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The structural relaxation and anisotropy of the spring-back in metals

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The spring-back phenomenon in rolled copper and aluminum sheets has been studied in experiment. The time dependences of strain at bending of specimens under permanent load have been described by power functions with a fractional index. The anisotropy of the spring-back phenomenon has been revealed. A mathematical model is proposed which describes the spring-back basing on fractal concepts of the relaxation processes under external permanent stress.

Экспериментально исследовано явление упругого последействия в прокатанных листах меди и алюминия. Зависимости деформации от времени при изгибе образцов под действием постоянной нагрузки описаны степенными функциями с дробным индексом. Установлена анизотропия явления упругого последействия. Предложена математическая модель, которая описывает явление упругого последействия на основе фрактальных представлений о релаксационных процессах под действием постоянного внешнего напряжения.

The purpose of this work is to study the influence of the straining ratio on the spring-back in aluminum and copper and to establish the fractal nature of the spring-back phenomenon.

The physical existence region of Hooke law is extremely limited in real solids [1]. Inelasticity, or a transition into an elasto-plastic deformation range, is observed at stresses well below the yield strength of a material. Therefore, besides of elastic processes, the effects of the material inelastic behavior are manifest themselves within limits of elastic strain. The spring-back is an example of such inelasticity phenomena. The spring-back phenomena are of a great practical importance: the direct spring-back plays an important role in elements like springs, membranes, and other elastic elements being under long-term load [2]. However, despite the importance of the spring-back phenomena, their theory is not quite complete. The mathematical model of creep in heterogeneous media based on the fractal concepts of non-equilibrium processes, was proposed for the first time in [3] where a relation between the fractal dimension and Andrade creep parameters [4, 5] was also established.

The rectangular specimens were cut out of copper and aluminum sheets were used as the research objects. The specimen axis were positioned through each 150 between the rolling direction (RD) and transversal direction (TD) after rolling to 60 and 80 % deformation in thickness. The investigations of the direct spring-back were carried out on a special testing machine [6]. The relative strain ε was calculated from the experimental deflection f of the strip having the thickness h and length l (distance between the rollers) according to the formula [6]:

$$\varepsilon = \frac{6hf}{l^2}. \qquad (1)$$

The measurement error did not exceed 1.5 %. The X-ray diffraction method was used to examine the crystallographic texture [7]. The analysis of texture research results has shown that a texture typical of rolled FCC metals and alloys was formed in sheets which can be described mainly as {110} <112> + {112} <111> [7].

The general strain of a solid body caused by constant external stress consists of two components:

$$\varepsilon(t) = \varepsilon_0 + \varepsilon_1(t). \qquad (2)$$

The experimental time dependences of the strain $\varepsilon(t)$ caused by the action of permanent flexion stresses was subjected to computerized regressive analysis that has shown that power functions of time with a fractional index are the best way to describe the experimental time dependence of strain:

$$\varepsilon(t) = \varepsilon_0 + ct^{\alpha}. \qquad (3)$$

The dependence $\varepsilon = \varepsilon(t)$ and values of c and α in equation (3) are given in Table and Figs. 1, 2. It is seen that the spring-back is anisotropic. The coefficients and power indices in Eq.(3) differ for different directions of the sheets. Influence of deformation also takes place. The coefficients of Eq.(3) as well as shapes of the curves differ for the different straining ratios.

A model of spring-back is proposed in what follows. The plasticity arises up not instantly in the whole solid. Regions which pass to the plastic state because of the solid microscopic level heterogeneity are accumulated gradually. According to the modern views, the micro plastic straining is the result of little and, as a rule, reversible motion of dislocations [8, 9]. If the stress applied to a specimen is constant, a fraction of dislocations (or other defects and their clusters) will be favorably oriented for a sliding (or for a moving). The location of other dislocations (or other defects) will be less favorable. If the stresses applied to a specimen are constant, dislocations (or other defects) will be in such state that a weak stroke only will be needed to onset their motion. Time fluctuations of thermal energy will cause motion of dislocations (or other defects). Hoverer, as dislocations (or other defects) will move, the process of thermally activated motion will gradually become decelerated because of various internal friction mechanisms. Thus, the straining speed will gradually go down.

Schematically, the transition into the micro-plastic state can be described as follows (Fig. 3a, b, c). At first, the single defects of l_0 linear size arise (zero scale level). Then the clusters of defects appear of l_1 linear size (first scale level), etc. Eventually, the area of the defects that penetrates the entire volume of the material will appear in time at the l_n scale level (Fig. 4d). In this case, the defects will create a self-similar set (fractal) of a mass M_f depending on the scale as follows:

$$M_f \sim l^{d_f}, \qquad (4)$$

where d_f is the fractal dimension of the specific set.

Let the deformation be consisting of two parts according to Eq.(1). Thus, ε_0 follows the applied stress σ with infinitesimal delay according to Hooke law, $\varepsilon_0 = s_0\sigma$, in the examined region. The time-dependent part of deformation $\varepsilon_1(t)$ lags behind the applied stress. Let the law of delay be as follows: if $\varepsilon^* = s_\infty \cdot \sigma$ is the maximum achievable strain value at a fixed σ, then at any moment, $\varepsilon_1(t)$ increases toward this value at a speed proportional to $s_\infty \cdot \sigma - \varepsilon_1(t)$. Here s_0 and s_∞ are the material parameters characterizing the specific material. In this case, the differential equation of relaxation can be written, according to [3], as follows:

Table. A spring-back in deformed metals.

Aluminium					Copper			
Angle with RD, °	$\varepsilon_0 \cdot 10^5$	$c \cdot 10^7$	$\alpha \cdot 10^3$	60 % reduction	Angle with RD, °	$\varepsilon_0 \cdot 10^5$	$c \cdot 10^7$	$\alpha \cdot 10^3$
0	124.7	64.7	522		0	214.7	50.7	444
45	125.8	79.6	429		45	219.8	179.6	327
90	127.9	50.4	467		90	203.1	56.7	429
0	216.9	27.6	488	80 % reduction	0	106.9	49.6	318
45	226.0	8.8	943		45	122.5	15.6	669
90	187.1	4.4	910		90	105.5	32.9	453

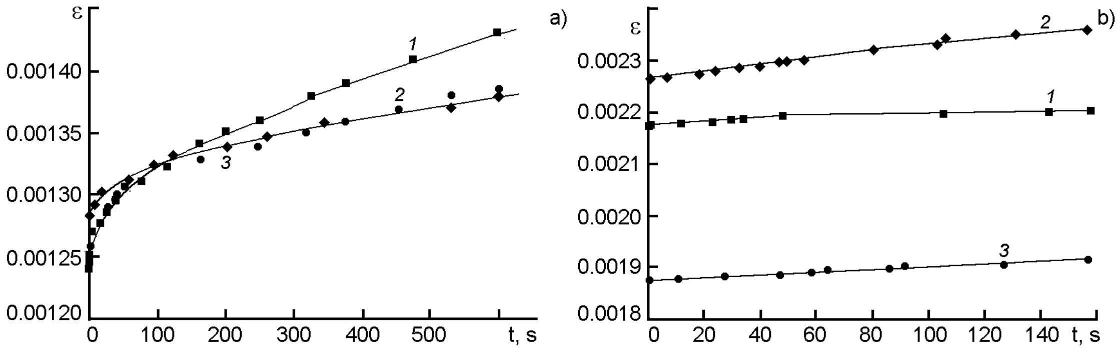

Fig. 1. A spring-back in the rolled aluminium. Reduction: 60 % (*a*), 80 % (*b*). The numerals indicate samples cut (*1*) along the rolling direction (RD), (*2*) at an angle of 45° to the RD, and (*3*) along the transversal direction (TD). The points stand for experimental data, and the lines correspond to relations of the type $\varepsilon(t) = \varepsilon_0 + ct^\alpha$ with coefficients given in the Table.

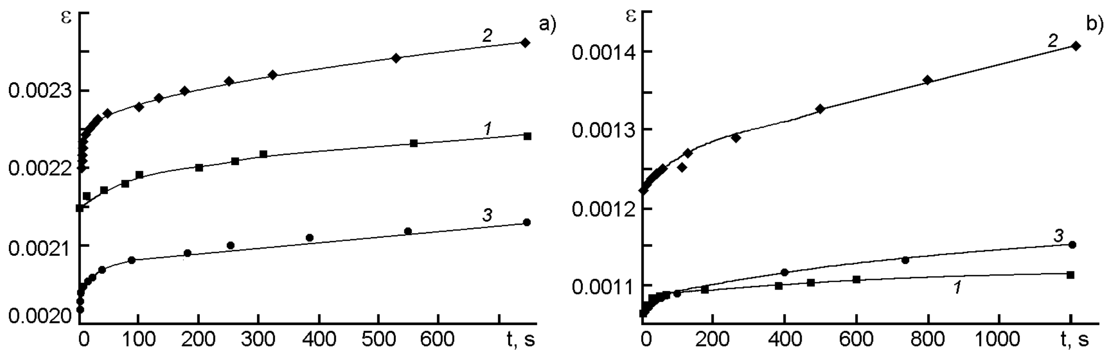

Fig. 2. A spring-back in the rolled copper. Reduction: 60 % (*a*), 80 % (*b*). The legend is identical to Fig.1.

$$\frac{d\varepsilon_1(t)}{dt} = \frac{1}{\tau}\left(s_\infty\sigma - \varepsilon_1(t)\right), \quad (5)$$

where τ is the relaxation time. The solution to this equation, taking (2) into account, is the following:

$$\varepsilon(t) = \varepsilon_0 + \varepsilon_1(t) = \left[s_0 + s_\infty(1 - \exp(-\frac{t}{\tau}))\right]\sigma. \quad (6)$$

Now let us consider the non-equilibrium state of a medium having a fractal nature. Let the non-equilibrium state be defined by set of periods of events, where a next event happens after a previous event has happened. In such a case, some periods are excluded from the continuous series of states. Such a process is produced by the fractal state with a preset fractal dimension d_f. The relaxation equation written using the operator of fractional differentiation $D^\alpha f(x)$ is similar to Eq.(5):

$$[1 + (\tau D)^\alpha]\varepsilon_1(t) = s_\infty\sigma, \quad (7)$$

where [10–12]

$$D^\alpha[f(t)] = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t (t-\tau)^{-\alpha} f(\tau)d\tau, \quad (8)$$

where $\Gamma(x)$ is the gamma function, α is the fractal dimension d_f. To solve the equation (7), we use the Laplace transformation defined as:

$$\bar{f}(p) = \int_0^\infty f(x)\exp(-px)dx, \quad (9)$$

where the original function is

$$f(x) = \frac{1}{2\pi}\int_{\alpha-i\infty}^{\alpha+i\infty} \bar{f}(p)\exp(px)dp, \quad \mathrm{Re}p \ge \alpha, \quad (10)$$

where Rep is the real part of complex value p.

The solution of equation (7) for the original function $\varepsilon_1(t)$ is:

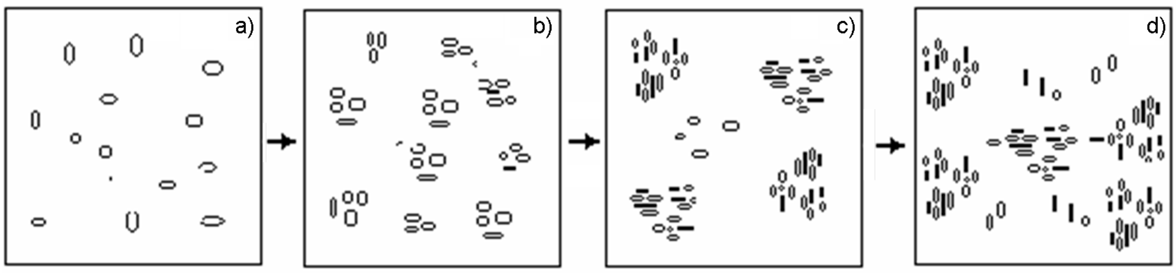

Fig. 3. The path of transition of a heterogeneous material from elastic into plastic state: (*a*), zero scale level; (*b*), first scale level; (*c*), second scale level; (*d*), the l_n-th scale level.

$$\varepsilon_1(t) = s_\infty \sigma \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{t}{\tau}\right)^{\alpha(n+1)}}{\Gamma[\alpha(n+1) + 1]} . \quad (11)$$

Then

$$\varepsilon(t) = \varepsilon_0 + \varepsilon_1(t) = \left\{ s_0 + s_\infty \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{t}{\tau}\right)^{\alpha(n+1)}}{\Gamma[\alpha(n+1)+1]} \right\} \sigma. \quad (12)$$

When $\alpha = 1$, then (12) is equivalent to (6). The spring-back takes place during much shorter time intervals than relaxation periods, $t << |\tau$. Therefore, Eq.(12) can be considered as an expansion in terms of small (t/τ). If only the first components of the series (12) (linear approximation) are taken into account, then the time dependence of strain is as follows:

$$\varepsilon(t) = \varepsilon_0 + a_1 t^\alpha + \ldots, \quad (13)$$

where $\varepsilon_0 = s_0 \cdot \sigma$, $a_1 = s_\infty \sigma (1/\tau)^\alpha / \Gamma(\alpha+1)$.

Thus, the transition from strictly exponential to anomalous dependence occurs at the transition from the continuous distributing ($\alpha = 1$) to the fractal distributing of relaxation periods ($0 < \alpha = d_f < 1$) at the spring-back.

Thus, the anisotropy of spring-back effect in metals has been revealed. The power functions of time with a fractional index are the best way to describe the experimental time dependence of strain at the spring-back. The fractal mathematical model of spring-back in strained metal is proposed. The differential equation of the fractal relaxation process at a spring-back is derived. The solution of the differential equation for relaxation coincides as a first approximation with the experimental time dependences of the strain, if the relaxation periods have the fractal distribution. The fractional index of power dependence characterizes the fractal dimensionality of relaxation process at a spring-back.

References

1. J.F.Nye, Physical Properties of Crystals, Clarendon Press, Oxford (1957).
2. Ya.B.Freedman, Mechanical Properties of Metals, Part 1: Deformation and Destruction, Mashinostroenie, Moscow (1974) [in Russian].
3. V.V.Novikov, *Ukr.Fiz. Zh.*, **49**, 117 (2001).
4. R.W.K.Honeycombe, The Plastic Deformation of Metals, Edward Arnold Publishers, Cambridge (1968).
5. E.N.da C.Andrade, *Phil.Mag.*, **7**, 2003 (1962).
6. A.F.Voyitenko, Yu.D.Skripnik, N.G.Solovyeva, G.N.Nadegdina, *Strength of Materials*, No.11, 83 (1982).
7. M.M.Borodkina, E.N.Spector, X-ray Diffraction Analysis of the Textures in Metals and Alloys, Metallurgia, Moscow, 1982 [in Russian].
8. A.B.Lebedev, *Fiz.Tverd.Tela*, **41**, 1214 (1999).
9. Bingen Yang, Stress, Strain and Structural Dynamics, Elsevier, Amsterdam (2004).
10. S.G.Samko, A.A.Kilbas, O.I.Marychev, Fractional, Integrals and Derivatives. Theory and Applications, Elsevier, Amsterdam (1993).
11. I.Poglubny, Fractional Differential Equations, Wiley & Sons, New York (1999).
12. Applications of Fractional Calculus in Physics, ed. by R.Hilfer, Elsevier Applied Science, London (2000).

Структурна релаксація та анизотропія пружної післядії у металах

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Виконано експериментальні дослідження явища пружної післядії у прокатаних листах міді і алюмінію. Залежності деформації від часу при вигині зразків під дією постійного навантаження описано ступеневими функціями з дробовим індексом. Встановлено анізотропію явища пружної післядії. Запропоновано математичну модель, яка описує явище пружної післядії на основі фрактальних уявлень про релаксаційні процеси під дією постійного зовнішнього напруження.