# ON THE DEPENDENCE OF THE EFFICIENCY OF STOCHASTIC MECHANISM OF CHARGED PARTICLE BEAM DEFLECTION IN A BENT CRYSTAL ON THE PARTICLE ENERGY

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The problem of stochastic deflection of high-energy charged particles was considered on the basis of analytical calculation and numerical simulation. It was shown that with increasing energy of charged particles the maximal deflection angle, achievable with a help of stochastic deflection mechanism decreases as $E^{-1/4}$, while the optimal radius of crystal curvature, which corresponds to this maximal deflection angle, increases as $E^{5/4}$.

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## 1. INTRODUCTION

If a high-energy charged particle penetrates through a crystal having a small angle $\psi$ between its momentum and one of the main crystallographic axes (let us call this axis as $z$-axis), correlations between successive collisions of the particle with neighboring atoms may occur. This happens when the angle $\psi$ is of the order of the critical angle of axial channeling $\psi_c$ [1]. In this case motion of the particle is defined by the continuous potential of atomic strings. In this potential particle motion in the plane $(x, y)$ that is orthogonal to the $z$-axis can be finite (axial channeling) or infinite (above-barrier motion). If the crystal is bent both axial channeling and above-barrier motion may cause a deflection of the direction of motion of the particle [2]. The main advantage of such deflection of high-energy charged particle in comparison with deflection in the field of electromagnet is compact sizes of the bent crystal. Strong intra-crystalline field provides an opportunity to deflect charged particles on angles that far exceed $\psi_c$ by a crystal with a thickness of several centimeters.

The mechanism of charged particle deflection in the field of bent atomic strings, that was proposed for above-barrier particles in [2], was later called as *stochastic deflection mechanism*. Such term was used due to a resemblance of particle motion in the $(x, y)$ plane during this regime of motion in the crystal with stochastic motion [3]. The main advantage of this mechanism over two other deflection mechanisms – planar channeling in a bent crystal [4, 5] and volume reflection from bent atomic planes [6] – is that stochastic deflection allows to deflect not only positively but also negatively charged particles on angles that far exceed $\psi_c$.

The possibility of negatively charged particle beam deflection by means of a bent crystal was experimentally confirmed in [7]. Later, in [8] the analysis of the influence of incoherent scattering on atomic thermal vibrations and electrons on the efficiency of stochastic deflection of high-energy negative particle beams in bent crystals was carried out by an example of 150 GeV/$c$ $\pi^-$-mesons. It was shown that incoherent scattering leads to the existence of a maximum in the dependence of deflection efficiency from the radius of curvature of the crystal. The radius of curvature that corresponds to the maximum in deflection efficiency was called as *optimal radius of curvature*. In this article we consider the dependence of the optimal radius of curvature from the energy of negatively charged particles.

## 2. MOTION OF NEGATIVELY CHARGED PARTICLES IN A FIELD OF BENT ATOMIC STRINGS

As it was written above, if for a high-energy charged particle $\psi \sim \psi_c$, crystal potential could be integrated over the $z$-axis and written as a sum of potentials of atomic strings, that are parallel to the $z$-axis. To obtain the potential of an atomic string let us take atomic potential in the Doyle-Turner approximation [9]. In this approximation the potential energy of a particle with a charge, that equals to the charge of an electron, in the field of an atomic string

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could be written as

$$U_{str}(\rho) = -\frac{8\pi^2\hbar^2}{m_e d}\sum_{k=1}^{4}\frac{\alpha_k}{\beta_k + B}e^{-\frac{4\pi^2\rho^2}{\beta_k+B}}, \quad (1)$$

where $m_e$ is an electron mass, $d$ is the distance between neighboring atoms in the atomic string, $\alpha_k$ and $\beta_k$ are coefficients found in [9] for a large number of elements, $B = 8\pi^2\langle {r_T}^2\rangle$ and $r_T$ is the rms atomic thermal vibration amplitude in one direction ($r_T \approx 0.075$ Å for Si at 293 K), $\rho$ is the distance from the atomic string.

Since $U_{str}(\rho)$ in Eq. (1) decreases rapidly with increasing distance from the atomic string, only a limited number of neighboring atomic strings determine the value of the potential at the selected point inside the crystal. This fact gives us a possibility to sum the potentials of atomic strings analytically and find the potential energy of a high-energy charged particle in the crystal as

$$U(\vec{\rho}) = \sum_{n=-\infty}^{\infty} U_{str}(\vec{\rho} - \vec{\rho}_n), \quad (2)$$

if the charged particle is far from the edge of the crystal[1]. In Eq. (2) vector $\vec{\rho}$ corresponds to the coordinates of the charged particle in the $(x, y)$ plane and vector $\vec{\rho}_n$ corresponds to the coordinates of the $n$-th atomic string in this plane.

If a high-energy particle with a charge, that equals to the charge of an electron, moves in a silicon crystal and the $z$-axis is parallel to the $\langle 110\rangle$ crystal axis, summation over atomic strings in Eq. (2) gives

$$U(\vec{\rho}) = -\frac{2\pi\hbar^2}{m_e d d_x d_y}\sum_{k=1}^{4}\alpha_k\left\{\theta_3\left(\pi\frac{x}{d_x}, e^{-\frac{\beta_k+B}{4d_x^2}}\right)\theta_3\left(\pi\frac{y}{d_y}, e^{-\frac{\beta_k+B}{4d_y^2}}\right) + \right.$$
$$+\theta_3\left(\pi\frac{x}{d_x}, e^{-\frac{\beta_k+B}{4d_x^2}}\right)\theta_3\left(\pi\left(\frac{y}{d_y}-\frac{1}{4}\right), e^{-\frac{\beta_k+B}{4d_y^2}}\right) + \theta_3\left(\pi\left(\frac{x}{d_x}-\frac{1}{2}\right), e^{-\frac{\beta_k+B}{4d_x^2}}\right)\theta_3\left(\pi\left(\frac{y}{d_y}-\frac{1}{2}\right), e^{-\frac{\beta_k+B}{4d_y^2}}\right) +$$
$$\left. + \theta_3\left(\pi\left(\frac{x}{d_x}-\frac{1}{2}\right), e^{-\frac{\beta_k+B}{4d_x^2}}\right)\theta_3\left(\pi\left(\frac{y}{d_y}-\frac{3}{4}\right), e^{-\frac{\beta_k+B}{4d_y^2}}\right)\right\}, \quad (3)$$

where $d_x = d = a/\sqrt{2}$, $d_y = a$, $a$ is the lattice constant which for Si is about 5.4307 Å, $\theta_3(u,q) = \sum_{n=-\infty}^{\infty} q^{n^2}e^{2nui}$ is the Jacobi theta function of the third kind [10], $i^2 = -1$.

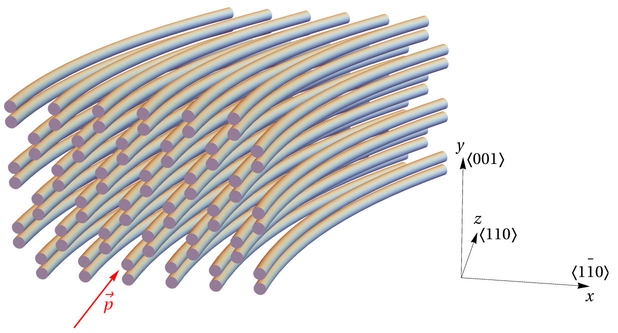


***Fig.1.*** *Orientation of the bent crystal with respect to the initial direction of motion of a high-energy charged particle.*

Let us now consider the motion of a high-energy charged particle with a charge, that equals to the charge of an electron, in a bent silicon crystal. Let the crystal be oriented along the $\langle 110\rangle$ axis with respect to the initial direction of motion of the particle, the $(x, z)$ plane is the plane of curvature and it coincides with the (001) crystallographic plane (see Fig. 1). If the thickness of the crystal is much smaller than the radius of curvature, the trajectory of the particle in the crystal can be found by solving equations of motion in the plane $(x', y')$ which inside the crystal is orthogonal to the current direction of the $\langle 110\rangle$ axis and coincides with $(x, y)$ plane when particle impinges on the crysral [2, 11]:

$$\ddot{x}' = -\frac{c^2}{E}\frac{\partial}{\partial x'}U(x', y') - \frac{v_z^2}{R} + f_{i,x}$$
$$\ddot{y}' = -\frac{c^2}{E}\frac{\partial}{\partial y'}U(x', y') + f_{i,y}, \quad (4)$$

where $E$ and $v$ are the energy and velocity of the particle, respectively, $f_{i,x}$ and $f_{i,y}$ are summands that correspond to incoherent scattering (scattering on atomic thermal vibrations, electrons, etc.).

In [12] it was shown that without an account of incoherent scattering stochastic deflection of particle beam takes place if

$$\overline{\psi^2} = \frac{lL}{R^2} \leq \psi_m^2, \quad (5)$$

where $l$ is the mean free path of the particle between consequent collisions with atomic strings, $L$ is the path traversed by the particle in the crystal, $R$ is the radius of curvature of the crystal, $\overline{\psi^2}$ is the value of a square of the angle between the particle momentum and current direction of the crystal axis, averaged over the beam, $\psi_m$ is the maximal value of $\psi$ for which particle take part in stochastic deflection ($\psi_m \sim \psi_c$). Experiments aimed at testing the possibility of using the stochastic mechanism for deflecting the direction of motion of high-energy charged particles have shown that this criterion works well for positively charged particles. At the same time, for negatively charged particles it is important to take into account incoherent scattering, since in the stochastic deflection regime they approach the atomic strings at closer distances than positively charged particles.

[1] the term ”far” here means distances that significantly exceed the lattice constant.

The influence of incoherent scattering on stochastic deflection of high-energy charged particles was taken into account in [8]. It was shown that condition (5) with account of incoherent scattering could be written as

$$\overline{\psi^2} = \frac{lL}{R^2} + \overline{\psi_i^2} \le \psi_m^2, \tag{6}$$

where $\overline{\psi_i^2}$ is the mean square angle of incoherent scattering. Conditions (5) and (6) were obtained for a simple approximation of the potential of an atomic string $U_{str}(\rho) = U_0\,(a/\rho)^2$, where $U_0$ and $a$ are constants.

In an amorphous medium $\overline{\psi_i^2} \propto L/E^2$ [13], thus by analogy we assume that in Si crystal $\overline{\psi_i^2} = \zeta L/E^2$. From Eq. (6) one could obtain the value of the bending angle of the crystal $\alpha_{st}$, up to which particles will be deflected by a bent crystal, in the next form:

$$\alpha_{st} = \frac{\psi_m^2}{l/R + \zeta R/E^2}. \tag{7}$$

From Eq. (7) we see that if $\zeta \ne 0$ the dependence of $\alpha_{st}$ from the radius of curvature of the crystal has a maximum at some radius of curvature, that in [8] was denoted as the optimal radius of curvature $R_{opt}$. Let us now consider the dependence of the optimal radius of curvature from the energy of high-energy charged particles.

From Eq. (7) we can obtain the value of the optimal radius of curvature as $R_{opt} = E\sqrt{l/\zeta}$. In the approximation $U_{str}(\rho) = U_0\left(\frac{a}{\rho}\right)^2$ one could obtain that $l \approx \frac{1}{4nda}\sqrt{\frac{E}{U_0}}$, where $n$ is the concentration of atoms in the crystal. Thus $R_{opt} \propto E^{5/4}$. Because of $\psi_m \sim \psi_c \propto E^{-1/2}$, we can obtain the dependence of the maximal value of $\alpha_{st}$ from the energy of particles as

$$\max(\alpha_{st}) = \frac{\psi_m^2}{l/R_{opt} + \zeta R_{opt}/E^2} \propto E^{-1/4}. \tag{8}$$

The dependence (8) was obtained for a simple approximation of atomic string potential $U_{str}(\rho) = U_0\left(\frac{a}{\rho}\right)^2$. For consideration of the dependence of the optimal radius of curvature and the maximal deflection angle from the energy of particles we carried out a numerical simulation of $\pi^-$-mesons motion in more realistic potential (3) of Si atomic strings that are parallel to the axis $\langle 110 \rangle$. The simulation code was the same as in Refs. [14, 15]. The code solves the equation of motion in the field of continuous atomic string potential through numerical integration of equations of motion (4). It takes into account the incoherent scattering on thermal vibrations of atoms and scattering on electrons. Other kinds of incoherent scattering were not taken into account considering the small crystal thickness.

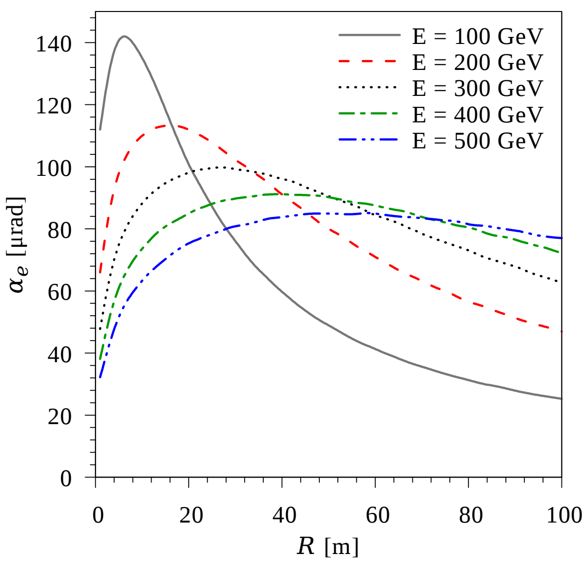


***Fig.2.*** *The dependence of the angle of the crystal bend $\alpha_e$ at which the number of $\pi^-$-mesons in the stochastic deflection regime decreases by a factor of $e$ on the radius of curvature $R$.*

To analyze the dependence of efficiency of stochastic deflection on the radius of curvature of the crystal we shown in Fig. 2 the dependence of the angle of the crystal bend $\alpha_e$, at which the number of $\pi^-$-mesons in the stochastic deflection regime decreases by a factor of $e$, on the radius of curvature. Before impinging on the crystal the beam of $\pi^-$-mesons had no angular divergence. To obtain Fig. 2 we assumed that $\psi_m = 1.5\psi_c$, because the angular acceptance of stochastic deflection for negatively charged particles is $\approx 1.5\psi_c$ (see Fig. 2 in [8] and a description of it in the text). Each of the five curves in Fig. 2 was built from two hundred of points and each of these points is a result of simulation of $5 \times 10^4$ $\pi^-$-mesons motion in the bent crystal with the radius of curvature $R$. From simulation for each of two hundred of values of $R$ in the range from zero to 100 m we obtained the length at which the number of $\pi^-$-mesons in the stochastic deflection regime decreases by a factor of $e$. Then we divided this length by the radius of curvature and thus obtained $\alpha_e$. Each of the five curves corresponds to different energy of particles. Gray solid curve corresponds to $\pi^-$-mesons with an energy of 100 GeV, red dashed curve – 200 GeV, black dotted curve – 300 GeV, green dash-dotted curve – 400 GeV and blue dash-double-dotted curve – 500 GeV. From Fig. 2 we see that with increasing particle energy the value of the optimal radius of curvature, which corresponds to a maximum of $\alpha_e$, increases while the value of maximal $\alpha_e$ decreases.

For a better understanding of the dependence of the optimal radius of curvature on the energy of $\pi^-$-mesons in the beam we plotted in Fig. 3 the values of $R_{opt}$, obtained from the simulation. In addition to the points that correspond to the maxima of the curves shown in Fig. 2, in Fig. 3 the values of the optimal radius of curvature for higher energies are also shown. Solid curve in Fig. 3 is a fit of points, obtained from the simulation, by the function $f_R(E) = k_R E^{5/4}$. The best agreement with obtained points is achieved with $k_R \approx 130$ [m/TeV]. In Fig. 3 we see that the dependence $E^{5/4}$ successfully describes the dependence of the optimal radius of curvature from the energy of particles in the beam not only for a simple potential of atomic string but also for the Doyle-Turner potential (1).

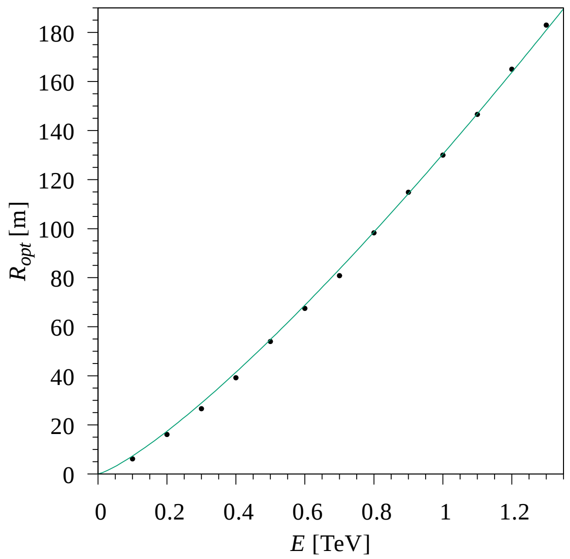


***Fig.3.*** *The dependence of the optimal radius of curvature on the particle energy.*

In the theory of channeling the critical angle of axial channeling $\psi_c$ is one of the main parameters. This parameter, for example, determines the angular acceptance of all three main beam deflection mechanisms: planar channeling, volume reflection and stochastic deflection. In the case of the volume reflection the deflection angle is proportional to $\psi_c$ [6] and thus decreases with particle energy increase as $E^{-1/2}$. In Fig. 2 we saw that with increasing particle energy $\alpha_e$ decreases. This therefore means that the maximal deflection angle achievable with stochastic deflection also decreases with increasing particle energy. Let us however consider the dependence of the ratio of $\alpha_e$ to the critical angle of axial channeling on the particle energy. This dependence is shown in Fig. 4. Points correspond to values of $\alpha_e$ obtained from the simulation. As it was written above, for the simple approximation of atomic string potential we obtained the dependence of $\max(\alpha_{st})$ on the particle energy as $E^{-1/4}$, thus $\max(\alpha_{st})/\psi_c \propto E^{1/4}$. That is why for fitting the data shown in Fig. 4 we used the function $f_\alpha(E) = k_\alpha E^{1/4} + C$. The additive constant $C$ corresponds to the fact that $\alpha_e$ shows the deflection angle of the crystal at which the number of particle in the stochastic deflection regime decreases by factor of $e$, but if the beam impinge on the crystal parallel to the crystal axis, most of particles take part in stochastic deflection at least until the ratio of crystal thickness to the radius of curvature is less than $\psi_c$. The solid curve shows the best fit of data by the function $f_\alpha(E)$. This fit corresponds to $k_\alpha \approx 3.75$[TeV$^{-1}$] and $C \approx 1$.

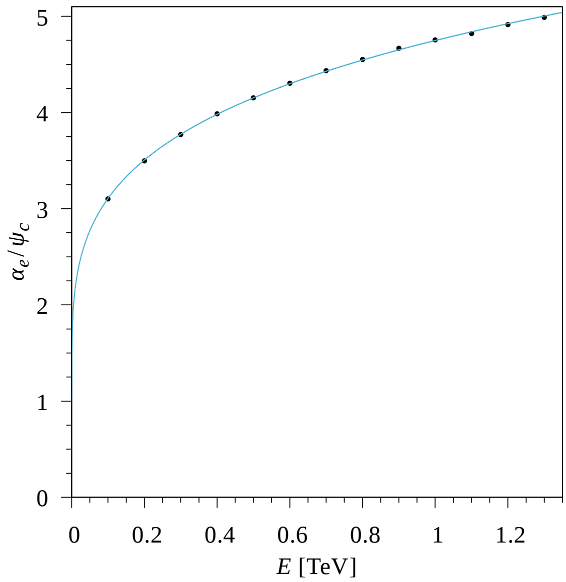


***Fig.4.*** *The dependence of the ratio of $\alpha_e$ to the critical angle of axial channeling on the particle energy.*

In Fig. 4 we see that the dependence $E^{1/4}$ successfully describes the dependence of $\alpha_e$ (and thus the maximal deflection angle) from the energy of particles in the beam not only for a simple potential of atomic string but also for the Doyle-Turner potential (1). This fact shows that, for example, in comparison with the volume reflection the deflection angle in the case of stochastic deflection increases as $E^{1/4}$ with particle energy increase.

## 3. CONCLUSIONS

The research made in the present work allowed to analyze the dependence of the optimal radius of curvature and maximal deflection angle that is achievable with a help of the stochastic deflection mechanism on the energy of charged particles. The analysis was done in the example of $\pi^-$-mesons and it shown that with increasing particle energy the ratio of the maximal deflection angle to the critical angle of axial channeling increases as $E^{1/4}$ while the optimal radius of curvature increases as $E^{5/4}$.

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## О ЗАВИСИМОСТИ ЭФФЕКТИВНОСТИ СТОХАСТИЧЕСКОГО МЕХАНИЗМА ОТКЛОНЕНИЯ ПУЧКА ЗАРЯЖЕННЫХ ЧАСТИЦ ИЗОГНУТЫМ КРИСТАЛЛОМ ОТ ЭНЕРГИИ ЧАСТИЦ

***И. В. Кириллин***

При помощи аналитических расчётов и численного моделирования была рассмотрена задача о стохастическом отклонении высокоэнергетических заряженных частиц изогнутым кристаллом. Было показано, что с ростом энергии заряженных частиц максимальный угол отклонения, достижимый при использовании стохастического механизма отклонения, уменьшается как $E^{-1/4}$, в то время как оптимальный радиус кривизны кристалла, соответствующий этому максимальному углу отклонения, растёт как $E^{5/4}$.

## ПРО ЗАЛЕЖНІСТЬ ЕФЕКТИВНОСТІ СТОХАСТИЧНОГО МЕХАНІЗМУ ВІДХИЛЕННЯ ПУЧКА ЗАРЯДЖЕНИХ ЧАСТИНОК ЗІГНУТИМ КРИСТАЛОМ ВІД ЕНЕРГІЇ ЧАСТИНОК

***І. В. Кириллін***

За допомогою аналітичних розрахунків і числового моделювання була розглянута задача про стохастичне відхилення високоенергетичних заряджених частинок зігнутим кристалом. Було показано, що з ростом енергії заряджених частинок максимальний кут відхилення, який є досяжним при використанні стохастичного механізму відхилення, зменшується як $E^{-1/4}$, в той час як оптимальний радіус викривлення кристалу, що відповідає цьому максимальному куту відхилення, зростає як $E^{5/4}$.