POLYHEDRONS AT THE NUCLEAR STRUCTURE

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The space disposition of nucleons at the light nuclei is given with the help of polyhedrons. At many cases it is the collection of embedded each other regular polyhedrons with nucleons at vertexes. But at the construction one meets other polyhedrons also. We give verification of construction by extremal properties of regular polyhedrons.

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1. INTRODUCTION

At the work one extends the consideration of space disposition of nucleons at light nuclei which begin at the work [1]. This question close connects with regular and almost semi-regular polyhedrons. Under construction we use data from the book [3, 4]. At framework of unlimited nuclear matter from [3] we consider the possibility of application of the polyhedral theory to calculation of nuclear spectrum. The construction of space nucleon disposition justified by extremal properties of regular polyhedrons.

2. THE NUCLEAR CONSTRUCTIONS OF Ca^{40} AND Fe^{56}

Chemical elements Ca^{40} and Fe^{56} have special places between other one. At the L.Aller book [2] on the page 254, figure 21 or at the I.P.Selinov book [4] on the Table III a the graphs of abundance of the elements in the Solar system on these elements have spade. It mean that these elements have larger abundance between close to it. The nucleus of Ca^{40} is twice "magic": it has 20 protons and 20 neutrons. On the nucleus of Fe^{56} the nuclear synthesis is almost finished. (As the Table III a shows that synthesis is going to Zn inclusively).

At the H.A.Bethe book [3] on the page 138 the shell structure is given for nucleus of Ca^{40}. Every shell 0s and 1s has 2 protons, the shell 0p has 6 protons and the shell 0d has 10 protons. From another side on the figure 25 of [3] the graphs of density spreading of protons and separately of neutrons almost coincide with each other.Hence we can suppose that the spreading of neutrons by shells is same as for protons, t.e. the shells 0s and 1s have two neutrons, the shell 0p has 6 neutrons and the shell 0d has 10 neutrons.Summing the numbers of protons and neutrons at every shell we obtain the sequence of shells with the following numbers of nucleons: 4,4,12,20.

Supposing that nucleons at every shell lie at the vertexes of regular polyhedrons (see chapter 6) we obtain the following sequence of polyhedrons which included each other with common center O:

$$\textbf{tetrahedron} \subset \textbf{tetrahedron} \subset \textbf{icosahedron} \subset \textbf{dodecahedron}. \quad (1)$$

The another possible construction we obtain by union of two first shells

$$\textbf{cube} \subset \textbf{icosahedron} \subset \textbf{dodecahedron}. \quad (2)$$

Remark that icosahedron and dodecahedron are dual figures. Therefore these two polyhedrons will take the symmetric positions relatively each other if the vertexes of icosahedron project from the center O at the centers of the dodecahedron faces. This precise position of icosahedron relatively dodecahedron gives the possibility to indicate all connections between vertexes of these figures: the icosahedron vertex connects with those vertexes of dodecahedron, which belong to face where lies the projection of the considering icosahedron vertex.

At the construction (1) the tetrahedron positions can be by two kinds: 1) the vertexes of inner tetrahedron project at vertexes of exterior tetrahedron, 2) the vertexes of inner tetrahedron project at the faces centers second one.

Remark that a tetrahedron is geometrical realization of α- particle. But under some extremal conditions as we suppose it can has form of two triangles with common side. Under this assuming in the article [1] we give the constructions of a icosahedron and dodecahedron by union of α- particles without intersections.

Keeping in the mind the α-particle construction we can write for Ca^{40} the following expansion

$$(2 + 3 + 5) \cdot 4 = 40. \quad (3)$$

Pass now to consideration of Fe^{56}. This nucleus has 26 protons and 30 neutrons. Suppose that nucleus is composed by union of α-particles and write the following expansion

$$(2 + 3 + 4 + 5) \cdot 4 = 56. \quad (4)$$

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The numbers in the round brackets denote the numbers of α-particles at shells which participate at construction of the nucleus. Hence, at construction of nucleus of Fe^{56} at compare with the nucleus of Ca^{40} appears one new shell with 4 α-particles. We put at correspondence to that shell some polyhedron with 16 vertexes, which we denote Λ_{16}. It is possible to construct polyhedron Λ_{16} with 10 faces at forms of squares and 8 isosceles triangles which close to equilateral triangles. Therefore, we can say that Λ_{16} is almost semi-regular. By analogy with (1) we give the following construction for nucleus of Fe^{56}

$$\textbf{tetrahedron} \subset \textbf{tetrahedron} \subset \textbf{icosahedron} \subset \boldsymbol{\Lambda_{16}} \subset \textbf{dodecahedron}. \quad (5)$$

But there is one important peculiarity at constructions of nucleus of Fe^{56}. If that nucleus was been forms precisely by all α-particles then the number of it protons will be 28. But the atom number of this element is 26. Hence, at the process of construction two protons were converted into two neutrons. It possible due to proton radiation of positron β^+.

Consider the formation process of the nucleus of Fe^{56}. It is natural to suppose that it was been forms non by exterior association α-particles with nucleus of Ca^{40}, because between icosahedron and dodecahedron must lie the polyhedron Λ_{16}. Therefore address to previous chemical elements. Let us consider the nucleus of ${}_{18}Ar^{36}$. According to Table III a ${}_{18}Ar^{36}$ has larger abundance that another isotopes. We can write the expansion for this element similar (3)

$$(2+3+4)\cdot 4 = 36. \quad (6)$$

The correspondent sequence of polyhedrons for ${}_{18}Ar^{36}$ is following

$$\textit{tetrahedron} \subset \textit{tetrahedron} \subset \textit{icosahedron} \subset \Lambda_{16}.$$

Consequently we have the beginning of nucleus Fe^{56} construction. Later with the help of α-particle addition one forms exterior cover as dodecahedron. At first we have the reaction

$${}_{18}Ar^{36} + 3{}_2He^4 \to_{22} Ti^{48} + 2\beta^+.$$

Exactly at this reaction 2 protons converted into 2 neutrons. The question why it take place at that moment is remain open. Later we have

$${}_{22}Ti^{48} +_2 He^4 \to_{24} Cr^{52},$$

$${}_{24}Cr^{52} +_2 He^4 \to_{26} Fe^{56}.$$

Hence the nucleus of Fe^{56} can be form from the nucleus of ${}_{18}Ar^{36}$ by sequential associations with α-particles.

At the conclusion of that chapter we can formulate the supposition that larger abundance of elements Ca^{40} and Fe^{56} at comparison with nearest elements justified by the circumstance that exterior shell of their nuclei is regular polyhedron, i.e. dodecahedron.

3. THE SEQUENCE OF LIGHT NUCLEI WITH EVEN NUMBERS OF PROTONS

Write the chain of nuclear constructions of chemical elements by sequential associations of α-particles. One can suppose that the nucleus of nonstable isotope of ${}_4Be^8$ has form: **tetrahedron** $\subset$ **tetrahedron** or **cube**. Later

$$3{}_2He^4 \to_6 C^{12},$$

icosahedron,

$${}_6C^{12} +_2 He^4 \to_8 O^{16}, \quad (1+3)\cdot 4 = 16,$$

tetrahedron $\subset$ **icosahedron**,

$${}_8O^{16} +_2 He^4 \to_{10} Ne^{20}, \quad (2+3)\cdot 4 = 20,$$

tetrahedron $\subset$ **tetrahedron** $\subset$ **icosahedron**

or **dodecahedron**,

$${}_{10}Ne^{20} +_2 He^4 \to_{12} Mg^{24}, \quad (1+2+3)\cdot 4 = 24,$$

tetrahedron $\subset$ **cube** $\subset$ **icosahedron**,

$${}_{12}Mg^{24} +_2 He^4 \to_{14} Si^{28}, \quad (2+5)\cdot 4 = 28,$$

cube $\subset$ **dodecahedron**,

$${}_{14}Si^{28} +_2 He^4 \to_{16} S^{32}, \quad (3+5)\cdot 4 = 32,$$

icosahedron $\subset$ **dodecahedron**,

$${}_{16}S^{32} +_2 He^4 \to_{18} Ar^{36}, \quad (2+3+4)\cdot 4 = 36.$$

Remark, that elements O^{16}, Ne^{20}, Si^{28} and S^{32} have almost same abundance as Fe^{56}. At that time Mg^{24} and Ar^{36} have same abundance as Ca^{40}. It is interesting to note on the Table III a from the I.P.Selinov book [4], 1990, the abundance of Ar^{36} is considerable larger that Ar^{40} , although at many early books the abundance of Ar^{40} is on the first place.

Can we continue the sequence in the round brackets in (4) ? Really , we can prolong it with indication of corresponding elements

$$(1+2+3+4+5)\cdot 4 = 60 \to Ni^{60},$$

$$(1+2+3+4+5+6)\cdot 4 = 84 \to Kr^{84},$$

$$(2+3+4+5+6+7)\cdot 4 = 108 \to Ag^{108}.$$

Later the sequence has prolongation, but some indetermination appears which connects with existence of numerous isotopes.

4. THE SEQUENCE OF LIGHT NUCLEI WITH ODD NUMBERS OF PROTONS

Remark, that at [1] we give representation for nucleus of ${}_3Li^7$ at form of octahedron with one neutron at the center.Such representation gives symmetric disposition for all nucleons.

Consider the sequence ${}_3Li^7 + n{}_2He^4$. We have

$${}_3Li^7 +_2 He^4 \to_5 B^{11},$$

$${}_5B^{11} +_2 He^4 \to_7 N^{15},$$

$$_7N^{15} +_2 He^4 \to_9 F^{19},$$
$$_9F^{19} +_2 He^4 \to_{11} Na^{23},$$
$$_{11}Na^{23} +_2 He^4 \to_{13} Al^{27},$$
$$_{13}Al^{27} +_2 He^4 \to_{15} P^{31},$$
$$_{15}P^{31} +_2 He^4 \to_{17} Cl^{35},$$
$$_{17}Cl^{35} +_2 He^4 \to_{19} K^{39},$$
$$_{19}K^{39} +_2 He^4 \to_{21} Sc^{43}.$$

At that sequence are represented isotopes which have larger abundance, except elements N and Sc , for which the largest abundance have $_7N^{14}$ and $_{21}Sc^{45}$. The curve of the abundance of chemical elements has form of saw with cogs directed up. The tops of cogs correspond to elements with even atomic numbers. And abysses correspond to elements with odd numbers. Write two lines:

$$_5B^{11},_7 N^{15},_9 F^{19},_{11} Na^{23},_{13} Al^{27},_{15} P^{31},_{17} Cl^{35};$$
$$_6C^{12},_8 O^{16},_{10} Ne^{20},_{12} Mg^{24},_{14} Si^{28},_{16} S^{32},_{18} Ar^{36}.$$

We see that nucleus at the first line has on one proton less than nucleus on the second line, which stand under it. The number of neutrons is same. Therefore, we can suppose that from exterior shell of nucleus from the first line is taken one proton. Under this operation the stability of shell is disordered and the abundance of elements falls. For example, the abundance of C^{12} is enough high, but for near standing element B^{11} it is exceptionally low.

However it is difficult to imaginer the removal reaction of one proton from nucleus. Therefore let us consider the formation of light nuclei by join of α-particles.

We can represent the construction of nucleus of $_3Li^7$ at form of octahedron with one neutron at the center. It is symmetric construction with one neutron at distinguished place.

The addition to nucleus of $_3Li^7$ of two α- particles gives the nucleus of $_7N^{15}$, to which corresponds the following construction

n $\subset$ octahedron $\subset$ cube.

The addition of three α-particles gives the nucleus of $_9F^{19}$ and the possible construction

n $\subset$ octahedron $\subset$ icosahedron.

The addition of 4 α-particles to nucleus of $_9F^{19}$ gives nucleus of $_{17}Cl^{35}$ and the construction

n $\subset$ octahedron $\subset$ icosahedron
$\subset$ polyhedron with 16- vertexes.

Then the addition to nucleus of $_9F^{19}$ of five α-particles gives the nucleus of $_{19}K^{39}$ with the following construction

n $\subset$ octahedron $\subset$ icosahedron
$\subset$ dodecahedron.

We remark that the element $_{19}K^{39}$ has almost same abundance as element $_{20}Ca^{40}$.

Thus the the difference at this construction of nuclei with odd numbers of protons consists at using of octahedron with neutron at the center at beginning of construction.

At the book [2] on the p. 348 author write that "the formation problem of visible quantity of light elements Be, Li, B remains as principal unsolvable difficulty of modern theory of element origin". On these elements the curve of abundance falls deeply down, that tails on singularity of its origin. At that time for the next elements with odd numbers of protons in nuclei the corresponding points on the curve are close to points of elements with even numbers of protons. The second deep cavity of the curve lies between points of Ca^{40} and Fe^{56}.

5. POLYHEDRONS AND SPECTRUM OF NUCLEI AT FRAMEWORK OF THE NUCLEAR MATTER THEORY

The density of heavy nuclei ρ is approximately constant and at the center $\rho = 0,17\frac{nucl.}{fm^3}$. At the H.Bothe's book [3] one considers the theory on nuclear matter which infinitely stretched and has constant density. For the volume energy E_v of such matter included at some domain with volume V will be $E_v = \rho V$.

Consider the polyhedron M as convex hull of all nucleons centers of nucleus. By analogy we can construct convex hull of inner shell also. If all faces of M are triangles then by I.Sabitov's theorem [6] the volume $V(M)$ is a root of some polynomial equation

$$V^k + a_{k-1}V^{k-1} + ... = 0, \qquad (7)$$

where the coefficients a_i are determined by squares of edge lengths. If some face is non triangle , then we divide it on triangles by diagonals of this face. The equation (6) has some number of real roots $V_1, V_2, ...$ Assume that M with volume V_i transforms under emanation or irradiation with preservation of edge lengths into polyhedron with volume V_j. Then the corresponding change of energy

$$E_{vi} - E_{vj} = \rho(V_i - V_j)$$

will give some spectral lines of emanation or new excited state of nucleus under irradiation. If the length of edges change and the polyhedron M_1 transforms to M_2 then we consider the difference

$$E_v(M_1) - E_v(M_2) = \rho(V_1 - V_2).$$

For example at the work [7] some cases of volume calculations are given for octahedron type polyhedrons. For this case the equation (6) is following

$$V^{16} + a_1(l^2)V^{14} + ... + a_8(l^2) = 0,$$

where every summand has form $a_i(l^2)V^{16-2i}$ and l^2 denotes the edge squares. The number of monomials for calculation of a_i can be very large (order 10^{10}). But if M has many edges with equal lengths then calculation will be possible.

6. EXTREMAL PROPERTIES OF REGULAR POLYHEDRONS

It is well known that for figures in 3-dimensional Euclidean space (convex or non convex) the following inequality has place

$$F^3 \geq 36\pi V^2,$$

where F is the surface area of the figure and V is its volume.The equality can be for a ball and only at this case.

There are many remarkable and difficult works devoted to extremal properties of regular polyhedrons at some class of convex polyhedrons. Enough detailed exposition of this question is given at the L.F.Tóth's book [8] . For some kind of polyhedrons one considers the fraction

$$\frac{F^3}{V^2}$$

and looks for a polyhedron of given kind for which this fraction is smallest. That polyhedron is called best. Lhuilier proved that between polyhedrons of the tetrahedron kind the best is tetrahedron.J. Steiner in [9] formulated the supposition that between polyhedrons which have type of some regular polyhedron the best is the regular polyhedron. He proved his supposition for kind of octahedron. Then M.Goldberg in [10] for polyhedrons with n faces discovered and partially proved the inequality

$$\frac{F^3}{V^2} \geq 54(n-2)tg\omega_n(\sin^2\omega_n - 1),$$

where $\omega_n = \frac{n}{n-2} \cdot \frac{\pi}{6}$ and equality has place only for regular polyhedrons which angles have 3 faces.

From here it follows that cube and dodecahedron are best. The complete proof was been given by L.F.Tóth at [11, 12]. The case of icosahedron don't have proof. But L.F.Toth told that probably for convex polyhedron with n vertexes the following inequality has place

$$\frac{F^3}{V^2} \geq \frac{27\sqrt{3}}{2}(n-2)(3tg^2\omega_n - 1),$$

and equality can be only for regular polyhedron with triangle faces.

With volume of figure one can connect the volume energy $E_v = \rho V$, where ρ is the density of nuclear matter, and the surface area F corresponds the surface energy E_f, (see [13],p 48,133 and 136). By formula of H.Bothe-C.F.Weizsäcker the nucleus energy is given by following expression

$$M(A,Z)c^2 = Zm_pc^2 + Nm_nc^2 - a_vA + a_fA^{2/3} +$$

$$a_c\frac{Z^2}{A^{1/3}} + a_a\frac{(N-Z)^2}{A},$$

where m_p, m_n are the proton and neutron masses, the third member is volume energy, the fourth member is surface energy, the fifth member is Coulomb's energy and the last member is symmetry energy. Here a_v, a_f, a_c, a_a are positive constants. For the light nuclei the last two members are very small, and we reject it.

The equilibrium state of a nucleus (under constant volume) determined by minimum of the free energy of nucleus. It means that in the considering case surface energy achieves minimum and the surface area F also. Therefore for equilibrium state of nucleus the fraction $\frac{F^3}{V^2}$ will have the minimal value among considering kind (and with given number of vertexes) of polyhedrons. By stated above it leads to regular polyhedrons.

Under some natural suppositions the combinator kind of a polyhedron can be reconstructed by the number of vertexes. Remember that by well known hypothesis every nucleon contains 3 quarks. Suppose that every quark from nucleon can be connect with one other nucleon of the nucleus. At our polyhedron this connection corresponds to one edge which is going from the vertex. So from every vertex can go out three edges.

Consider, for example, the polyhedron with 20 vertexes from every which are going 3 edges. Hence the number of all edges is $\frac{3\cdot 20}{2} = 30$. By Euler formula we obtain the number m of faces: $20 - 30 + m = 2$. Consequently, the number of faces is $m = 12$.Suppose that all faces are same type polygons with l sides. Calculating the number of all edges of the polyhedron we obtain $\frac{l\cdot 12}{2} = 30$, that is $l = 5$. So, we have polyhedron with 20 vertexes, 30 edges and 12 faces every which is a pentagon.

Draw the image of the polyhedron on a plane begin from some pentagon. At first adjoin to it 5 pentagons and then again 5 pentagons keeping the condition that from every vertex 3 sides are going out. Exterior 5 sides of the last pentagons form the boundary of exterior domain which is image of the last face. Hence we have on the drawing image of $1+5+5+1 = 12$ faces. But such image the dodecahedron has also. Hence our polyhedron has combinator kind of a dodecahedron.

Similar consideration can be given for polyhedrons with 4 or 8 vertexes.

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МНОГОГРАННИКИ В СТРУКТУРЕ АТОМНЫХ ЯДЕР

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Пространственное расположение нуклонов в легких ядрах описано с помощью многогранников. Во многих случаях это есть набор вложенных друг в друга правильных многогранников с нуклонами в вершинах. Но в этой конструкции тоже можно заметить и другие многогранники. Мы приводим обоснование конструкции с помощью экстремальных свойств правильных многогранников.

БАГАТОГРАННИКИ В СТРУКТУРІ АТОМНИХ ЯДЕР

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Розташування нуклонів легких ядрах у просторі описано за допомогою багатогранників. У значній кількості випадків це є множина вкладених одного в інший правильних багатогранників з нуклонами у вершинах. Але в цій конструкції теж можливо помітити і інші багатогранники. Мы приводимо обгрунтування конструкції за допомогою екстремальних властивостей багатогранників.