic in the momentum space, with the anisotropy being quite general and including both d-wave and extended s-wave symmetries.

4. Discussion

The inhomogeneities leading to the spread of Δ magnitudes over the sample may be of different nature. As the possible driven forces of these structural and/or electronic domains in high-T_c oxides one should mention (i) composition irregularities, especially the inherent disorder in oxygen vacancy positions, observed, e.g., for $BaPb_{1-x}Bi_xO_3$ [51,54], $La_{2-x}[Sr(Ba)]_xCuO_{4-y}$ [54,55] and $YBa_2Cu_3O_{7-x}$ [26,54,56] and (ii) the phase separation of the electronic origin with impurity atoms frozen because of

the kinetic barriers [57]. In oxides both mechanisms apparently act together [45,56,58].

Vacancy disorder comprise point-like defects. As was indicated in Sec. 3, an attempt to allow for such irregularities was made in Ref. [48], where it was shown that only for an anomalously great dispersion W of the site order parameters Δ_i it is possible to obtain the gapless-like behavior of the quasiparticle density of states. Considering the condition $W >> \Delta_i^{\max}$ for the maximal quantity $\Delta_i^{\max}$ very improbable, the cited authors, in order to explain the experimental data, argued that $YBa_2Cu_3O_{7-x}$ is a d-wave object.

The inhomogeneities attributed above to the second group are of typical sizes exceeding the coherence length, the latter being extremely small in cuprates. The experimental evidence exists of the minority phase domains in $La_{2-x}Sr_xCuO_4$ being as large as several hundred Angströms in size [58]. For $YBa_2Cu_3O_{7-x}$ the x-ray and neutron diffraction measurements supplemented by the lattice gas Monte Carlo simulations revealed not only tetragonal and ortho-I phases with the long range order but also a rich variety of structural phases with anisotropic correlation lengths of mesoscopic size [56]. The domain finiteness preserved even after annealing, and kinetic barriers turned out to be large enough to secure the logarithmic time ordering. The crystal field neutron spectra of $ErBa_2Cu_3O_{7-x}$ [59] and the Raman spectra of $YBa_2Cu_3O_{7-x}$ [60] which reflect the local region properties also revealed oxygen structure domains, indicating the phase separation and the percolation character of conductivity and superconductivity. It was pointed out in Ref. [61] as well that the percolative network of intermediate size hole-induced polarons (clusters) may lead to the difference between local and global (crystal) symmetry.

In contrast to $YBa_2Cu_3O_{7-x}$, the electron-doped superconducting oxide $Nd_{2-x}Ce_xO_{4-y}$ is a random alloy [62]. Such an atomic-scale disorder may prevent the formation of structural domains, thus making our hypothesis of averaging inapplicable in this case. On the other hand, the in-plane coherence length in $Nd_{2-x}Ce_xO_{4-y}$ is $\xi_{ab} \approx 70-80$ Å, which exceeds substantially $\xi_{ab} \approx$ 10-15 Å in $YBa_2Cu_3O_{7-x}$ [62]. Also, making allowance that superconductivity of the former substance exists in the narrow range $0.14 < x < 0.15$ and $y \le 0.01$ [63], it is natural to conclude that the spread of Δ assumed in our model is not large enough to validate the averaging procedure. Thus, $Nd_{2-x}Ce_xO_{4-y}$ should manifest its intrinsic exponential low-T asymptotics which is indeed the case [62].

At the same time, tunnel spectra of $YBa_2Cu_3O_{7-x}$ show a large spread of Δ magnitudes [64] which is favorable for our interpretation. The growth with x of structural domains with different nonoptimal (for a nominal stoichiometry) Δ's and the attended widening of the distribution function $f(\Delta)$ may explain the increase of the numerical factor in the observed linear-T term of $\delta\lambda_L(T)$ for $YBa_2Cu_3O_{7-x}$ [65]. Another important source of the Δ scatter is the CDW emergence in superconducting oxides [2,15,16,26,51]. All factors listed above, taken together or separately, may be responsible for the transition from the exponential to the power-law behavior of the quantities under consideration.

To summarize the comparison with experiment, we may state that our theory accurately describes the respective power-law exponents. At the same time, at the quantitative level it is at least not worse than the d-wave picture of the low-T asymptotics. To show this, let us compare our results for $\langle\delta\lambda_L\rangle$ with those of the d-wave approach. The choice of this quantity was made because it serves as a sensitive probe to test different pairing mechanisms. According to Ref. [4], $\delta\lambda_L \approx (T \ln 2)/\Delta_0$, which is consistent with the experiments for $YBa_2Cu_3O_{7-x}$ [29], and corresponds to the value $B_0 \approx 1.96$ in Eq. (16). This value of B_0 is reproduced for $f_E(\Delta_0)$ and $\alpha \approx 0.69$, whereas $B_0 \approx 0.29$ for $f_G(\Delta)$ if $\sigma = 1$, and $B_0 \equiv 1$ for $f_U(\Delta)$. These values of α and σ seem quite realistic. The observations of different exponents for $\delta\lambda_L(T)$ in various samples [1,29–33] may reflect their dissimilar nonhomogeneous structures, leading to a change-over from one $f_0(\Delta_0)$ to another with different k_0's.

Note that there is also another approach [3], valid both for s- and d-order parameter symmetry, which is based on the proximity effect in the S–N layer structures of cuprates and fits the experimental data on $\lambda_L(T)$. A possibility of the transformation of the dependence (1) into the power-law one with $M \le 1$ due to the proximity effect was demonstrated in Ref. [66] for Nb/Al bilayer films.

Of course, the theory outlined above can be applied not only to cuprate oxides but to other materials as well. But the main requirement for them to exhibit power-law asymptotics remains the same, namely, the mesoscopic nonhomogeneity characteristic size ought to exceed the coherence length. Otherwise, the exponential behavior for $T << T_c$ would survive and the critical temperatures would be slightly renormalized in full accordance with the Anderson theorem [23].

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