Calculation of the photocount probability distribution of the onemode stochastic radiation

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In frameworks of the Mandel model, the method of probability calculation of n-photon registration by the quantum photocounter of low-intensive one-mode electromagnetic noise is developed. The accuracy of method is provided by the smallness of the registration time value in comparison with the relaxation time.

В рамках модели Манделя, разработан метод вычисления с гарантируемой вероятности n-фотонной регистрации квантовым счётчиком низкоинтенсивного шумового одномодового электромагнитного излучения. Точность метода обеспечивается малостью величины времени наблюдения по сравнению со временем затухания.

For the description of the registration process of low-intensive electromagnetic radiation, it is necessary to take into account its quantum nature. In this case, the field is registered by separate portions consisting of photon groups. As low its intensity and as more the resolution of quantum counter as the registration of separate photons is more probable. Their number registered during the time T is random with the necessity. This randomness may be as the sequence of two reasons. First is the quantum nature of the registered electromagnetic radiation. Second is connected with the fact that the field may have the stochastic (noisy) constituent besides the regular (signal) one. Since the photon number is random, the problem of the registration process description consists of the prediction of the probability distribution of the registered photons. If the registered electromagnetic field contains the stochastic constituent then its quantum state is statistically *mixed* and it is described by the density matrix. In this case the concrete structure of the density matrix is closely connected with the choice of the adequate mathematical model of the electromagnetic noise. Thus, in the general case, the desired probability distribution of the registered photon number is "quantum"and it is defined by the diagonal of the density matrix in the filling number representation.

Simplification of the problem of the registration process description arises at sufficiently large typical frequencies of the registered electromagnetic field. In this case, one may use the quasi-classical approximation as it is shown in [1]. Therefore, the probability of registered photon number may be calculated on the basis of the ordinary classical probability distribution

$$P_n \equiv \Pr\{\tilde{n} = n\} = \frac{1}{n!}\langle \tilde{J}^n \exp[-\tilde{J}]\rangle \qquad (1)$$

representing the so-called *composite Poisson distribution*. It is referred to the Mandel distribution in quantum optics [2]. Here $\tilde{J}$ is the random value representing the energy of electromagnetic field absorbed during the registration time T. Brackets denote the averaging on its probability distribution.

Let us consider the model of the quantum photocounter [1] of the onemode electromagnetic radiation being completely noisy. In this case, the appropriate model of the electromagnetic noise is the complex Ornstein-Uhlenbeck process as it is proposed in the photodetection theory. The random value $\tilde{J}$ is represented by the formula [1]

$$\tilde{J} \equiv J[\tilde{\zeta}] = \int_0^T \left|\tilde{\zeta}(s)\right|^2 ds$$

where $\tilde{\zeta}(s) = \tilde{\xi}(s) + i\tilde{\eta}(s)$, $s \in \mathbb{R}$ are trajectories of the complex process connected with real Ornstein-Uhlenbeck's processes $\{\xi(t); t \in \mathbb{R}\}, \{\eta(t); t \in \mathbb{R}\}$ being stochastically equivalent and independent. From the physical point of view, they correspond accordingly to electric and magnetic constituents of the noise electromagnetic field. Ornstein-Uhlenbeck's processes are markovian and gaussian and they are completely characterized by these properties and their stationary condition. This class of processes is parametrizated by two numbers $\nu > 0$, $\sigma > 0$. Each Ornstein-Uhlenbeck process is completely determined by the following formula of the conditional probability density $w(x_0, t_0|x, t)$ of the transition from the point $x_0 \in \mathbb{R}$ at $t_0 \in \mathbb{R}$ to the point $x \in \mathbb{R}$ at $t \in \mathbb{R}$ which depends on parameters ν, σ,

$$w(x_0, t_0|x, t) = \left(\frac{\nu}{\pi\sigma\left(1 - e^{-2\nu|t-t_0|}\right)}\right)^{1/2} \exp\left(-\frac{\nu\left[x - x_0 e^{-\nu|t-t_0|}\right]^2}{\sigma\left(1 - e^{-2\nu|t-t_0|}\right)}\right). \quad (2)$$

In this case the one-point distribution density $w(x)$, $x \in \mathbb{R}$ of the process is determined by the formula

$$w(x) = \lim_{t_0 \to -\infty} w(x_0, t_0|x, t) = \left(\frac{\nu}{\pi\sigma}\right)^{1/2} \exp\left(-\frac{\nu x^2}{\sigma}\right). \quad (3)$$

The characteristic function $Q(-i\lambda)$, $\lambda \in \mathbb{R}$ of the random variable $J[\tilde{\xi}]$ of the process $\tilde{\xi}$ is given by the known Ziegert formula [4],

$$Q_{\tilde{\xi}}(\lambda) = \langle \exp(-\lambda J[\tilde{\xi}]) \rangle = \left(\frac{4r\nu\exp(\nu T)}{(r+\nu)^2\exp(rT) - (r-\nu)^2\exp(-rT)}\right)^{1/2} \quad (4)$$

where $r = \sqrt{\nu^2 + 2\lambda\sigma}$. Since processes $\{\tilde{\xi}(t)\}, \{\tilde{\eta}(t)\}$ are independent and equivalent, the generating function of the random variable $J[\tilde{\zeta}]$ is found on the basis of equalities

$$Q(\lambda) = Q_{\tilde{\xi}}(\lambda) Q_{\tilde{\eta}}(\lambda) = Q^2_{\tilde{\xi}}(\lambda).$$

The Mandel distribution P_n determined by the formula (1) and by the probability distribution of the random variable $\tilde{J}$ induced by the probability distribution of the process $\tilde{\zeta} = \{\zeta(t); t \in \mathbb{R}\}$ are very complex. It is easy to obtain the asymptotic formula of the probability distribution P_n at $T \to 0$. It has the form $P_n / P_n^{(0)} \to 1$ where $P_n^{(0)}$ is the following Poisson distribution [1]

$$P_n^{(0)} = \frac{1}{n!}\left(\frac{\sigma T}{\nu}\right)^n \exp\left(-\frac{\sigma T}{\nu}\right).$$

For the correction of this formula, it is necessary to find such a method of the calculation of probabilities P_n which permits to determine them with the guaranteed accuracy. In this work we propose such a calculation procedure. It permits to find probabilities P_n with any accuracy on the basis of formulas (1) - (4) at the sufficiently small T value. The approximation obtaining is based on the representation of the Mandel distribution in the form of the series expansion

$$P_n = \frac{1}{n!}\sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \langle \tilde{J}^{n+l} \rangle \quad (5)$$

where the averaging is fulfilled on the probability distribution of the random process $\{\zeta(t); t \in \mathbb{R}\}$. Since each moment $\langle \tilde{J}^n \rangle$, $n \in \mathbb{N}$ of the random value $\tilde{J}$ is proportional to T^n at $T \to 0$, then one may expect that just such an expansion is appropriate for the solution of the above-mentioned problem. The problem consists mathematically of the calculation of these moments and of the remainder estimation connected with the finite part of the series (5).

1. Calculation of random value $\tilde{\mathbf{J}}$ moments

Let us decompose the generation function $Q(\lambda) = \langle e^{-\lambda \tilde{J}} \rangle$ of the random value

$$\tilde{J} = \int_0^T \left| \tilde{\zeta}(t) \right|^2 dt$$

into the power series on λ. Introducing the "dimensionless"parameter νT instead of T, we designate hereafter by the same letter T if it will not cause a misunderstanding. We represent the function $Q(\lambda)$ by the formula

$$Q(\lambda) = e^T G^{-1}(\lambda)\,.$$

It is valid the following expansion for the function $G(\lambda)$ into the series on powers of the variable $q = (1+z)^{1/2}$, $z = 2\lambda\sigma/\nu^2$,

$$G(\lambda) = \sum_{n=0}^{\infty} \frac{T^{2n}}{(2n)!} q^{2n} \left[1 + \frac{T}{(2n+1)} \left(1 + \frac{\lambda\sigma}{\nu^2} \right) \right]\,.$$

After some transformations it may be represented in the form of the expansion on λ powers,

$$G(\lambda) = \sum_{m=0}^{\infty} \frac{z^m}{m!} \left[u_m + \frac{m}{2} v_{m-1} + v_m \right]$$

where

$$u_m = \sum_{n=m}^{\infty} \frac{T^{2n}}{(2n)!} \frac{n!}{(n-m)!}\,, \quad m \in \mathbb{N}_+\,, \tag{6}$$

$$v_m = \sum_{n=m}^{\infty} \frac{T^{2n+1}}{(2n+1)!} \frac{n!}{(n-m)!}\,, \quad m \in \mathbb{N}_+\,. \tag{7}$$

For the convenience, we introduce the functional $\mathsf{F}[z|A]$ depending on sequences $A = \{a_n; n \in \mathbb{N}_+\}$. This functional represents the ordinary power series

$$\mathsf{F}[z|A] = \sum_{n=0}^{\infty} z^n a_n$$

with sequence A components as its coefficients. It possesses the obvious property consisting of the fact that it is multiplicative relative to the production of two sequences A and B according to the convolution rule

$$(A \circ B)_n = \sum_{m=0}^{n} a_m b_{n-m}\,,$$

i.e. $\mathsf{F}[z|A \circ B] = \mathsf{F}[z|A]\mathsf{F}[z|B]$. Besides, it is additive by obvious way relative to the addition of sequences A and B. We consider the sequence $W = \langle w_m; m \in \mathbb{N} \rangle$ with components $w_0 = 0$ and

$$w_m = \frac{1}{m!} e^{-T} (u_m + m v_{m-1}/2 + v_m)\,, \quad m \in \mathbb{N}\,. \tag{8}$$

Since $e^{-T}(u_0 + v_0) = 1$, the expansion (7) of the function $G(\lambda)$ into the series on λ powers may be represented in the following form

$$G(\lambda) = e^T \left[1 + \sum_{m=1}^{\infty} z^m w_m \right] = e^T \left(1 - \mathsf{F}[z| - W] \right) , \quad z = \frac{2\lambda\sigma}{\nu^2} . \tag{9}$$

On the basis of multiplicative and additive properties above-formulated, the following expansion of the function $G^{-1}(\lambda)$ on λ powers is obtained

$$G^{-1}(\lambda) = e^{-T} \left(1 - \mathsf{F}[z| - W] \right)^{-1} = e^{-T} \mathsf{F} \left[z|X \right] ,$$

where $X = \langle x_n; n \in \mathbb{N}_+ \rangle$, $x_0 = 1$,

$$x_n = \sum_{l=1}^{n} (-1)^l \left(W_\circ^l \right)_n , \quad n \in \mathbb{N} \tag{10}$$

and $W_\circ^l$, $l = 1, 2, ...$ are l-tipled convolutions of the sequence W. Substituting this expansion and the expression of the function $Q(\lambda)$, we find

$$Q(\lambda) = e^T G^{-1}(\lambda) = \mathsf{F}[z|X] . \tag{11}$$

According to the moment definition of the random value $\tilde{J}$, on the basis of the production function $Q(\lambda)$, we obtain from Eq.(11) the formula

$$M_n = (-1)^n \left(\frac{\partial^n Q(\lambda)}{\partial \lambda^n} \right)_{\lambda=0} = (-1)^n n! \left(\frac{2\sigma}{\nu^2} \right)^n x_n . \tag{12}$$

2. Moment estimations

For the solution of the accuracy estimation of successive approximations of the Mandel probability distribution, it is necessary to find some *a priori* estimations of moments M_n of the random variable $\tilde{J}$. They are based on the following inequality

$$\frac{1}{(2n)!} \leq \frac{en}{2^{2n}(n!)^2} . \tag{13}$$

The proof of its validity is based on the identity

$$(2n)! = 2^n n!(2n-1)!! == 2^{2n}(n!)^2 \exp \left(\sum_{l=1}^{n} \ln(1 - (2l)^{-1}) \right) . \tag{14}$$

The logarithm of the righthand side is estimated from below,

$$\ln(1 - (2l)^{-1}) \geq -l^{-1} , \quad l = 1, 2, ..., n . \tag{15}$$

We estimate the sum $\sum_{l=1}^{n} 1/l$ from above considering it as the integral sum of the function α^{-1},

$$\sum_{l=1}^{n} \frac{1}{l} < 1 + \int_1^n \frac{d\alpha}{\alpha} = 1 + \ln n . \tag{16}$$

Applying inequalities (15) and (16) for the below estimation of the righthand side of Eq.(14), we find Eq.(13).

On the basis of the inequality (13), we make sure that the following above estimation for coefficients u_m, $m = 1, 2, ...$ is valid,

$$u_m \le e\frac{(T/2)^{2m}}{(m-1)!}\,\mathrm{I}_0(T)\,, \quad \mathrm{I}_0(T) = \sum_{n=0}^{\infty}\frac{(T/2)^{2n}}{(n!)^2}\,, \tag{17}$$

where $\mathrm{I}_0(T)$ is the zero order Bessel function of the imaginary variable. Indeed, from the formula (6), using the inequality (13), we have for any $m \in \mathbb{N}$ that it is valid

$$u_m \le e\sum_{n=m}^{\infty}\frac{(T/2)^{2n}}{(n-1)!(n-m)!} \le e\frac{(T/2)^{2m}}{(m-1)!}\sum_{n=m}^{\infty}\frac{(T/2)^{2(n-m)}}{[(n-m)!]^2}\,.$$

Eq.(17) follows from here.

Notice that $v_m \le [T/(2m+1)]u_m$, we obtain the above estimation for coefficients v_m, $m = 1, 2, ...$,

$$v_m \le \frac{e}{m!}\,(T/2)^{2m+1}\,\mathrm{I}_0(T)\,.$$

It is convenient to change the Bessel function $\mathrm{I}_0(T)$ in obtained estimations by more simple one not making worse these estimations seriously. Namely, on the basis of the famous integral representation of the function $\mathrm{I}_0(T)$, we have $\mathrm{I}_0(T) < e^T$ at $T > 0$.

Since the Bessel function

$$\mathrm{I}_1(T) = \sum_{m=0}^{\infty}\frac{(T/2)^{2m-1}}{m!(m-1)!}$$

of the first order justifies to the inequality $\mathrm{I}_1(T) \le \mathrm{I}_0(T)$ at $T > 0$, further it may change also by e^T at the above estimation obtaining.

Summarizing, we may state that the estimation

$$w_m < \frac{e}{m!(m-1)!}\,(T/2)^{2m}\,\varphi(m)\,, \quad \varphi(m) = \left[1 + \frac{T}{2m} + \frac{m}{T}\right]\,. \tag{18}$$

is valid for coefficients w_m.

Now we obtain the *a priori* estimations for moments of the Mandel distribution. We proof the following statement.

For moments M_n, it is valid the inequality

$$M_n = n!\left(\frac{2\sigma}{\nu^2}\right)^n |x_n| < \frac{e\psi(T)}{e\psi(T) - T}\,n!\left(\frac{e\sigma T\psi(T)}{2\nu^2}\right)^n$$

where the function $\psi(T)$ is defined by the formula

$$\psi(T) \equiv T(e^2 - 1)(1 + T/2) + e^2\,.$$

For the proof of it at $m > 1$, we rewrite the formula (10) by the following way

$$x_m = -w_m + \sum_{k=1}^{m-1}(-1)^{k-1}\sum_{\substack{l_1,\dots,l_k>0\\ l_1+\dots+l_k<m}} w_{m-l_1-\dots-l_k}\prod_{j=1}^{k} w_{l_j}\,. \tag{19}$$

Then, using the estimation (18), we have

$$|x_m| \le |w_m| + \sum_{k=1}^{m-1}\sum_{\substack{l_1,\dots,l_k>0\\ l_1+\dots+l_k<m}} |w_{m-l_1-\dots-l_k}|\prod_{j=1}^{k}|w_{l_j}| \le$$

$$\le (T/2)^{2m}\sum_{k=1}^{m}\sum_{\substack{l_1,\dots,l_k>0\\ l_1+\dots+l_k=m}}\prod_{j=1}^{k}\frac{e}{l_j!(l_j-1)!}\varphi(l_j) \le$$

$$\leq (T/2)^{2m} \sum_{k=1}^{m} e^k \left(\sum_{l=1}^{\infty} \frac{\varphi(l)}{l!(l-1)!} \right)^k .$$

The sum in brackets in the last expression, according to definitions of Bessel function and the function φ is equal to $I_1(2) + T\left(I_0(2) - 1\right)/2 + I_0(2)/T$. Therefore, using estimations of the Bessel function on the basis of the exponent, we obtain that this expression does not exceed $\psi(T)/T$. Then, taking in mind of the obvious inequality $e\psi(T) > 1$, we find at $m > 1$

$$|x_m| < (T/2)^{2m} \sum_{k=1}^{m} e^k \left(\psi(T)/T\right)^k = \frac{e\psi(T)}{e\psi(T) - T} \left(eT\psi(T)/4\right)^m . \tag{20}$$

From here, using the formula (12), the validity of the formulated statement follows.

3. Approximations of the Mandel distribution and estimations of their accuracy

We consider the Mandel distribution representing it in the form of the expansion on moments

$$P_n = \frac{1}{n!} \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} M_{l+n}$$

or, on the basis of the representation (9),

$$P_n = \frac{(-1)^n}{n!} \sum_{l=0}^{\infty} \frac{(n+l)!}{l!} \left(\frac{2\sigma}{\nu^2}\right)^{n+l} x_{l+n} . \tag{21}$$

We determine the sequence of approximations $P_n^{(N)}$, $N = 1, 2, ...$ of the probability distribution which present it with the accuracy up to the Nth power of the parameter $(\sigma T/\nu^2)$ at $n \leq N$,

$$P_n^{(N)} = \frac{(-1)^n}{n!} \sum_{l=0}^{N-n} \frac{(n+l)!}{l!} \left(\frac{2\sigma}{\nu^2}\right)^{n+l} x_{l+n} . \tag{22}$$

Further, we estimate the deviation of the $(N-1)$th approximation from the exact distribution (1). On the basis Eq.(21), using the estimation (20), we have

$$|P_n - P_n^{(N-1)}| \leq \frac{1}{n!} \frac{e\psi}{e\psi - T} \sum_{l=N}^{\infty} \frac{l!}{(l-n)!} \left(\frac{2\sigma}{\nu^2}\right)^l \left(\frac{eT\psi}{4}\right)^l . \tag{23}$$

Let us consider now the series

$$R_N(\zeta, n) = \sum_{l=N}^{\infty} \frac{l!}{(l-n)!} \zeta^l$$

It permits the strict summation,

$$R_N(\zeta, n) = \zeta^n \frac{d^n}{d\zeta^n} \frac{\zeta^N}{1-\zeta} = \frac{\zeta^N}{1-\zeta} \sum_{l=0}^{n} n! \binom{N}{l} \left(\frac{\zeta}{1-\zeta}\right)^{n-l} \leq$$

$$= n! \frac{(2\zeta)^N}{1-\zeta} \sum_{l=0}^{n} \left(\frac{\zeta}{1-\zeta}\right)^{n-l} \leq n! \frac{(2\zeta)^N}{1-2\zeta} .$$

Applying the obtained estimation to the inequality (23), we find that the Nth approximation $P_n^{(N)}$ of the Mandel distribution has the guaranteed accuracy defined by the inequality

$$|P_n - P_n^{(N)}| \leq \left(\frac{e\psi}{e\psi - T}\right) \frac{(2\zeta)^N}{1-2\zeta}, \quad \zeta = \frac{e\sigma T\psi}{2\nu^2} < \frac{1}{2} .$$

Now, it is necessary to find the effective algorithm of the building of consecutive approximations $P_n^{(N)}$. For this, it is necessary to point out the method of the component calculation of the sequence X. Since these components, according to the definition (23), are built by means of convolutions of the sequence W, it is sufficient to learn to calculate successively all components of this sequence. The key point of such a problem is the following assertion which is proved easily by the induction on the parameter m.

Namely, for each $m = 0, 1, 2, ...$, the formula

$$\left(\frac{\partial^n}{\partial \alpha^m} \exp(\pm \alpha^{1/2} T) \right)_{\alpha=1} = m! e^{\pm T} R_m^{\pm}(T) \tag{24}$$

takes place where polynomials $R_m^{\pm}(T)$ of the m degree on the variable T are determined by the recurrent relation

$$R_{m+1}^{\pm}(T) = \pm \frac{T}{2(m+1)} \sum_{l=0}^{m} (-1)^l \frac{(2l)!}{2^{2l}(l!)^2} R_{m-l}^{\pm}(T)\,, \quad R_0^{\pm}(T) = 1\,.$$

Now, on the basis of the formula (24) and the definition of coefficients u_m, we find their in the form

$$u_m = \left(\frac{\partial^m}{\partial \alpha^m} \mathrm{ch}(\alpha^{1/2} T) \right)_{\alpha=1} = \frac{1}{2} m! \left(e^T R_m^+(T) + e^{-T} R_m^-(T) \right)\,. \tag{25}$$

Analogously to the formula (24), the following identity

$$\left(\frac{\partial^m}{\partial \alpha^m} \left[\alpha^{-1/2} \exp\left(\pm \alpha^{1/2} T \right) \right] \right)_{\alpha=1} = \frac{2}{T} (m+1)! e^{\pm T} R_{m+1}^{\pm}(T).$$

is proved. Based on it, from the definition of coefficients v_m, we have

$$v_m = \left(\frac{\partial^m}{\partial \alpha^m} \frac{\mathrm{sh}(\alpha^{1/2} T)}{\alpha^{1/2}} \right)_{\alpha=1} = \frac{1}{T} (m+1)! \left(e^T R_{m+1}^+(T) + e^{-T} R_{m+1}^-(T) \right)\,. \tag{26}$$

Then, on the basis of formulas (25), (26) and the formula (8), components of the sequence W are represented by the following way

$$w_m = \frac{1}{2} R_m^+(T) \left(1 + \frac{m}{T} \right) + \frac{m+1}{T} R_{m+1}^+(T) +$$
$$+ e^{-2T} \left[\frac{1}{2} R_m^-(T) \left(1 + \frac{m}{T} \right) + \frac{m+1}{T} R_{m+1}^-(T) \right]\,, \quad m \in \mathbb{N}\,. \tag{27}$$

Formulas (10), (27) and (22) permit to calculate successively all approximations $P_n^{(N)}$ of probabilities P_n. In this section we give the example of such a calculation in 3d approximation on the parameter T.

At first, the explicit form of polynomials $R_m^{\pm}(T)$, $m = 0, 1, 2, 3, 4$ is found,

$$R_0^{\pm}(T) = 1\,, \quad R_1^{\pm}(T) = \pm T/2\,, \quad R_2^{\pm}(T) = (T/2^3)(T \mp 1),$$
$$R_3^{\pm}(T) = \pm (2^4 \cdot 3)^{-1} T \left(T^2 \mp 3T + 3 \right)\,,$$
$$R_4^{\pm}(T) = (2^7 \cdot 3)^{-1} T \left(T^3 \mp 6T^2 + 15T \mp 15 \right)\,.$$

After that, we calculate components w_m, $m = 1, 2, 3$, on their basis,

$$w_1 = T/2\,, \quad w_2 = 2^{-4} \left(2T^2 - 2T + 1 - e^{-2T} \right)\,, \tag{28}$$

$$w_3 = (3 \cdot 2^5)^{-1} \left(2T^3 - 6T^2 + 9T - 6 + 3(T+2) e^{-2T} \right)\,. \tag{29}$$

At last, components x_m, $m = 1, 2, 3$ are found by the formula (19),

$$x_0 = 1\,, \quad x_1 = -w_1\,, \quad x_2 = -w_2 + w_1^2\,, \quad x_3 = -w_3 + 2 w_1 w_2 - w_1^3\,,$$

or, after the substitution (28), (29),

$$x_1 = -T/2, \quad x_2 = 2^{-4}\left(2T^2 + 2T - 1 + e^{-2T}\right) ,$$

$$x_3 = -(3 \cdot 2^5)^{-1}\left[2T^3 + 6T^2 + 3T - 6 + 3(3T+2)e^{-2T}\right] .$$

Explicit expressions of components x_m, $m = 0,1,2,3$ permit to write the approximated expressions of n-photon registration probabilities for $n = 0,1,2,3$ up to the third order ($N = 0,1,2,3$). All probabilities $P_n^{(N)}$ of more high values $n > 3$ are equal to zero in this approximation.

In third approximation on T at $N = 3$, we obtain the approximate formulas

$$P_0^{(3)} = 1 - \frac{\sigma T}{\nu^2} + \left(\frac{\sigma}{2\nu^2}\right)^2 (2T^2 + 2T - 1 + e^{-2T}) -$$
$$- \frac{2}{3}\left(\frac{\sigma}{2\nu^2}\right)^3 \left[2T^3 + 6T^2 + 3T - 6 + 3(3T+2)e^{-2T}\right] ,$$

$$P_1^{(3)} = \frac{\sigma T}{\nu^2} - 2\left(\frac{\sigma}{2\nu^2}\right)^2 \left(2T^2 + 2T - 1 + e^{-2T}\right) +$$
$$+ 2\left(\frac{\sigma}{2\nu^2}\right)^3 \left[2T^3 + 6T^2 + 3T - 6 + 3(3T+2)e^{-2T}\right] ,$$

$$P_2^{(3)} = \left(\frac{\sigma}{2\nu^2}\right)^2 \left(2T^2 + 2T - 1 + e^{-2T}\right) -$$
$$- 2\left(\frac{\sigma}{2\nu^2}\right)^3 \left[2T^3 + 6T^2 + 3T - 6 + 3(3T+2)e^{-2T}\right] ,$$

$$P_3^{(3)} = \frac{2}{3}\left(\frac{\sigma}{2\nu^2}\right)^3 \left[2T^3 + 6T^2 + 3T - 6 + 3(3T+2)e^{-2T}\right] .$$

In this work, we have solved principally the old problem of quantum optics [4], i.e. we have built the calculation algorithm for n-photon registration probabilities of the noisy electromagnetic irradiation in frameworks of the proposed problem setting. It is necessary, however, to point out on a lack of our solution. We devote attention on the fact that obtained formulas permit to calculate the registration probability for each concrete photon number n. But they does not permit to calculate the registration probability in the case when this number is indefinite, i.e. it is not known strictly and, therefore, it is the parameter in the problem. It is connected with the fact that their analytical presentation is very tedious when its order increasing. In particular, in connection with this fact, it is impossible to calculate distribution moments with guaranteed accuracy on the basis of obtained formulas.

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Розрахунок розподілу імовірностей для числа фотонів одномодового стохастичного випромінювання

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На підставі моделі Манделя побудовано метод обчислення імовірності n-фотоної реєстрації квантовим приймачем низько інтенсивного одномодового електромагнітного шуму. Точність методу зумовлена малим значенням часу реєстрації у порівнянні з часом релаксації.