SUPERRADIANT EMISSION REGIMES OF THE SYSTEM OF STATIONARY OSCILLATORS

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The mathematical model of superluminescence of a finite system of nonlinear oscillators is considered. With each oscillator generating its own field, the oscillators interact through integral field of radiation. The influence of nonlinearity of oscillators due to relativistic effects is taken into account. Characteristics of the synchronization process are discussed. Regimes of field generation under conditions of radiation into an external space are considered. The influence of the energy loss due to external radiation on the generation efficiency is discussed.

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INTRODUCTION

Works [1, 2] describe the emitting of Langmuir waves by the extended and wavelength-sized beams of non-relativistic electrons. The last case corresponds to the process of superradiance of a beam and is qualitatively equal to a dissipative regime of generation. In this case the process of emitters' synchronization, which creates conditions for the coherent radiation, is defined by their spatial grouping.

But for particles with fixed centers of oscillations, spatial grouping is impossible and phase synchronization is difficult [3, 4]. However, nonlinear oscillators of such kind can be synchronized by the field, in particular – in a single-mode regime of generation [4]. But nevertheless, even in this case this process is rather slow.

Note that it is nonlinearity of oscillators that allows their grouping into bunches in the gyrotron-like devices. In this case their centers of oscillation slowly move along the direction of the wave which propagates with the decreased speed under conditions of operation near cutoff frequency [5 - 7].

Below we pay particular attention to the connection between descriptions (in different physical implementations) of moving emitters and fixed oscillators (both being in the field of the excited wave). We show general character of wave generation by the systems of emitters and oscillators and note the main characteristics of this process under dissipative conditions (e.g. radiation to the external space).

1. WAKEFIELD GENERATION BY A SHORT ELECTRON BEAM IN PLASMA

In a simplest one-dimensional case the movement of a short beam of electrons in plasma with the length b and initial velocity v_0 can be described with the following system of equations [1, 2]:

$$\partial \mathrm{E} / \partial \tau = -\theta \cdot \mathrm{E} + N^{-1} \cdot \sum_{i=1}^{N} \mathrm{Cos}\{2\pi\xi_i + \varphi\}, \qquad (1)$$

$$\partial \varphi / \partial \tau = -N^{-1} \cdot \sum_{i=1}^{N} \mathrm{Sin}\{2\pi\xi_i + \varphi\}, \qquad (2)$$

$$2\pi \cdot d^2\xi_j / d\tau^2 = -\left\{ \begin{array}{c} \mathrm{E} \cdot \mathrm{Cos}\{2\pi\xi_j + \varphi\} \\ (N \cdot \theta)^{-1} \sum_{i=1}^{N} \cos[2\pi\, g_i(\xi_j - \xi_i)] \cdot U(\xi_i - \xi_j) \end{array} \right\}, \qquad (3)$$

where $2\pi\xi = kz - \omega t$, $\tau = t \cdot \gamma$, $\omega_{pe,b}^2 = 4\pi e^2 n_{0e,b} / m_e$, e, m_e are electron charge and mass, $n_{0e,b}$ – densities of the plasma and the beam, effective logarithmic decrement of the oscillations in the beam δ_D can be defined as the ratio of the energy leaving the beam to the whole oscillatory energy in its volume, i.e. $\delta_D = V_0 / a$; $\theta = \delta_D / \gamma |_{\delta=0} = (V_0 / a \cdot \omega_{pe})(\omega_{be} / \omega_{pe})^{-2/3}$, $\gamma^3 = (n_{0b} / n_{0e}) \cdot \omega_{pe}^3$, $M = n_{pb} \cdot b$ – number of particles in the beam, $g_i = (1 + (v - v_0) / v_0)^{-1} = (1 + V\omega_{pe} / \gamma)^{-1}$, which can be easily derived from the equation: $2\pi d\xi / d\tau = k(v - v_0) / \gamma = V$. Here $U(x) = 1; x \ge 0$ and $U(x) = 0; x < 0$. Parameter $\theta = \delta_D / \gamma$ corresponds to the ratio of the oscillation decrement without disturbing element (e.g. beam) δ_D to γ – the maximal decrement of non-dissipative instability. The upper term in the right-hand side of (3) should be used when describing extended beam with integral field $\mathrm{E} \cdot \mathrm{Cos}\{2\pi\xi_j + \varphi\}$ accumulating in its volume. The lower term of (3) describes the aggregated field of particles of rather short beam with large θ. Note that this term, generally, defines spontaneous radiation of a Langmuir wave with frequency ω_{pe} by individual particles of the beam. Integral field $\mathrm{E} \cdot \exp\{2\pi i\xi_j + i\varphi\}$ is formed by initial perturbation or appears because of high-frequency energy accumulation in a relatively extended beam. The particles in the beam do not directly interact with each other and are connected only through the field. Such field usually arises in the tasks of generation or amplification of induced radiation. When θ is small (i.e. beam is extended) this field accumulates around the beam, and the lower term in (3) can be discarded.

When we have large $\theta >> 1$, contrariwise, equations (1) and (2) altogether with the upper term of (3) can be excluded. In this case interaction of particles becomes significant: particles moving ahead of others affect those behind them, but not vice versa. When the particles form groups in the beam and the phase synchronization appears, so called superradiance becomes possible – which is described by the lower term of (3).

In this case (i.e. when $\theta >> 1$ and/or the beam is sufficiently short), the energy of the field leaves the volume of the beam within period of time $v_0 / b << \gamma^{-1}$. In this case the amplification of the field is defined only by the second term of (3) and is caused by natural grouping of the particles and the increase of coherence of their radiation – which eventually forms the superradiant field. In [1, 2] it was noted that increment of the superradiance process is similar to the increment of the dissipative beam instability $\gamma_D \approx \omega_{pb}(\omega_{pe}/\delta_D)^{1/2} = \omega_{pb}(kb)^{1/2}$, when energy is lost due to its flow out of the volume of the beam. Spatial modulation of the particles, similar in both cases, lead to their phase synchronization and to the increase of the radiation coherence. The restriction of the amplitude is caused by the particles trapping in the potential trough of the wave, which corresponds to the case when oscillation frequency of trapped particles is equal to the increment of the process: $\Omega = \sqrt{ekE/m} \approx \gamma_D$. Maximal field amplitude for the beams of several Langmuir wavelengths is $E_{\max} \approx 2\pi eM$. It depends only on the number of particles. For the beams much shorter than that, this amplitude can reach $2E_{\max}$ in some points (this is the case of all particles being placed in one point); this happens because of self-profiling of the beam [8]. This process of self-profiling is of particular interest for the plasma's particles acceleration.

It can be easily seen that in superradiance regime for the beams of several Langmuir wavelengths, the maximal amplitude can reach the value

$$P_{\sup} = v \cdot E_{\max}^2 / 4\pi = \frac{\pi e^2 v}{4} M^2 \propto M^2,$$

i.e. it is proportional to the second power of the total number of particles in the beam $M = n_0 \cdot b$. Note that the intensity of the spontaneous radiation of similar beam $P_{spon} = v \cdot E_{spon}^2 / 4\pi = \pi e^2 v M \propto M$ is proportional to the total number of particles.

2. SYSTEM OF FIXED OSCILLATORS IN THE FIELD OF THE WAVE

Let the wave and oscillators frequency be the same and equal to ω. The wave vector is $\vec{k} = (0,0,k)$, the field is represented as $\vec{E} = (E,0,0)$, $\vec{B} = (0,E,0)$; $E = |E| \cdot \exp\{-i\omega t + ikz + i\varphi\}$. Oscillators lie along the OZ axis, and there are N oscillators per the wavelength $2\pi/k$. Mass of an oscillator is *m*, charge is $-e$, frequency of oscillation is equal to the wave's frequency ω. Initial amplitude is a_0. Assume that oscillators move along OX axis. This allows us to neglect the influence of the magnetic field of the wave on their dynamics [4].

For extended systems, or in a case of small group velocity of the wave, the energy can accumulate around oscillators, even when there is energy loss due to radiation. Here we neglect the reflection effects on the boundary of the oscillator system. In this case the effective absorption decrement is $\delta_D^{'} = 2c_{eff}/b$ and $\theta = \delta_D^{'}/\gamma' = 2c_{eff}/b \cdot \gamma'$, where γ' is the increment of instability in a system without wave energy loss. Now we need to consider two waves of the induced radiation propagating in both directions. As to the spontaneous radiation from each oscillator, as well as from the whole system of oscillators, it always goes in both directions. Now the motion equations of the oscillators are:

$$\begin{aligned}&\frac{d}{d\tau_1}A_j = \\ &-E_+Cos\{\varphi_+ + 2\pi Z_j - \psi_j\} - E\_Cos\{\varphi_- - 2\pi Z_j - \psi_j\} \\ &-(\theta N)^{-1}\sum_{s=1}^{N} A_s[Cos\{-\psi_j + 2\pi(Z_j - Z_s) + \psi_s\}U(Z_j - Z_s) \\ &+Cos\{-\psi_j - 2\pi(Z_j - Z_s) + \psi_s\}U(Z_s - Z_j)],\end{aligned} \tag{4}$$

$$\begin{aligned}&A_j[\frac{d}{d\tau_1}\psi_j - \Delta_j] = \\ &-E_+Sin\{\varphi_+ + 2\pi Z_j - \psi_j\} - E\_Sin\{\varphi_- - 2\pi Z_j - \psi_j\} \\ &-(\theta N)^{-1}\sum_{s=1}^{N} A_s[Sin\{-\psi_j + 2\pi(Z_j - Z_s) + \psi_s\}U(Z_j - Z_s) \\ &+Sin\{-\psi_j - 2\pi(Z_j - Z_s) + \psi_s\}U(Z_s - Z_j)],\end{aligned} \tag{5}$$

and for the integral fields $\mathrm{E}_\pm$ propagating in either direction:

$$\frac{\partial}{\partial\tau_1}E_\pm + \theta \cdot E_\pm = \frac{1}{N}\sum_{j=1}^{N} A_j \cdot \mathrm{Cos}\{\psi_j \mp 2\pi Z_j - \varphi_\pm\}, \tag{6}$$

$$E_\pm \frac{\partial\varphi}{\partial\tau_1} = \frac{1}{N}\sum_{j=1}^{N} A_j \cdot \mathrm{Sin}\{\psi_j \mp 2\pi Z_j - \varphi_\pm\}, \tag{7}$$

where $\mathrm{E} = eE/\omega m\gamma' a_0$, $\tau = \gamma' t$, $A_i = |x_i|/a_0$, $(\gamma')^2 = \pi e^2 n_0/m$, $kz_i = Z_i \subset (0,2\pi)$. In a non-relativistic case $\Delta_i = 0$. With relativism we need to take into account the nonlinearity of the oscillators: $\Delta_i = \alpha \cdot (A_i^2 - A_{i0}^2)$, $\alpha = 3\omega(k \cdot a_0)^2/4$. If we substitute $\psi_j \to -\psi_j$, $\varphi \to -\varphi$, $Z_j \to -Z_j$, and $\alpha \to -\alpha$ the system remains invariant, i.e. the sign of α does not change the dynamic of the process, it only makes the phases of the wave and particles to change in opposite direction. It can also be shown that the waves with opposite polarization can be described with exactly the same equations (assuming that the particles move parallel to the vector E). Particles density over length is $n_0 = M/b$, where *M* is the total number of particles in the beam, each super-particle contains *M/N* real ones, $\pi e^2 M / 2mc = \gamma'_D$. Obviously, the first terms in the right-hand side of (4) and (5) correspond to the spontaneous (at least at an initial moment) radiation. Energy density at the ends of the system can be estimated as:

$$w = (m\omega_0^2 a_0^2 n_0 / 2) \cdot W, \tag{8}$$

$$\begin{aligned}W &= \{|E(0,\tau)|^2 (\gamma_D/\gamma)^2 + \mathrm{E}_+^2 + \mathrm{E}_-^2\} = \\ &= 2\{\theta^{-2} \cdot |E(0,\tau)|^2 + \mathrm{E}_+^2 + \mathrm{E}_-^2\}.\end{aligned} \tag{9}$$

Note that the total field of individual oscillators $|E(0,\tau)|$ at the ends of the system equals to the first term in the right-hand side of (4). The density of the energy flow from the system can be also derived from the relation $p = c \cdot w$.

3. CONNECTION BETWEEN THE MODEL OF EXCITATION OF INTEGRAL FIELD (4) - (7) AND THE MODEL OF CYCLOTRON INSTABILITIES

Consider the excitation of electromagnetic waves of different polarization with the frequency ω and the wave vector $\vec{k} = (k_{ms}, 0, k_z)$ in a smooth metallic cylindrical waveguide with radius r_w by a beam of electrons in resonance: $\omega \approx k_z \cdot v_z + n \cdot \omega_B$, where v_z is velocity and ω_B is the angular cyclotron frequency of the electrons. Static magnetic field is $\vec{B} = (0, 0, B)$. The electron beam occupies cylindrical layer in a cross-section of the waveguide, which we will treat as sufficiently thin. All the centers of the Larmor rotation of electrons (with the radius r_B) are located at the same distance r_C from the waveguide's axis.

TE wave. The equations that describe TE-field of the wave (where longitudal component of E is zero) can be written as follows (see e.g. [9]):

$$\frac{d\mathrm{E}_e}{d\tau} + \theta_e \cdot \mathrm{E}_e = N^{-1} \cdot \sum_{j=1}^{N} a_j \cdot J_n^{'}(a_j) \cdot \mathrm{Sin}(2\pi\zeta + \varphi_e), \quad (10)$$

$$\frac{d\varphi_e}{d\tau} - \Delta_e = (\mathrm{E}_e N)^{-1} \cdot \sum_{j=1}^{N} a_j \cdot J_n^{'}(a_j) \cdot \mathrm{Cos}(2\pi\zeta_j + \varphi_e). \quad (11)$$

The motion equations of the electrons in this wave in the presence of external uniform magnetic field are:

$$2\pi \frac{d\zeta_i}{d\tau} = \eta_i + n\mathrm{E}_e \cdot J_n(a_i) \cdot \{1 - \frac{n^2}{a_i^2}] \cdot \mathrm{Cos}(2\pi\zeta_i + \varphi_e), \quad (12)$$

$$d\eta_i / d\tau = -R_e \cdot \mathrm{E}_e \cdot a_i \cdot J_n^{'}(a_i) \cdot \mathrm{Sin}(2\pi\zeta_i + \varphi_e), \quad (13)$$

$$da_i / d\tau = -n \cdot \mathrm{E}_e \cdot J_n^{'}(a_i) \cdot \mathrm{Sin}(2\pi\zeta_i + \varphi_e), \quad (14)$$

where $\tau = \delta_e t$,

$$\delta_e^2 = 4e^2 \cdot \omega_B \cdot \mathrm{N}_{b0} \cdot [m_e \cdot c \cdot k_{ms}^2 \cdot r_w \cdot J_m^2(x_{ms}) \cdot (1 - m^2 / x_{ms}^2) \cdot D_\omega]^{-1} \cdot J_{m-n}^2(k_{ms} \cdot r_C),$$

$$D_\omega = \frac{\partial D}{\partial \omega} = \partial\{[\omega^2 - (k_z^2 + k_{ms}^2)c^2] / [\omega^2 - k_z^2 c^2]\} / \partial\omega |_{D=0},$$

$$R_e = k_z^2 \cdot \omega_B / k_{ms}^2 \cdot \delta_e, \; \mathrm{E}_e = e \cdot b \cdot J_{m+n}(k_{ms} \cdot r_C) / m_e \cdot c \cdot \delta_e,$$

$$\eta = (k_z \cdot v_z - \omega + n \cdot \omega_B) / \delta_e, \; a = k_{ms} r_B = k_{ms} v_\Phi / \omega_B,$$

$\omega_B = eB / m_e c$, N_{b0} is the number of particles of undisturbed beam per unit length, *b* is the amplitude of the wave. Axial component of the magnetic field of the wave is $B_z = b \cdot J_m(k_{ms} r) \cdot \exp\{-i\omega t + ik_z z + im\vartheta\}$ (in cylindrical coordinates (r, ϑ, z)), $J_m(x)$ and $J_m^{'}(x) = dJ_m(x)/dx$ – Bessel function of the order *m* and its derivative. The requirement of the tangential component of the field being equal to zero on the boundary of the waveguide defines the values of the transverse wave number $k_\perp = k_{ms} = x_{ms} / r_w$, where x_{ms} denotes the *s*-th root of the equation $dJ_m(x)/dx = 0$.

TM wave. Equations describing the field of the TM wave (which has no magnetic field in the direction of propagation) can be written as follows (see e.g. [6]):

$$\frac{d\mathrm{E}_h}{d\tau} + \theta_h \cdot \mathrm{E}_h = N^{-1} \cdot \sum_{j=1}^{N} J_n(a_j) \cdot \mathrm{Cos}(2\pi\zeta + \varphi_h), \quad (15)$$

$$\frac{d\varphi_h}{d\tau} - \Delta_h = (\mathrm{E}_h N)^{-1} \cdot \sum_{j=1}^{N} J_n(a_j) \cdot \mathrm{Sin}(2\pi\zeta_j + \varphi_h). \quad (16)$$

The equations of motion of the electrons in this wave are:

$$2\pi \frac{d\zeta_i}{d\tau} = \eta_i + \frac{n}{a_i} \cdot \mathrm{E}_h \cdot J_n^{'}(a_i) \cdot \mathrm{Sin}(2\pi\zeta_i + \varphi_h), \quad (17)$$

$$d\eta_i / d\tau = -R_h \cdot \mathrm{E}_h \cdot a_i \cdot J_n(a_i) \cdot \mathrm{Cos}(2\pi\zeta_i + \varphi_h), \quad (18)$$

$$da_i / d\tau = -(n / a_j) \cdot \mathrm{E}_h \cdot J_n(a_i) \cdot \mathrm{Cos}(2\pi\zeta_i + \varphi_h), \quad (19)$$

where $\tau = \delta_h t$,

$$\delta_h^2 = 4e^2 \cdot \omega_B \cdot \mathrm{N}_{b0} \cdot [m_e \cdot r_w^2 \cdot J_m^{'2}(x_{ms}) \cdot D_\omega]^{-1} \cdot (k_{ms}^2 / \omega_B k_z^2) \cdot J_{m-n}^2(k_{ms} \cdot r_C),$$

$$R_e = k_z^2 \cdot \omega_B / k_{ms}^2 \cdot \delta_h, \; \eta = (k_z \cdot v_z - \omega + n \cdot \omega_B) / \delta_h,$$

$\mathrm{E}_h = e \cdot h \cdot J_{m+n}(k_{ms} \cdot r) \cdot (k_{ms}^2 / \omega_B k_z) / m_e \cdot \delta_h$, h is the amplitude of the wave, with the longitudal component of the electric field $E_z = h \cdot J_m(k_{ms} r) \cdot \exp\{-i\omega t + ik_z z + im\vartheta\}$ (in the cylindrical coordinates (r, ϑ, z)). Other symbols are the same as were used in the description of the TE wave.

Conserved quantities. Note that for these two waves the following conservation equations hold:

$$R \cdot a^2 - 2n \cdot \eta = Const, \quad (20)$$

$$|\mathrm{E}|^2 - (2/R) \cdot N^{-1} \sum_{j=1}^{N} \eta_j = Const, \quad (21)$$

$$|\mathrm{E}|^2 - n^{-1} \cdot N^{-1} \sum_{j=1}^{N} a_j^2 = Const, \quad (22)$$

with the last integral (22) being correct only when $\theta = 0$; if $\theta \neq 0$, its right-hand side should read as $Const + \theta \cdot \int_0^t dt' |\mathrm{E}(t')|^2$. It should be noted that the consequence of these integrals in the absence of energy dissipation ($\theta = 0$) is the following [6]: the change of the energy of transverse movement of the particles $\Delta W_\perp = \frac{\omega_B \cdot m_e \cdot \mathrm{N}_{b0}}{2k_{ms}^2} N^{-1} \sum_{j=1}^{N} (a_j^2 - a_{j0}^2)$ and the change of corresponding energy of the longitudal movement $\Delta W_\parallel = \frac{v_{z0} \cdot m_e \cdot \mathrm{N}_{b0}}{k_z} N^{-1} \sum_{j=1}^{N} (\eta_j - \eta_{j0})$ relate as $\Delta W_\perp / \Delta W_\parallel = = n\omega_B / k_z v_{z0}$. Also the change of the energy of the field and the energy of the transverse movement of the particles relate as $\omega / n\omega_B$. The case when $n < 0$ and the particles move in the same direction as the wave, corresponds to the normal Doppler effect; it does not appear for particles which do not move in that direction. Note that the system of equations for TM wave can be transformed to a well-known system of equations for gyrotron even in the presence of a low-density plasma [5, 6].

Introduction of relativism (negative mass effect, see e.g. [7]) leads to the non-linearity in the motion equations (12) and (17): η_i should be substituted by $\eta_i - \alpha(a_j^2 - a_{j0}^2)$, where

$$a_{j0}^2 = a_j^2 |_{\tau=0}, \gamma_0 = (1 - v_\Phi^2 / c^2)^{-1/2} |_{\tau=0}, \alpha = n\omega_B^3 \cdot \gamma_0^2 / 2k_{ms}^2 c^2 \delta_h.$$

It turns out, that for linear oscillators which do not move in the direction of the wave, systems of equations for TE and TM waves (10) - (14) and (15) - (19) in some particular cases are equal to the equations (4) - (7) (with appropriate selection of α and the sign of the field). This holds when $n = 1$, $\theta = 0$, $\Delta = 0$, $\varphi \to -\varphi$, $\psi \leftrightarrow 2\pi\zeta + \pi/2$, $R \to 0$ at small a_i ($J_1(a_i)_{a_i \to 0} \to a_i / 2$, $J_0(a_i)_{a_i \to 0} \to 1$, $J'_1(a_i)_{a_i \to 0} \to 1/2$).

4. CONNECTION BETWEEN DISSIPATIVE MODE OF INSTABILITY AND SUPERRADIANCE

From the motion equations (3) - (5) follows that with the same values of $\theta > 0$ the regimes of dissipative instability (see upper terms in the right-hand sides of corresponding equations) and of superradiance (the lower term) appear similar. Dissipative mode of beam instability in plasma has been studied quite well [1]; also the details of the dissipative mode of the system of fixed oscillators were presented in the work [10]. The characteristic time of these processes in any physical realization is of the same order. Maximal values of the wave's amplitude are of the same order too.

In a case of superradiance, the amplitude of the wave is proportional to the number of oscillators $M = n \cdot b$ and inversely proportional to the group velocity of the excited waves. If dielectric ε_0 and magnetic μ_0 constants in the region around oscillators are not equal to 1, the group velocity of the waves can be dramatically lowered: $c_{eff} = v_g = kc^2 / \omega\mu_0\varepsilon_0$. In the gyrotron-like devices it is achieved by the selection of the frequency of the wave near its cutoff value. Obviously, the intensity of the superradiance will be maximal in the direction of the largest dimension of the system – which is observed in the experiments.

If the dimensions of the system are large enough or the group velocity of the excited waves is sufficiently small (i.e. $\theta << 1$), the wave energy begins to accumulate in the system, and a mode with a fixed phase is formed due to reflection from the system's boundaries. This mode is described by the equations (1), (2) and (6), (7). In the right-hand sides of the equations (3), (4), and (5) the upper terms should be used. Under such conditions the superradiance regime is replaced by the generation regime. Particularly, for a short beam (see section 2) the wave's amplitude is determined by the condition $\sqrt{ekE/m} \approx \gamma$ and becomes significantly larger.

Interestingly, that even in dissipative instability ($\theta > 1$), where the decrement arises because of energy flow out of the system, the wave's amplitude is also proportional to the number of emitters and inversely proportional to the group speed of the oscillations. This allows to talk about similarity of the dissipative instability with effective increment γ_D and the superradiant regime.

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РЕЖИМЫ СВЕРХИЗЛУЧЕНИЯ СИСТЕМЫ НЕПОДВИЖНЫХ ОСЦИЛЛЯТОРОВ

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Обсуждается математическая модель суперлюминисценции ограниченной в пространстве системы нелинейных осцилляторов. Каждый осциллятор генерирует свое поле. Взаимодействие осцилляторов происходит через интегральное поле излучения. Проведен учет влияния нелинейности осцилляторов за счет релятивистских эффектов. Обсуждаются особенности процесса синхронизации осцилляторов. Рассмотрены режимы генерации в условиях излучения во внешнюю среду. Учитывается влияние потерь энергии системы за счет внешнего излучения на эффективность генерации.

РЕЖИМИ НАДВИПРОМІНЮВАННЯ СИСТЕМИ СТАЦІОНАРНИХ ОСЦИЛЯТОРІВ

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Обговорюється математична модель суперлюмінісценції, обмеженої в просторі системи нелінійних осциляторів. Кожен осцилятор генерує власне поле. Взаємодія осциляторів відбувається за допомогою інтегрального поля випромінювання. Враховано вплив нелінійності осциляторів за рахунок релятивістських ефектів. Обговорюються особливості процесу синхронізації осциляторів. Розглянуто режими генерації в умовах випромінювання в зовнішній простір. Враховано вплив втрати енергії системи за рахунок зовнішнього випромінювання на ефективність генерації.