INTERACTION OF RELATIVISTIC PARTICLES WITH CRYSTALS AND MATTER

ANGULAR DENSITY OF DIFFRACTED TRANSITION RADIATION GENERATED IN A COMPOSITE TARGET BY A RELATIVISTIC ELECTRON BEAM

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The paper considers coherent X-ray radiation from a beam of relativistic electrons crossing a three-layer structure consisted of three layers: amorphous, vacuum, and single crystal. The possibility of constructive interference of transitional radiation waves from different target boundaries contributing to the diffracted transition radiation is shown. It is shown that small but appreciable changes in the thicknesses of the target layers do not lead to change the constructive character of interference of the TR waves from the amorphous layer and the front surface of the monocrystalline layer, i.e. the conditions for constructive interference are sufficiently stable to use them in practice.

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INTRODUCTION

When an electron moving rectilinearly with steady speed crosses a boundary between two media the transition radiation (TR) arises, propagating along the electron velocity. The great interest in the transition radiation of the relativistic electron is due to the possibility of its application as an alternative source of X-ray radiation [2]. When the charged particle crosses a single-crystal plate, the TR photons arising on entrance surface of the plate diffract on a system parallel atomic planes of the crystal forming in a narrow spectral range the diffracted transition radiation (DTR) [3, 4] propagating in the direction of Bragg scattering.

The transition radiation only from the entrance surface gives the contribution to DTR. The transition radiation from the outlet surface of the single-crystal plate doesn't take part in DTR formation. It does not allow using the interference of these radiations to increase the DTR radiation output. In the works [5 - 7] the theory for coherent radiation generated by a relativistic electron crossing various combined structures has been developed. It was shown in these works that in the mentioned structures the spectral-angular density of DTR can be considerably increased. The influence of the electron beam divergence on the DTR spectral-angular density for single-crystal target has been considered in the works [8, 9]. In the present work, the process of coherent radiation in a three-layer combined target consisted of amorphous, vacuum and single-crystal layers by the beam of relativistic electrons has been considered in two-wave approximation of dynamic theory of diffraction.

1. GEOMETRY OF RADIATION PROCESS

Let us consider a beam of relativistic electrons passing through a three-layer structure consisted of amorphous layer, vacuum (or air) and single-crystal layer (Fig. 1) with thicknesses correspondently *c*, *a* and *b*. Let us denote the dielectric susceptibility of amorphous medium as χ_c, the average dielectric susceptibility of the crystal as χ_0 and the coefficient of Fourier expansion of the crystal dielectric susceptibility over the reciprocal lattice vectors $\mathbf{g}$:

$$\chi(\omega,\mathbf{r}) = \sum_{\mathbf{g}} \chi_{\mathbf{g}}(\omega)\exp(i\mathbf{gr}) = \sum_{\mathbf{g}} \left(\chi'_{\mathbf{g}}(\omega) + i\chi''_{\mathbf{g}}(\omega)\right)\exp(i\mathbf{gr}) . \quad (1)$$

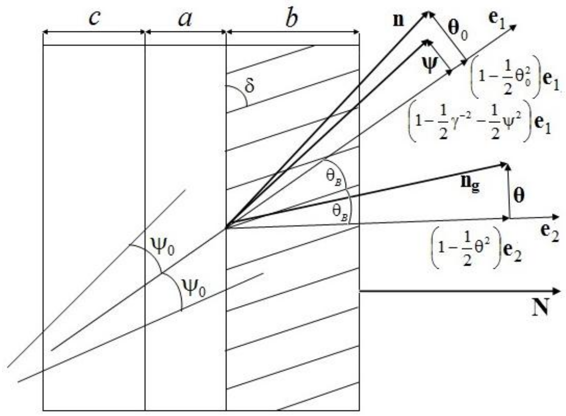

Fig. 1. Radiation process geometry

Let us denote the angular variables $\boldsymbol{\psi}$, $\boldsymbol{\theta}$ and $\boldsymbol{\theta_0}$ as consistent with the definition of velocity vector $\mathbf{V}$ of relativistic electron and unit vectors $\mathbf{n}$ (in the direction of the momentum of photon radiated along the electron velocity direction) and $\mathbf{n_g}$ (in Bragg scattering direction):

$$\mathbf{V} = \left(1 - \frac{1}{2}\gamma^{-2} - \frac{1}{2}\psi^2\right)\mathbf{e}_1 + \boldsymbol{\psi}, \quad \mathbf{e}_1\boldsymbol{\psi} = 0,$$

$$\mathbf{n} = \left(1 - \frac{1}{2}\theta_0^2\right)\mathbf{e}_1 + \boldsymbol{\theta}_0, \; \mathbf{e}_1\boldsymbol{\theta}_0 = 0, \quad \mathbf{e}_1\mathbf{e}_2 = \cos 2\theta_B,$$

$$\mathbf{n_g} = \left(1 - \frac{1}{2}\theta^2\right)\mathbf{e}_2 + \boldsymbol{\theta}, \; \mathbf{e}_2\boldsymbol{\theta} = 0, \quad (2)$$

where $\boldsymbol{\theta}$ is the radiation angle counted from the detector axis $\mathbf{e}_2$, $\boldsymbol{\psi}$ is the angle of deviation of the electron from the electron beam axis $\mathbf{e}_1$, $\boldsymbol{\theta_0}$ is the angle between the propagation direction of incident pseudo photon of coulomb field of the electron and the beam axis $\mathbf{e}_1$, $\gamma = 1/\sqrt{1-V^2}$ is Lorentz factor of the electron. The angular variables can be resolved into rectangular components parallel and perpendicular to plane of the picture: $\mathbf{\theta} = \mathbf{\theta}_{\|} + \mathbf{\theta}_{\perp}$, $\mathbf{\theta}_0 = \mathbf{\theta}_{0\|} + \mathbf{\theta}_{0\perp}$, $\mathbf{\psi} = \mathbf{\psi}_{\|} + \mathbf{\psi}_{\perp}$.

We will consider the radiation of a separated electron in the beam crossing the three-layer structure at the angle $\mathbf{\psi}(\psi_{\perp}, \psi_{//})$ to electron beam axis $\mathbf{e}_1$.

2. ANGULAR DENSITY OF THE RADIATION

To reveal and analyze the effects which are not connected with the absorption let us consider a simple case of thin nonabsorbing target: $\chi_0'' = \chi_c'' = 0$. This consideration is valid when the TR photon path in the amorphous medium $L_f^c = \dfrac{c}{\sin(\delta - \theta_B)}$ is considerably less than the photon absorption length $L_{abs}^c = \dfrac{1}{\omega\chi_c''}$, i.e. $L_f^c << L_{abs}^c$, and the path of the DTR photos in the single-crystal layer of the composite target $L_f^b = \dfrac{b}{\sin(\delta + \theta_B)}$ is considerably less than the photon absorption length $L_{abs}^c = \dfrac{1}{\omega\chi_c''}$ in the single-crystal layer, i.e. $L_f^c << L_{abs}^c$. We will consider the X-ray waves generation and propagation processes in a single crystal in the framework of two-wave approximation of the dynamic diffraction theory. By means of the procedures analogous to presented in the works [5 - 9] we have obtained the expression describing the DTR angular density generated by relativistic electron in the considered three-layer target:

$$\frac{dN_{\mathrm{DTR}}^{(s)}}{d\Omega} \equiv F_{DTR}^{(s)} = F_1^{(s)} + F_2^{(s)} + F_{\mathrm{int}}^{(s)}, \quad \text{(3a)}$$

where
$$F_1^{(s)} = \frac{e^2 |\chi_{\mathbf{g}}'| C^{(s)}}{2\pi^2 \sin^2\theta_B} \Omega^{(s)2} \left(\frac{1}{\Lambda_c} - \frac{1}{\Lambda}\right)^2 \sin^2\left(\frac{c\omega_B}{4\sin(\delta - \theta_B)} \cdot \Lambda_c\right) \int_{-\infty}^{\infty} R_{DTR}^{(s)} d\xi, \quad \text{(3b)}$$

$$F_2^{(s)} = \frac{e^2 |\chi_{\mathbf{g}}'| C^{(s)}}{8\pi^2 \sin^2\theta_B} \Omega^{(s)2} \left(\frac{1}{\Lambda_0} - \frac{1}{\Lambda}\right)^2 \int_{-\infty}^{+\infty} R_{DTR}^{(s)} d\xi, \quad \text{(3c)}$$

$$F_{\mathrm{int}}^{(s)} = \frac{e^2 |\chi_{\mathbf{g}}'| C^{(s)}}{4\pi^2 \sin^2\theta_B} \Omega^{(s)2} \left(\frac{1}{\Lambda_0} - \frac{1}{\Lambda}\right)\left(\frac{1}{\Lambda} - \frac{1}{\Lambda_c}\right) \times$$
$$\left[\cos\left(\frac{a\omega_B}{2\sin(\delta - \theta_B)} \cdot \Lambda\right) - \cos\left(\frac{a\omega_B}{2\sin(\delta - \theta_B)} \cdot \Lambda + \frac{c\omega_B}{2\sin(\delta - \theta_B)} \Lambda_c\right)\right] \int_{-\infty}^{+\infty} R_{DTR}^{(s)} d\xi, \quad \text{(3d)}$$

$$R_{DTR}^{(s)} = \frac{4\varepsilon^2}{\xi^{(s)2} + \varepsilon} \sin^2\left(b^{(s)} \frac{\sqrt{\xi^{(s)2} + \varepsilon}}{\varepsilon}\right), \quad \text{(4)}$$

$$\Omega^{(1)} = \theta_{\perp} - \psi_{\perp}, \ \Omega^{(2)} = \theta_{//} + \psi_{//},$$

$$\Lambda_0 = (\theta_{\perp} - \psi_{\perp})^2 + (\theta_{//} + \psi_{//})^2 + \gamma^{-2} - \chi_0', \ \Lambda_c = (\theta_{\perp} - \psi_{\perp})^2 + (\theta_{//} + \psi_{//})^2 + \gamma^{-2} - \chi_c',$$

$$\Lambda = (\theta_{\perp} - \psi_{\perp})^2 + (\theta_{//} + \psi_{//})^2 + \gamma^{-2}, \ b^{(s)} = \frac{1}{2\sin(\delta - \theta_B)} \frac{b}{L_{ext}^{(s)}}, \ L_{ext}^{(s)} = \frac{1}{\omega \cdot |\chi_g'| \cdot C^{(s)}},$$

$$\xi^{(s)}(\omega) = \eta^{(s)}(\omega) + \frac{1-\varepsilon}{2\nu^{(s)}}, \ \eta^{(s)}(\omega) = \frac{2\sin^2\theta_B}{V^2 |\chi_{\mathbf{g}}'| C^{(s)}} \left(1 - \frac{\omega(1 - \theta_{//}\cot\theta_B)}{\omega_B}\right), \quad \text{(5)}$$

$$\varepsilon = \frac{\sin(\delta + \theta_B)}{\sin(\delta - \theta_B)}, \quad C^{(1)} = 1, \ C^{(2)} = \cos 2\theta_B, \quad \omega_B = gV / 2\sin\theta_B \text{ is Bragg frequency.}$$

When s=1 the expressions (3) describe the σ-polarized waves and when s=2 then π-polarized ones.

The functions $F_1^{(s)}$ and $F_2^{(s)}$ describe the DTR angular densities corresponding to the waves of transition radiation generated in the amorphous layer and on the entrance surface of the single-crystal layer correspondently, and the function $F_{\mathrm{int}}^{(s)}$ describes the interference of these waves. The function $R_{DTR}^{(s)}$ describes the DTR spectrum. The parameter ε in the expression (4) describing the degree of asymmetry relative to the target surface of electrical field reflection in the single-crystal layer of the target is important for our theory. The expression for ε parameter includes the variable δ, which denote the angle between the target surface and system of diffracting atomic planes in the single-crystal layer.

The constructive interference of TR waves arising on different boundaries of amorphous layer in vicinity of Bragg frequency can cause the considerably increase in angular density of DTR. The conditions of constructive interference following from (3,b) can be written as:

$$\frac{c\omega_B}{4\sin(\delta - \theta_B)} \cdot \Lambda_c = (2n+1)\frac{\pi}{2}, \ (n = 0,1,2.....). \quad \text{(6a)}$$

An additional increase in spectral-angular density of DTR can be reached at the expense of the constructive interference of the transition radiation waves from the amorphous layer and the entrance surface of the single-crystal layer, which will take the place under fulfillment of the condition

$$\frac{a\omega_B}{2\sin(\delta-\theta_B)}\cdot\Lambda=(2m+1)\pi,\ (m=0,1,2.....). \quad (6b)$$

In the work [6] for DTR generated by a relativistic electron crossing rectilinearly such a three-layer structure there was shown that the constructive interference of TR waves leads to nine-fold increase in spectral-angular density of DTR as compared with DTR from only single-crystal layer in the case when each the layer of the target has the same dielectric susceptibility.

The expressions (3) describe the angular density of DTR of relativistic electron crossing the combined three-layer target at angle $\boldsymbol{\psi}(\psi_\perp,\psi_{//})$ in relation to electron beam axis $\mathbf{e}_1$. By the averaging of these expressions over two-dimension function of the angular distribution of the electrons in the beam we can find the angular density of DTR normalized to one electron.

Let us average the expressions (3) over all possible directions of the electron movement in the beam, for example, according to two-dimensional distribution function of Gauss:

$$f(\psi)=\frac{1}{\pi\psi_0^2}e^{-\frac{\psi^2}{\psi_0^2}}, \quad (7)$$

where ψ_0 is the parameter, which we will call by divergence of the beam of the radiating electrons (see Fig. 1). The angle ψ_0 defines a cone limiting the part of the beam where the beam density decreases less than 1/e times in comparison with the density on the axis of the beam. Let us write averaged expressions for DTR angular densities corresponding to the waves of DTR generated in amorphous layer $\left\langle F_1^{(s)}\right\rangle$ and on entrance surface of crystalline layer $\left\langle F_2^{(s)}\right\rangle$ and their interference term $\left\langle F_{\mathrm{int}}^{(s)}\right\rangle$:

$$\left\langle\frac{dN_{\mathrm{DTR}}^{(s)}}{d\Omega}\right\rangle\equiv\left\langle F_{DTR}^{(s)}\right\rangle=\left\langle F_1^{(s)}\right\rangle+\left\langle F_2^{(s)}\right\rangle+\left\langle F_{\mathrm{int}}^{(s)}\right\rangle, \quad (8a)$$

$$\left\langle F_1^{(s)}\right\rangle=\frac{1}{\pi\psi_0^2}\iint d\psi_\perp d\psi_{//}e^{-\frac{\psi^2}{\psi_0^2}}F_1^{(s)}, \quad (8b)$$

$$\left\langle F_2^{(s)}\right\rangle=\frac{1}{\pi\psi_0^2}\iint d\psi_\perp d\psi_{//}e^{-\frac{\psi^2}{\psi_0^2}}F_2^{(s)}, \quad (8c)$$

$$\left\langle F_{\mathrm{int}}^{(s)}\right\rangle=\frac{1}{\pi\psi_0^2}\iint d\psi_\perp d\psi_{//}e^{-\frac{\psi^2}{\psi_0^2}}F_{\mathrm{int}}^{(s)}. \quad (8d)$$

3. NUMERICAL CALCULATION

We carried out the numerical calculations using the obtained expressions (8). The calculation was carried out for the beam of the relativistic electrons with energy E=250 MeV crossing the three-layer structure consisted of the amorphous carbon layer C, of vacuum layer and of a single-crystal Si(111) layer. We assumed that the electron beam crosses the single-crystal layer under conditions of symmetric reflection, i.e. the system of reflecting atomic planes in the crystal situated perpendicularly to the crystal layer surface ($\delta=\pi/2$, $\varepsilon=1$) and ω_B=8 keV, $\theta_B\approx14.5°$. We considered σ-polarized waves (s=1, $\theta_{//}$=0). Under the conditions indicated above we considered the dependence of DTR angular density on electron beam divergence. We have chosen the thicknesses of the layers as c=4.1 μm for amorphous layer, a=19 μm for vacuum layer which satisfy the condition (6a) (under n=0) and the condition (6b) (under condition m=0) for constructive interference of TR waves in the maximum of DTR angular density ($\theta=\gamma^{-1}\approx2$ mrad). In this case, we have $L_f^c\approx4.2$ μm, $L_{abs}^c\approx700$ μm, $L_f^b\approx2.1$ μm, $L_{abs}^b\approx71$ μm, which imply that: $L_f^c<<L_{abs}^c$ and $L_f^b<<L_{abs}^b$, i.e. we have a right to ignore the photon absorption in this case and use (3).

The curves in Fig. 2 plotted by formula (8,a) with considering (3) demonstrate the dependence of angular density of DTR in the considered three-layer structure on value of divergence angle ψ_0.

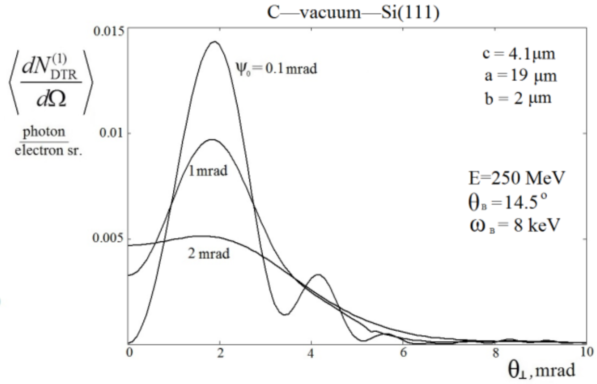

Fig. 2. DTR angular density under different values of the electron beam divergence ψ_0

In Fig. 3 the curves plotted by formula (8,a) with taking into account (3) demonstrate the contribution of TR from the amorphous layer $\left\langle F_1^{(1)}\right\rangle$ and the front surface of single-crystal layer $\left\langle F_2^{(1)}\right\rangle$, and summand describing their interference $\left\langle F_{\mathrm{int}}^{(1)}\right\rangle$ to the DTR angular density $\left\langle F_{DTR}^{(1)}\right\rangle$ under ψ_0=0.1 mrad (see Fig. 2).

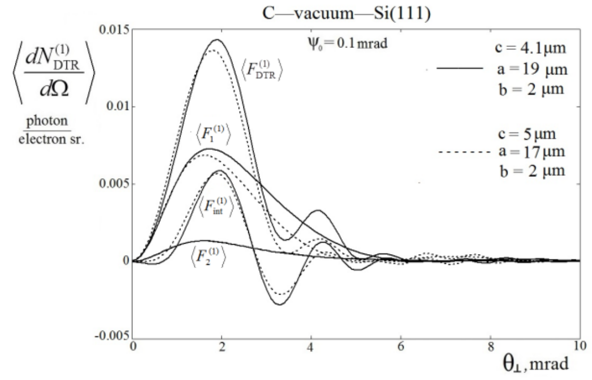

Fig. 3. The contributions of TR from amorphous layer $\left\langle F_1^{(1)}\right\rangle$, from front boundary of the single-crystal layer $\left\langle F_2^{(1)}\right\rangle$ and their interference summand $\left\langle F_{\mathrm{int}}^{(1)}\right\rangle$ to DTR angular density $\left\langle F_{DTR}^{(1)}\right\rangle$

One can see in Fig. 3 the considerable contribution of TR from amorphous layer and from forward surface of crystalline to the angle density of DTR. Solid curves are plotted under the condition (6,a) and (6,b) which correspond to the case of constructive interference of

TR waves. The dashed curves in Fig. 3 are plotted under condition (c=5 μm and a=17 μm) slightly changing from condition of constructive interference. As it is seen from Fig. 3 these changes in thicknesses of layers not lead to sharp change in character of interference from constructive to destructive.

So, small but appreciable changes in the thicknesses of the target layers do not lead to such a change in the character of the waves interference from the amorphous layer and the front surface of the monocrystalline layer that to turn it from constructive into destructive. Because of this fact, we can expect the possibility of experimental observation and investigation of interference effects in the coherent radiation in the considered three-layer target by the beams of relativistic electrons.

CONCLUSIONS

In the present work, the diffracted transition radiation generated by a beam of relativistic electrons crossing a three-layer target consisted of an amorphous layer, a layer of vacuum and a single-crystal layer has been considered. The expression describing the DTR angular density has been derived. The conditions of constructive interference of the TR wave from different boundaries of the layered target have been defined. The averaging of the expression of DTR angular density over all possible trajectories of electron in the beam has been carried out. The calculations of the distribution of DTR angular density have been done for the electron beam with angular distribution described by the two-dimensional distribution function of Gauss. It has been shown, that slight changes in the thicknesses of the target layers do not destroyed the constructive character of the waves interference radiated from the amorphous layer and front surface of the monocrystalline layer.

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УГЛОВАЯ ПЛОТНОСТЬ ДИФРАГИРОВАННОГО ПЕРЕХОДНОГО ИЗЛУЧЕНИЯ ПУЧКА РЕЛЯТИВИСТСКИХ ЭЛЕКТРОНОВ В СОСТАВНОЙ МИШЕНИ

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Рассматривается когерентное рентгеновское излучение пучка релятивистских электронов, пересекающих трехслойную структуру, состоящую из трех слоев: аморфного, вакуумного и монокристаллического. Показана возможность конструктивной интерференции волн переходного излучения от разных границ мишени, дающих вклад в дифрагированное переходное излучение. Показано, что небольшие изменения толщин слоев структуры не приводят к существенному изменению конструктивной интерференции волн ПИ из аморфного слоя и передней поверхности монокристаллического слоя, то есть условия конструктивной интерференции достаточно стабильны для того, чтобы использовать их на практике.

КУТОВА ЩІЛЬНІСТЬ ДИФРАГОВАНОГО ПЕРЕХІДНОГО ВИПРОМІНЮВАННЯ ПУЧКА РЕЛЯТИВІСТСЬКИХ ЕЛЕКТРОНІВ У СКЛАДЕНІЙ МІШЕНІ

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Розглядається когерентне рентгенівське випромінювання пучка релятивістських електронів, що перетинають тришарову структуру, яка складається з трьох шарів: аморфного, вакуумного і монокристалічного. Показана можливість конструктивної інтерференції хвиль перехідного випромінювання від різних меж мішені, що дають вклад у дифраговане перехідне випромінювання. Показано, що невеликі зміни товщини шарів структури не призводять до істотної зміни конструктивної інтерференції хвиль ПІ з аморфного шару і передньої поверхні монокристалічного шару, тобто умови конструктивної інтерференції досить стабільні для того, щоб використати їх на практиці.