

Equilibrium shapes of director in nematic with cylindrical particles

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Equilibrium shapes of the director arising around a cylindrical particle in nematic liquid crystal are studied in their dependence upon the value and type of anchoring between the director and the lateral surface of the particle. Perpendicular and parallel orientations of the particle axis with respect to the non-perturbed nematic director are considered. In the case of strong anchoring between the director and the lateral surface of the particle the equilibrium distance between the axis of the particle and a disclination line is obtained as a function of value anchoring energy. In the case of weak anchoring for nematic in external electric field the analytical expression for the distribution of the director field around the cylindrical particle is obtained.

Исследованы равновесные конфигурации директора, возникающие вокруг цилиндрической частицы в нематическом жидком кристалле, в зависимости от величины и типа сцепления директора с ее боковой поверхностью. Рассмотрены перпендикулярная и параллельная ориентации оси частицы относительно невозмущенного директора нематика. В случае сильного сцепления директора с боковой поверхностью частицы найдено равновесное расстояние дисклинационной линии от оси частицы как функцию величины энергии сцепления. При слабом сцеплении, когда рассматриваемый нематик находится во внешнем электрическом поле, получено аналитическое выражение распределения поля директора вокруг цилиндрической частицы.

In the recent years, so-called liquid-crystal systems, in which the interaction of liquid crystal with the surface of the solids is important, are widely used. Among them the most well-known are the systems, in which the liquid crystal, in the form of drops, is situated in the polymeric matrix [1, 2]. But it is not long ago that the research of another kind of heterogeneous liquid-crystal systems is started. Such systems are the mixture of nematic liquid crystal (NLC) with small ($\sim$ 100 Å in diameter) silicon particles (aerosil), which are named filled nematics [3-7]. The aerosil particles, interacting with the liquid crystal, form heterogeneities of detector field structure, which cause a certain kind of incident light diffusion and essentially surpress the detector heat fluctuations. Electric field being applied, such a sample becomes transparent at a lower voltage, than in case of aerosil absence.

So, studying of equilibrium shapes of director field around the particles of various forms, placed in a nematic matrix, rises much interest from the theoretical as well as from the experimental points of view. The rate of detector field heterogeneousness is obvious to sufficiently depend upon the anchoring of the liquid crystal director with the particles surface. In the research [8] the distribution of NLC field around a spherical particle at weak adherence of director with its surface was found out. If the adherence if strong enough, then there can appear disclination-like structure defects of director field around the spherical particle [9-11].

In this paper the structure of director field, appearing around a cylindric particle in nematic liquid crystal in external electric field is studied. Let us note that the problem in case of no electric field and weak director anchoring with the particle surface was considered in the paper [12].

We consider two types of nematic director anchoring with the lateral surface of cylindric particle: a) homeotropic — the direction of simple director orientation vector $\mathbf{e}$ is perpendicular to the lateral surface of the particle; b) circular — the vector $\mathbf{e}$ direction is perpendicular to the cylindric axis of the particle and tangent to its lateral surface. Besides, the cases of two different orientations of cylindric axis $\mathbf{l}$ of the particle as to the perturbed director $\mathbf{n}_o$ of nematic — perpendicular ($\mathbf{l} \perp \mathbf{n}_o$) and parallel ($\mathbf{l} \parallel \mathbf{n}_o$) ones, which correspond to the two equilibrium shapes of director field, are considered.

Let us consider homogenous nematic liquid crystal, in which a cylindric particle of radius R is placed. The length of the particle L we consider a little larger than R, so that we shall neglect the fringe effects at the cylinder bases. Let such an NLC containing the particle be situated in external homogenous electrostatic field with field vector $\mathbf{E}$, directed along the non-perturbed nematic director $\mathbf{n}_o$.

In rectangular Cartesian coordinates system with z axis directed along the cylindrical axis of the particle, we shall characterize the director field $\mathbf{n}$ in the volume of NLC by two angles: polar $\Omega(\mathbf{r})$ — the angle between the director $\mathbf{n}$ and the positive direction of z axis and azimuthal $\Phi(\mathbf{r})$ — the angle between the vector projection $\mathbf{n}$ onto plane xy and the positive direction of x axis. Then in the coordinate system chosen

$$\mathbf{n} = \sin\Omega(\mathbf{r})\cos\Phi(\mathbf{r}) \cdot \mathbf{e}_x + \sin\Omega(\mathbf{r})\sin\Phi(\mathbf{r}) \cdot \mathbf{e}_y + \cos\Omega(\mathbf{r}) \cdot \mathbf{e}_z\,. \tag{1}$$

The full free energy of NLC, containing the particle, in the electric field can be written as follows

$$F = \int\limits_V f_{el}\,dV + \int\limits_V f_E\,dV + \int\limits_S f_S\,dS\,,$$

$$f_{el} = \frac{1}{2}\left\{K_1\,(\operatorname{div}\mathbf{n})^2 + K_2\,(\mathbf{n}\cdot\operatorname{rot}\mathbf{n})^2 + K_3\,[\mathbf{n}\times\operatorname{rot}\mathbf{n}]^2\right\}\,,$$

$$f_E = -\frac{\varepsilon_a}{8\pi}\left(\mathbf{n}\cdot\mathbf{E}\right)^2, \tag{2}$$

$$f_S = -\frac{W_\theta}{2}\cos^2(\gamma-\gamma_o) - \frac{W_\varphi}{2}\cos^2(\delta-\delta_o)\,, \quad W_\theta > 0, W_\varphi > 0\,.$$

Here f_{el} is the cubic density of Frank elastic energy; f_E is the cubic density of anisotropic contribution into the energy of NLC interaction with the electric field; f_S is the density of nematic free surface energy, written in the form of Rapini potential [13]; $\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp > 0$ is the anisotropy of static permittivity; W_θ, W_φ are the polar and azimuthal energies of director anchoring with the lateral surface of the cylindric particle respectively; γ_o is the polar angle between the simple director orientation vector $\mathbf{e}$ on the lateral surface of the particle and the external normal direction to this surface; δ_o is the azimuthal angle in the plane of tangent lateral surface of the particle between the vector $\mathbf{e}$ projection onto this plane and the direction of a certain polar axis; γ, δ are the polar and azimuthal angles, setting the director $\mathbf{n}$ direction on the lateral surface of the particle respectively.

Let us introduce a polar coordinate system (r,φ) in the plane xy and, minimizing the free energy (2) on the angles Ω and Φ, we shall get the following equations

$$\frac{\partial(f_{el}+f_E)}{\partial\chi} - \frac{1}{r}\sum_\alpha \partial_\alpha\, r\frac{\partial f_{el}}{\partial\,\partial_\alpha\chi} = 0\,, \tag{3}$$

and, respectively, the boundary conditions for them

$$\left(\frac{\partial f_S}{\partial\chi} - \frac{\partial f_{el}}{\partial\,\partial_r\chi}\right)_{r=R} = 0\,, \tag{4}$$

where $\chi = \Omega, \Phi\,;\ \partial_\alpha \equiv \dfrac{\partial}{\partial\,\alpha}\,;\ \alpha = r, \varphi, z\,.$

Let the cylindrical axis $\mathbf{l}$ of the particle be perpendicular to the non-perturbed director of $\mathbf{n}_o$ nematic. Let us direct axis x of the Cartesian coordinate system along the non-perturbed director. Due to the homogenousness of the system in the direction of axis z for homeotropic as well as for circular type of director anchoring to the cylindrical particle lateral surface the polar angle is $\Omega(\mathbf{r}) = \pi/2$, and the director field distortion around the particle, according to (1), will be flat. In this case, there are no torsional deformations ($\mathbf{n} \cdot \mathrm{rot}\,\mathbf{n} = 0$), and in the approximation $K_1 = K_3 = K$ we obtain the following equation to determine the azimythal angle $\Phi(r,\varphi)$ of the director from the correlations (3) and (4), taking into account (1)

$$\Delta\Phi - \frac{\varepsilon_a E^2}{8\pi K}\sin 2\Phi = 0\,, \tag{5}$$

and the boundary condition for it

$$\left[2K\,\frac{\partial\Phi}{\partial r} - W_\theta \sin 2(\Phi - \Phi_o)\right]_{r=R} = 0\,, \tag{6}$$

where Φ_o is the azimuthal angle of simple director orientation vector $\mathbf{e}$ on the lateral surface of the cylindrical particle. It is obvious that in case of absolutely rigid anchoring of the director with the particle lateral surface, the boundary condition (6) takes the form $\Phi(R,\varphi) = \Phi_o$.

Let the anchoring of the director with the particle lateral surface be homeotropic, then the azimuthal angle Φ_o of vector $\mathbf{e}$ is

$$\Phi_o = \begin{cases} \varphi, & \text{if } \; -\dfrac{\pi}{2} < \varphi < \dfrac{\pi}{2}\,, \\[2ex] \varphi - \pi, & \text{if } \; \dfrac{\pi}{2} < \varphi < \dfrac{3\pi}{2}\,. \end{cases} \tag{7}$$

Let us consider the case when the external electric field is absent ($E = 0$), and the polar anchoring energy W_θ of the director with the particle lateral surface is infinite ($W_\theta = \infty$). It is easy to see that (see Fig.1a) in the plane yz there appear particles, symmetrical to the cylindric axis, and two disclination force lines "$-1/2$". From the reasons of symmetry, we can also write

$$\Phi(r,-\varphi) = -\Phi(r,\varphi), \quad \Phi(r,\pi-\varphi) = -\Phi(r,\varphi). \tag{8}$$

Then the solution $\Phi(r,\varphi)$ of Laplace's equation (see(5)) is to be found in the first quarter of complex plane $s = x + iy$ in the exterior of the circle with the radius R (in the region $D = \{r \geqslant R, 0 \leqslant \varphi \leqslant \pi/2\}$). Obviously, on the boundary of the region under consideration, the sought function $\Phi(r,\varphi)$ must meet the following conditions:

$$\Phi_S = \begin{cases} 0, & \text{if } \; R < r < +\infty,\; \varphi = 0\,, \\[1ex] \varphi, & \text{if } \; r = R,\; 0 \leqslant \varphi \leqslant \dfrac{\pi}{2}\,, \\[1ex] \dfrac{\pi}{2}\,, & \text{if } \; R < r < a,\; \varphi = \dfrac{\pi}{2}\,, \\[1ex] 0, & \text{if } \; a < r < +\infty,\; \varphi = \dfrac{\pi}{2}\,, \end{cases} \tag{9}$$

where a is the distance from the disclination line to the cylindrical particle axis ($a > R$).

Using the function

$$w(s) = \frac{1}{2}\left(\left(\frac{s}{R}\right)^2 + \left(\frac{R}{s}\right)^2\right) \tag{10}$$

let us conformally map the considered region D of the complex plane s to the upper half-plane of the complex plane $w = u + iv$. As a result of such mapping, the function $\Phi(r,\varphi) \equiv \Phi(s)$ grades into the function $\tilde{\Phi}(w)$, and the boundary points of the region under consideration D ($r = R, \infty; \varphi = 0, \pi/2$) grade into the points of real axis u in the complex plane w. The boundary conditions (9) for the function $\Phi(r,\varphi)$ after the transformation (10) give the values of function $\tilde{\Phi}(w)$ in the real axis. So, for the harmonic

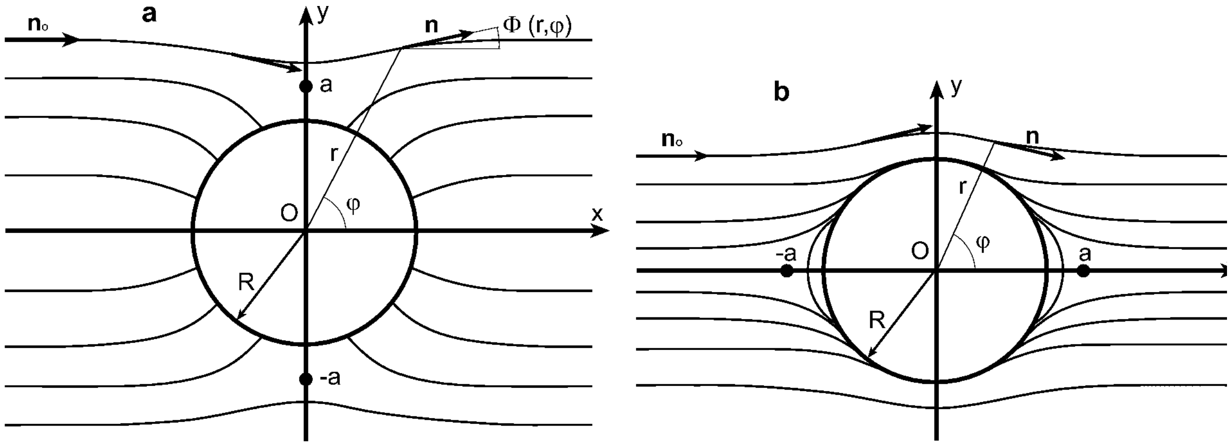

Fig.1. The structure of director field around the cylindrical particle at rigid homeotropic (a) and circular (b) director anchoring with its lateral surface in case $\mathbf{l} \perp \mathbf{n}_o$.

function $\tilde{\Phi}(w)$ we have Dirichlet's problem for the upper half-plane $\mathrm{Im}\, w \geqslant 0$, the solution of which can be written with Poisson integral [14, 15]: $\tilde{\Phi}(w) = \dfrac{1}{\pi} \displaystyle\int\limits_{-\infty}^{+\infty} \dfrac{v\, \tilde{\Phi}(t)\, dt}{(t-u)^2 + v^2}$.

Substituting the values $\tilde{\Phi}(w)$ on the real axis and integrating, we have

$$\tilde{\Phi}(r,\varphi) = \tilde{\Phi}(w) = \frac{1}{2}\,\mathrm{arctg}\,\frac{b+u}{v} + \frac{1}{2\pi}\int\limits_0^{\pi} \mathrm{arctg}\,\frac{\cos p - u}{v}\, dp\,, \tag{11}$$

where

$$u = \frac{1}{2}\left(\left(\frac{r}{R}\right)^2 + \left(\frac{R}{r}\right)^2\right)\cos 2\varphi\,,\quad v = \frac{1}{2}\left(\left(\frac{r}{R}\right)^2 - \left(\frac{R}{r}\right)^2\right)\sin 2\varphi\,,\quad b = \frac{1}{2}\left(\left(\frac{a}{R}\right)^2 + \left(\frac{R}{a}\right)^2\right).$$

The value of equilibrium distance a from the disclination line to the cylindrical particle axis should correspond to the minimum of free energy F of NLC containing the particle. Minimizing the dependence $F(a)$, we have $a^* = 1,27R$.

In case of strong (but not absolutely rigid) anchoring of the director with the particle lateral surface $\left(\varepsilon_\theta = \dfrac{W_\theta R}{K} \gg 1\right)$ at $E = 0$ the solution of Laplace's equation for the function $\Phi(r,\varphi)$ (see (5)) should be sought in the form

$$\Phi(r,\varphi) = \tilde{\Phi}(r,\varphi) + \phi(r,\varphi)\,, \tag{12}$$

where $|\phi(r,\varphi)| \ll |\tilde{\Phi}(r,\varphi)|$, and the function $\tilde{\Phi}(r,\varphi)$ is defined by the correlation (11). It is obvious that the function $\phi(r,\varphi)$ also meets the Laplace's equation $\Delta\phi = 0$ and the boundary condition, following from (6) and written in the linear approximation by $\phi(r,\varphi)$

$$\left(\varepsilon_\theta \phi - R\,\frac{\partial \phi}{\partial r}\right)_{r=R} = R\,\left.\frac{\partial \tilde{\Phi}}{\partial r}\right|_{r=R}. \tag{13}$$

The solution of Laplace's equation for function $\phi(r,\varphi)$, taking into consideration the symmetry correlations (8) and its limitation at $r \to \infty$, has the following form

$$\phi(r,\varphi) = \sum_{k=1}^{\infty} B_{2k}\left(\frac{R}{r}\right)^{2k} \sin(2k\varphi)\,, \tag{14}$$

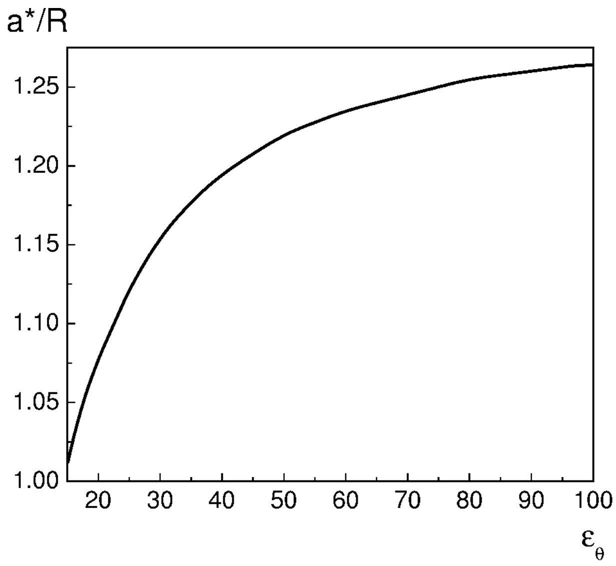

Fig.2. The dependence of equilibrium distance a^*/R of disclination line to the cylindrical particle axis on the value of polar energy ε_θ of homeotropic (circular) director anchoring with its lateral surface at $\mathbf{l} \perp \mathbf{n}_o$.

where the coefficients B_{2k} found, respectively, from the boundary condition (13), take the form

$$B_{2k} = \frac{4R}{\pi(\varepsilon_\theta + 2k)} \int_0^{\pi/2} \left.\frac{\partial \tilde{\Phi}}{\partial r}\right|_{r=R} \sin(2k\varphi)\, d\varphi \,. \tag{15}$$

Minimizing the free energy (2) of NLC, containing the particle, for (12) solution, taking into account the correlations (14) and (15), we obtain the dependence of equilibrium distance a^* , shown in the Fig. 2 (the disclination line from the cylindrical axis of the particle) on the value of anchoring ε_θ polar energy for the director with the particle lateral surface. Let us note that at the anchoring polar energy value $\varepsilon_\theta < 10$ disclination lines are not formed, and a weakly deformed structure, considered in [12], appears.

Let the director anchoring with the particle lateral surface is weak ($\varepsilon_\theta \ll 1$), and the NLC under consideration is situated in external homogenous electrostatic field with field vector $\mathbf{E} = (E, 0, 0)$.

Solving the equation (5) in the linear approximation by $\Phi(r,\varphi)$, taking into account symmetry correlations (8) and the limitation of the solution at the infinity, we obtain

$$\Phi(r,\varphi) = \sum_{m=1}^{\infty} D_{2m}\, K_{2m}\Big(\frac{r}{l_E}\Big)\, \sin(2m\varphi)\,, \tag{16}$$

where $K_{2m}(x)$ are modified Bessel functions [16], $l_E = \sqrt{\dfrac{4\pi K}{\varepsilon_a E^2}}$ is so called electric coherent length, and D_{2m} are some coefficients.

Substituting the solution (16) obtained into the boundary condition (6), in which (7) is taken into account, in the linear approximation by $\Phi(r,\varphi)$, we obtain the following equation

$$\sum_{m=1}^{\infty} D_{2m}\, K'_{2m}(x_R) \sin(2m\varphi) - \frac{\varepsilon_\theta}{x_R} \sum_{m=1}^{\infty} D_{2m}\, K_{2m}(x_R) \sin(2m\varphi) \cos 2\varphi = -\frac{\varepsilon_\theta}{2x_R} \sin 2\varphi \,,$$

where $x_R = R/l_E$, the stroke in the modified Bessel functions means argument derivative.

Due to linear independence of functions $\sin(2m\varphi)$ on the interval $[0, 2\pi]$, we have an infinite system of algebraic equations for defining the coefficients D_{2m}:

$$\begin{aligned} D_2\, K_2'(x_R) - \frac{\varepsilon_\theta}{2x_R}\, D_4\, K_4(x_R) &= -\frac{\varepsilon_\theta}{2x_R}\,, \\ D_{2m}\, K_{2m}'(x_R) - \frac{\varepsilon_\theta}{2x_R}\Big(D_{2m+2}\, K_{2m+2}(x_R) + D_{2m-2}\, K_{2m-2}(x_R)\Big) &= 0\,, \quad m \geqslant 2. \end{aligned} \tag{17}$$

It follows from the equations (17), that at large m such correlations hold: $\dfrac{D_{2m}}{D_{2m-2}} = O\Big(\dfrac{1}{m^3}\Big)$, and, so, the series (16) converges by m absolutely and uniformly. Then in the expression (16) in the sum by m we can limit ourselves with a finite terms number calculation with any desired accuracy, and this leads to a finite number of equations (17) and the problem becomes technically soluble . Then, limiting ourselves, for simplicity sake, by calculation the terms with $m \leqslant 3$, we have

$$D_2 = -\frac{\varepsilon_\theta}{2x_R}\,\frac{1}{K_2'(x_R)}\,, \qquad D_4 = -\frac{\varepsilon_\theta^2}{4x_R^2}\,\frac{K_2(x_R)}{K_2'(x_R)K_4'(x_R)}\,, \qquad D_6 = -\frac{\varepsilon_\theta^3}{8x_R^3}\,\frac{K_2(x_R)K_4(x_R)}{K_2'(x_R)K_4'(x_R)K_6'(x_R)}\,.$$

The substitution of the found expressions for the coefficients D_2, D_4 and D_6 into (16) gives the distribution of director NLC field around the cylindrical particle in the electric field.

Let us consider the circular type of NLC director anchoring with the cylindrical particle lateral surface. In case of absolutely rigid anchoring $W_\theta = \infty$ at external electric field absence, in the plane xz two symmetric disclination lines appear parallel to the particle cylindrical axis in the distance a from it (see Fig.1b). Taking into account the symmetry correlations (8) we seek a harmonic function $\Phi(r, \varphi)$ in the region $D = \{r \geqslant R, 0 \leqslant \varphi \leqslant \pi/2\}$ of the complex plane $s = x + iy$, to meet the following conditions on the boundary of the region under consideration:

$$\Phi_S = \begin{cases} 0, & \text{if} \quad a < r < +\infty,\ \varphi = 0\,, \\ -\dfrac{\pi}{2}, & \text{if} \quad R < r < a,\ \varphi = 0\,, \\ \varphi - \dfrac{\pi}{2}, & \text{if} \quad r = R,\ 0 \leqslant \varphi \leqslant \dfrac{\pi}{2}\,, \\ 0, & \text{if} \quad R < r < +\infty,\ \varphi = \dfrac{\pi}{2}\,. \end{cases} \tag{18}$$

Having conformally mapped the region D by the transformation (10) on the upper half-plane of the complex plane w and solving in it the Dirichlet problem for function $\tilde{\Phi}(w)$, we obtain

$$\tilde{\Phi}(r, \varphi) = -\frac{1}{2}\,\mathrm{arctg}\,\frac{b-u}{v} + \frac{1}{2\pi}\int_0^\pi \mathrm{arctg}\,\frac{\cos p - u}{v}\, dp\,. \tag{19}$$

Minimizing the free NLC energy (2) for (19) solution, we obtain the same result, as in case of homeotropic director anchoring with the particle lateral surface, the equilibrium distance $a^* = 1,27R$ from the disclination line to the particle cylindrical axis.

In case of strong anchoring $\varepsilon_\theta \gg 1$ of the director with the particle lateral surface at $E = 0$, the solution (12) with the consideration of (14) preserves its form with the difference that in the expression (15) for the coefficients B_{2k} the function $\tilde{\Phi}(r, \varphi)$ is defined now by the expression (19). The dependence $a^*(\varepsilon_\theta)$ of the disclination line equilibrium distance to the particle cylindrical axis on the value of anchoring polar energy ε_θ of the director with the particle lateral surface coincides with the presented in Fig.2.

If the director anchoring with the particle lateral surface is weak, and the NLC is situated in the external electric field, then, solving the problem similarly to the case of homeotropic anchoring, we obtain the expression (16) for the azimuthal director angle, where

$$D_2 = \frac{\varepsilon_\theta}{2x_R}\,\frac{1}{K_2'(x_R)}\,, \quad D_4 = -\frac{\varepsilon_\theta^2}{4x_R^2}\,\frac{K_2(x_R)}{K_2'(x_R)K_4'(x_R)}\,, \quad D_6 = \frac{\varepsilon_\theta^3}{8x_R^3}\,\frac{K_2(x_R)K_4(x_R)}{K_2'(x_R)K_4'(x_R)K_6'(x_R)}\,.$$

In the extreme case of electric field absence ($E = 0$), the distributions (16) of nematic director field around the cylindrical particle, obtained in 3.1c and 3.2c for both types of director anchoring with its lateral surface, are in accord with the results of the paper [12].

Let the cylindrical axis $\mathbf{l}$ of the particle is parallel to the non-perturbed director $\mathbf{n}_o = (0,0,1)$, and the NLC in question is situated in external homogenous electrostatic field $\mathbf{E} = (0,0,E)$. Regarding the director anchoring with the particle lateral surface as homeotropic, let us represent the NLC director in the form

$$\mathbf{n} = \cos\varphi \sin\Omega(r) \cdot \mathbf{e}_x + \sin\varphi \sin\Omega(r) \cdot \mathbf{e}_y + \cos\Omega(r) \cdot \mathbf{e}_z \,. \tag{20}$$

In the approximation $K_1 = K_3 = K$ the free energy (2) of NLC containing the particle is

$$F = \pi K L \int\limits_R^\infty \left[r^2 \left(\frac{d\Omega}{dr} \right)^2 + r \frac{d\Omega}{dr} \sin 2\Omega + \sin^2\Omega - \frac{\varepsilon_a E^2}{4\pi K} r^2 \cos^2\Omega \right] \frac{dr}{r} - \pi K L \varepsilon_\theta \sin^2\Omega \big|_{r=R} \,.$$

From the correlations (3) and (4), we obtain the following equation for the polar angle $\Omega(r)$

$$r^2 \frac{d^2\Omega}{dr^2} + r \frac{d\Omega}{dr} - \frac{1}{2} \sin 2\Omega - \frac{\varepsilon_a E^2}{8\pi K} r^2 \sin 2\Omega = 0 \tag{21}$$

and the boundary condition to it

$$\left[R \frac{d\Omega}{dr} + \frac{1}{2}(1+\varepsilon_\theta) \sin 2\Omega \right]_{r=R} = 0 \,. \tag{22}$$

At weak anchoring $\varepsilon_\theta \ll 1$ of the director with the particle lateral surface, we obtain, solving the equation (21) with the boundary condition (22) in the linear $\Omega(r)$ approximation:

$$\Omega(r) = \frac{K_1(r/l_E)}{K_1(x_R)} \sqrt{\frac{3}{2} \left(1 + \frac{x_R}{1+\varepsilon_\theta} \cdot \frac{K_1'(x_R)}{K_1(x_R)} \right)} \,.$$

After the substitution of the found value of the polar angle $\Omega(r)$ into (20), we obtain the director distribution in the volume of NLC containing the cylindrical particle.

In case of director circular anchoring with particle lateral surface, the director in the nematic volume director has the form

$$\mathbf{n} = -\sin\varphi \sin\Omega(r) \cdot \mathbf{e}_x + \cos\varphi \sin\Omega(r) \cdot \mathbf{e}_y + \cos\Omega(r) \cdot \mathbf{e}_z \,. \tag{23}$$

In the approximation $K = K_1 = K_3 \neq K_2$ the free NLC energy is

$$F = \frac{\pi K L}{1+\nu} \int\limits_R^\infty \left[r^2 \left(\frac{d\Omega}{dr} \right)^2 + r \frac{d\Omega}{dr} \sin 2\Omega + \sin^2\Omega + \nu \sin^4\Omega - \frac{\varepsilon_a E^2}{4\pi K}(1+\nu) r^2 \cos^2\Omega \right] \frac{dr}{r} -$$

$$-\pi K L \varepsilon_\varphi \sin^2\Omega \big|_{r=R} \,,$$

where $\varepsilon_\varphi = \dfrac{W_\varphi R}{K}$, $\nu = \dfrac{K - K_2}{K_2}$.

As a result of free energy F minimization by the angle Ω, we obtain the following equation

$$r^2 \frac{d^2\Omega}{dr^2} + r \frac{d\Omega}{dr} - \left(\frac{1}{2} + \nu \sin^2\Omega \right) \sin 2\Omega - \frac{\varepsilon_a E^2}{8\pi K} (1+\nu) r^2 \sin 2\Omega = 0 \tag{24}$$

and the boundary condition for it

$$\left[R \frac{d\Omega}{dr} + \frac{1}{2} \left(1 + \varepsilon_\varphi (1+\nu) \right) \sin 2\Omega \right]_{r=R} = 0. \tag{25}$$

Let us consider weak anchoring $\varepsilon_\varphi \ll 1$ of the director with the particle lateral surface. Solving the equation (24) with the boundary condition (25) in the linear approximation by $\Omega(r)$, we have

$$\Omega(r) = \frac{K_1(r\sqrt{1+\nu}/l_E)}{K_1(x_R\sqrt{1+\nu})}\sqrt{\frac{3}{2}\left(1+\frac{x_R\sqrt{1+\nu}}{1+\varepsilon_\varphi(1+\nu)}\cdot\frac{K_1'(x_R\sqrt{1+\nu})}{K_1(x_R\sqrt{1+\nu})}\right)}.$$

Substituting the value of $\Omega(r)$ obtained into (23), we find the director field around the particle. Let us note that in case of electric field absence, the director distributions (20) and (23), obtained coincide with the results of [12].

References

1. R.Eidenschink, W.H. de Jeu, *Electron. Lett.*, **27**, 1195 (1991).
2. P.S.Drzaic, Liquid Crystal Dispersions, World Scientific, Singapore (1995).
3. J.W.Doane, N.A.Var, B.-G.Wu, S.Zumer, *Appl. Phys. Lett.*, **48**, 269 (1989).
4. M.Kreuser, T.Tschudi, W.H. de Jeu, R.Eidenschink, *Appl. Phys. Lett.*, **62**, 1712 (1993).
5. G.Ya.Guba, Yu.A.Reznikov, N.Yu.Lopukhovich et al., *Mol. Cryst. Liq. Cryst.*, **251**, 303 (1994).
6. O.V. Yaroshchuk, G. Ya. Guba, N. Yu. Lopukhovich et al., *Ukr. Fiz. Zh.*, **39**, 990 (1995).
7. M.Kreuser and R.Eidenschink, in: Liquid Crystals in Complex Geometries, ed. by G.P.Crawford and S.Zumer, Taylor & Francis, London (1995).
8. I.P.Pinkevich, V.Yu.Reshetnyak, *Mol. Cryst. Liq. Cryst.*, **321**, 145 (1998).
9. O.V.Kuksenok, S. V. Shiyanovsky, *Ukr. Fiz. Zh.*, **41**, 190 (1996).[in Russian]
10. O.V.Kuksenok, R.W.Ruhwandl, S.V.Shiyanovskii, E.M.Terentjev, *Phys. Rev.*, **E 54**, 5198 (1996).
11. T.C.Lubensky, D.Pettey, N.Currier, H.Stark, *Phys. Rev.*, **E 57**, 610 (1998).
12. S.V.Burylov, Yu.L.Raikher, *Phys. Rev.*, **E 50**, 358 (1994).
13. A.Rapini, M.Papolar, *J.Phys.Collod.*, **30**, 54 (1969).
14. M. A. Lavrentyev, B. V. Shabat, The Methods of Complex Variable Functions Theory, Nauka, Moscow (1973).[in Russian]
15. Yu. V. Sidorov, M. V. Fedoryuk, M. I. Shabunin, Lection on Complex Variable Functions Theory, Nauka, Moscow (1989).[in Russian]
16. M. Abramovitz, I. Stigan, Reference-Book on Special Functions with Formulae, Graphs and Mathematic Tables, Nauka, Moscow (1979).[in Russian]

Рівноважні конфігурації директора у нематику з циліндричними включеннями

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Досліджено рівноважні конфігурації директора, які виникають навколо циліндричної частинки у нематичному рідкому кристалі залежно від величини й типу зчеплення директора з її бічною поверхнею. Розглянуто перпендикулярну та паралельну орієнтації осі частинки відносно незбуреного директора нематика. У разі сильного зчеплення директора з бічною поверхнею частинки знайдено рівноважну відстань дисклінаційної лінії від осі частинки як функцію величини енергії зчеплення. Для слабкого зчеплення, коли досліджуваний нематик перебуває у зовнішньому електричному полі, отримано аналітичний вираз розподілу поля директора навколо циліндричної частинки.