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Rayleigh light scattering on the director static inhomogeneities in filled flexoelectric nematic crystal

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Director equilibrium configurations are obtained in the neighborhood of spherical and cylindrical impure particles in flexoelectric nematic liquid crystal under external electric field. Weak homeotropic and circular director anchoring with particle surface has been considered. Expressions obtained for the director are used for numerical calculations of the angle distribution of the light scattering cross-section on the director structure inhomogeneities in the Rayleigh-Gan's approximation. All possible kinds of the light scattering are considered and dependence of cross-section on value of flexopolarization, external field and kind of the director anchoring is studied. In particular, it is shown that scattering cross-section essentially changes in the presence of flexopolarization.

Получены равновесные конфигурации директора в окрестности сферической и цилиндрической примесных частиц во флексоэлектрическом нематическом жидком кристалле в присутствии внешнего электрического поля. Рассмотрены слабые гомеотропное и циркулярное сцепления директора с поверхностью частиц. Полученные выражения для директора использованы для численного расчета в приближении Релея-Ганса углового распределения сечения рассеяния света на неоднородностях структуры директора. Рассмотрены все возможные типы рассеяния и исследована зависимость сечения от величины флексополяризации, внешнего поля, характера сцепления директора. В частности, показано, что наличие флексополяризации в жидком кристалле приводит к существенному изменению сечения рассеяния.

Heterogeneous systems act the important part in electro-optical applications. At present time only the polymer-dispersed liquid crystals(LC) which contain the LC droplets in polymer matrix have been sufficiently investigated [1, 2]. Another interesting type of the heterogeneous LC, the so-called filled nematics, consist of the mixture of the nematic liquid crystal (NLC) and aerosil (small impurity silicium particles of diameter $\sim$ 100 – 1000 Å) [3-7]. Impurity particles create the inhomogeneous director distribution around them. The character of the director field disturbance is substantially determined by the director anchoring with the impurity particles surface. In the case of weak director anchoring the nematic director field distribution was studied near the particles of spherical [8, 9] and cylindrical [10] forms. At the strong director anchoring such defects of the director structure as disclinations can appear near the particles [9, 11, 12]. Static inhomogeneities of the director field induced by impurities cause the light scattering additional to the scattering by the thermal director fluctuations. Such scattering was studied in the Rayleigh-Gans approximation [13, 14] and anomalous diffraction approximation [15, 16], but in the absence of flexoelectricity in LC.

In present paper we study the equilibrium configurations of the director field in flexoelectric LC which contains the impurity particles of spherical and cylindrical form under external electric field in approximation of weak director anchoring with the particles surface. Light scattering on the director field structure inhomogeneities which appear in this case is studied in the Rayleigh-Gans approximation.

NLC free energy and equations for the director. Let we have the homeotropic cell of the flexoelectric NLC with infinitely rigid director anchoring at the cell surfaces under external stationary homogeneous electric field $\mathbf{E}$, directed along the undisturbed nematic director $\mathbf{n}_0$. The cell contains the spherical (or cylindrical) particle of radius R. Free energy of NLC which contains the impurity particle under electric field can be written in the form

$$F = F_{el} + F_E + F_d + F_S\,,$$

$$F_{el} = \frac{K}{2}\int\limits_V \left[(\mathrm{div}\,\mathbf{n})^2 + (\mathrm{rot}\,\mathbf{n})^2\right] dV - \frac{K_{24}}{2}\int\limits_V \mathrm{div}\Big(\mathbf{n}\,\mathrm{div}\,\mathbf{n} + [\mathbf{n}\times\mathrm{rot}\,\mathbf{n}]\Big)\, dV,$$

$$F_E = -\frac{\varepsilon_a^0}{8\pi}\int\limits_V (\mathbf{n}\cdot\mathbf{E})^2\, dV, \qquad (1)$$

$$F_d = -\int\limits_V \Big\{e_1(\mathbf{n}\cdot\mathbf{E})\,\mathrm{div}\,\mathbf{n} + e_3\left([\mathrm{rot}\,\mathbf{n}\times\mathbf{n}]\cdot\mathbf{E}\right)\Big\}\, dV,$$

$$F_S = -\frac{W}{2}\int\limits_S (\mathbf{n}\cdot\mathbf{e})^2\, dS.$$

Here $\mathbf{n}$ is the director, F_{el} is the Frank's elastic energy written in the one constant approximation (but $K \neq K_{24}$), F_E, F_d are anisotropic and flexoelectric parts of interaction energy of NLC with electric field, correspondingly, F_S is contribution to the free energy from the nematic director interaction with the particle surface taken in the form of Rapini potential [17], K, K_{24} are the elastic constants, $\varepsilon_a^0 = \varepsilon_{\|}^0 - \varepsilon_{\perp}^0 > 0$ is the static permittivity anisotropy, e_1, e_3 are the flexoelectric coefficients, W is the energy of director anchoring with the particle surface, $\mathbf{e}$ is the unit vector along the director easy axis at the particle surface.

Minimizing the free energy (1) by $\mathbf{n}$, and taking into account the condition $\mathbf{n}^2 = 1$, we can get the Euler-Lagrange stationary equation for the director

$$\begin{gathered} K(\Delta\mathbf{n} - (\mathbf{n}\Delta\mathbf{n})\mathbf{n}) + \frac{\varepsilon_a^0}{4\pi}\Big((\mathbf{nE})\mathbf{E} - (\mathbf{nE})^2\mathbf{n}\Big) + \\ +(e_1 - e_3)\left(\big(\mathbf{E} - (\mathbf{nE})\mathbf{n}\big)\mathrm{div}\,\mathbf{n} - \nabla(\mathbf{nE}) + \mathbf{n}(\mathbf{n}\nabla)(\mathbf{nE})\right) = 0 \end{gathered} \qquad (2)$$

and boundary conditions at the particle surface

$$\begin{gathered} (K - K_{24})\left[\Big(1 - (\mathbf{ne}_\tau)^2\Big)\mathrm{div}\,\mathbf{n} + (\mathbf{ne}_\tau)\Big(\mathbf{n}\,[\mathbf{e}_\tau\times\mathrm{rot}\,\mathbf{n}]\Big)\right] + K_{24}\Big[\mathbf{e}_\tau(\mathbf{e}_\tau\nabla)\mathbf{n} - (\mathbf{ne}_\tau)\mathbf{n}(\mathbf{e}_\tau\nabla)\mathbf{n}\Big] - \\ -W(\mathbf{ne})(\mathbf{ee}_\tau) + W(\mathbf{ne})^2(\mathbf{ne}_\tau) - e_1(\mathbf{nE})\Big(1 - (\mathbf{ne}_\tau)^2\Big) - e_3(\mathbf{ne}_\tau)\Big(\mathbf{n}\big[\mathbf{e}_\tau\times[\mathbf{n}\times\mathbf{E}]\big]\Big) = 0, \end{gathered} \qquad (3)$$

where $\mathbf{e}_\tau$ is a unit vector along the external normal to the particle surface.

We shall consider the only case of weak nematic anchoring with the particle surface so that the parameter $\xi = WR/K \ll 1$.

Director field around the spherical particle. Let we have the spherical particle. We put the Cartesian frame of reference in the particle center and direct the axis Oz along the undisturbed director. The azimuth symmetry of the system allow us to take the director in the form

$$\mathbf{n} = \mathbf{n}_0 + \mathbf{n}_\perp = \mathbf{e}_z + n_\perp(r,\theta)\left[\mathbf{e}_x\cos\varphi + \mathbf{e}_y\sin\varphi\right], \quad |n_\perp(r,\theta)| \ll 1, \qquad (4)$$

where $\mathbf{n}_\perp$ is the component of the vector $\mathbf{n}$ perpendicular to the vector $\mathbf{n}_0$; r, θ, φ are the spherical coordinates of the LC point with respect to the particle center.

Then leaving in (2) only terms linear in $\mathbf{n}_\perp$ we get equation

$$\Delta\mathbf{n}_\perp - \frac{\varepsilon_a^0 E^2}{4\pi K}\mathbf{n}_\perp = 0, \qquad (5)$$

which in spherical coordinates takes the next form

$$x^2\frac{\partial^2 n_\perp}{\partial x^2}+2x\frac{\partial n_\perp}{\partial x}+\frac{\partial^2 n_\perp}{\partial \theta^2}+\operatorname{ctg}\theta\,\frac{\partial n_\perp}{\partial \theta}-\frac{n_\perp}{\sin^2\theta}-x^2 n_\perp=0. \tag{6}$$

Putting $\mathbf{n}_\perp$ and ξ to be small values of the same order the boundary condition (3) can be written in the form

$$\left[x\frac{\partial n_\perp}{\partial x}+(1-k_{24})n_\perp+\left(\frac{2}{\sqrt{\nu_1}}+\frac{1}{\sqrt{\nu_3}}\right)xn_\perp\cos\theta\pm\frac{\xi}{2}\sin 2\theta-\frac{x\sin\theta}{\sqrt{\nu_1}}\right]_{x=x_E}=0. \tag{7}$$

Here and further the upper sign corresponds to director homeotropic anchoring with the particle surface ($\mathbf{e}=\mathbf{e}_r$) and the lower one corresponds to the circular anchoring ($\mathbf{e}=\mathbf{e}_\theta$); here the next dimensionless values are introduced: $k_{24}=\dfrac{K_{24}}{K}$, $x=\dfrac{r}{l_E}$, $x_E=\dfrac{R}{l_E}$, $\nu_i=\dfrac{\varepsilon_a^0 K}{4\pi e_i^2}$, where $\mathbf{e}_r$, $\mathbf{e}_\theta$ are the unit vectors of spherical system of reference, $l_E=\sqrt{\dfrac{4\pi K}{\varepsilon_a^0 E^2}}$ is the electric coherence length, $i=1,3$.

Finite at $x\to\infty$ solution to equation (6) takes a form

$$n_\perp(x,\theta)=\sum_{m=1}^{\infty}A_m\sqrt{\frac{\pi}{2x}}K_{m+1/2}(x)P_m^1(\cos\theta), \tag{8}$$

where $K_{m+1/2}(x)$ is the modified spherical Bessel function [18], $P_m^1(x)$ is the associated Legendre polynomial [18], A_m are the coefficients of general solution.

Substituting expression (8) into boundary condition (7) and putting parameters $\nu_i\gg 1$ ($i=1,3$), we obtain that at $m\geqslant 3$ the coefficients $A_m=0$. Then solution in the cases of the director homeotropic and circular anchoring with the spherical particle surface takes a form

$$n_\perp(r,\theta)=\chi_1(r)\sin\theta+\chi_2(r)\sin 2\theta, \tag{9}$$

where

$$\chi_1(r)=\frac{x_E}{\sqrt{\nu_1}}\sqrt{\frac{R}{r}}\frac{K_{3/2}(r/l_E)}{x_E K'_{3/2}(x_E)+(0.5-k_{24})K_{3/2}(x_E)},$$

$$\chi_2(r)=\mp\frac{\xi}{2}\sqrt{\frac{R}{r}}\frac{K_{5/2}(r/l_E)}{x_E K'_{5/2}(x_E)+(0.5-k_{24})K_{5/2}(x_E)}.$$

Here a prime at modified spherical Bessel function indicates the derivative with respect to argument.

Formula (9) together with formula (4) give us the director field distribution in the NLC bulk with impurity particle. Note, that in the case of the director homeotropic anchoring with the particle surface in the absence of flexopolarization ($\nu_i=\infty$) obtained above the director field distribution coincides with the result of paper [8].

Director field around the cylindrical particle Let the impurity particle is cylindrical and its axis $\mathbf{l}$ is perpendicular to the undisturbed nematic director $\mathbf{n}_0$. We put the particle length L to be much more than its radius R, so that we can neglect boundary effects at the cylindrical bases. Let the Ox axis is directed along the undisturbed director and the Oz axis coincides with the particle axis. Due to the homogeneity of the system along the Oz axis the disturbance of the director field around the particle will be plane for both the homeotropic ($\mathbf{e}=\mathbf{e}_\rho$) and circular ($\mathbf{e}=\mathbf{e}_\varphi$) director anchoring with the cylindrical particle side surface:

$$\mathbf{n}=\mathbf{n}_0+\mathbf{n}_\perp=\mathbf{e}_x\cdot\cos\Phi(\rho,\varphi)+\mathbf{e}_y\cdot\sin\Phi(\rho,\varphi), \tag{10}$$

where Φ is the director deviation angle from its undisturbed direction, ρ, φ are the polar coordinates of point in the plane xOy, $\mathbf{e}_\rho$, $\mathbf{e}_\varphi$ are the unit vectors of the polar system of reference.

Linearized in $\mathbf{n}_\perp$ equation (2) for the director coincides with equation (5). Introducing the dimensionless length $x = \rho/l_E$ and using the polar system of reference we can write equation in the form

$$x^2\frac{\partial^2\Phi}{\partial x^2} + x\frac{\partial\Phi}{\partial x} + \frac{\partial^2\Phi}{\partial\varphi^2} - x^2\Phi = 0. \quad (11)$$

To the values of the first order in $\mathbf{n}_\perp$ and ξ, boundary condition (3) takes a form

$$\left[x\frac{\partial\Phi}{\partial x} \pm \frac{\xi}{2}\sin 2\varphi - \frac{1}{\sqrt{\nu_1}}x\sin\varphi + \left(\frac{1}{\sqrt{\nu_1}} + \frac{1}{\sqrt{\nu_3}}\right)x\Phi\cos\varphi\right]_{x=x_E} = 0, \quad (12)$$

where the upper sign corresponds to the director homeotropic anchoring with the particle side surface and the lower one corresponds to the circular anchoring.

Solution to equation (11), taking into account its periodicity $\Phi(x,\varphi+2\pi) = \Phi(x,\varphi)$, symmetry $\Phi(x,-\varphi) = -\Phi(x,\varphi)$ and finiteness at $x \to \infty$, takes the following form

$$\Phi(x,\varphi) = \sum_{m=1}^{\infty} B_m K_m(x)\sin m\varphi, \quad (13)$$

where $K_m(x)$ are the modified Bessel function [18], B_m are the coefficients of general solution.

Substituting obtained solution into boundary condition (12) at the flexoelectric parameters values $\nu_i \gg 1$ $(i = 1,3)$ we get that

$$\Phi(\rho,\varphi) = \mp\frac{\xi}{2x_E}\frac{K_2(\rho/l_E)}{K_2'(x_E)}\sin 2\varphi + \frac{1}{\sqrt{\nu_1}}\frac{K_1(\rho/l_E)}{K_1'(x_E)}\sin\varphi. \quad (14)$$

Here a prime at modified Bessel function indicates the derivative with respect to argument.

Formulas (10) and (14) describe the director field distribution around the cylindrical particle. In the absence of flexopolarization the obtained director field distribution coincides with the result of paper [12].

Let now the cylindrical particle axis $\mathbf{l}$ is parallel to the undisturbed director $\mathbf{n}_0$. If the director anchoring with the particle side surface is circular then putting the Oz axis along the particle axis one can write the director in the form

$$\mathbf{n} = -\mathbf{e}_x\cdot\sin\varphi\sin\Omega(\rho) + \mathbf{e}_y\cdot\cos\varphi\sin\Omega(\rho) + \mathbf{e}_z\cdot\cos\Omega(\rho), \quad (15)$$

where $\Omega(\rho)$ is the angle between the director $\mathbf{n}$ and the positive direction of the Oz axis.

It is easy to check that in this case the flexoelectric contribution F_d to the NLC free energy (1)is absent so that the director field distribution around the cylindrical particle can be described by expression obtained before in paper [12].

In the case of the director homeotropic anchoring with the particle side surface one can present the director in the form

$$\mathbf{n} = \mathbf{e}_x\cdot\cos\varphi\sin\Omega(\rho) + \mathbf{e}_y\cdot\sin\varphi\sin\Omega(\rho) + \mathbf{e}_z\cdot\cos\Omega(\rho). \quad (16)$$

In this case it follows from (2), (3) that dimensionless equation for the director and boundary condition take the form

$$x^2\frac{d^2\Omega}{dx^2} + x\frac{d\Omega}{dx} - \frac{1}{2}(1+x^2)\sin 2\Omega = 0, \quad (17)$$

$$\left[x\frac{d\Omega}{dx} + \frac{1}{2}(1 - k_{24} + \xi)\sin 2\Omega + \left(\frac{1}{\sqrt{\nu_1}} + \frac{1}{\sqrt{\nu_3}}\right)x\sin^2\Omega - \frac{x}{\sqrt{\nu_1}}\right]_{x=x_E} = 0. \quad (18)$$

Solution to equations (17), (18) takes a form

$$\Omega(\rho) = \frac{K_1(\rho/l_E)}{K_1(x_E)}\left\{\sqrt{\frac{3}{2}\left(1 + \frac{x_E}{1-k_{24}+\xi}\cdot\frac{K_1'(x_E)}{K_1(x_E)}\right)} + \frac{3x_E}{4(1-k_{24}+\xi)}\times\right.$$
$$\left.\times\left[\frac{1}{\sqrt{\nu_1}} + \frac{1}{\sqrt{\nu_3}} - \frac{1}{\sqrt{\nu_1}}\frac{1}{\frac{3}{2}\left(1 + \frac{x_E}{1-k_{24}+\xi}\frac{K_1'(x_E)}{K_1(x_E)}\right)}\right]\right\}. \quad (19)$$

Formulas (16) and (19) describe the director field distribution in the NLC bulk with cylindrical particle if its axis is parallel to the undisturbed director. In the absence of flexopolarization the director distribution coincides with that obtained in paper [12].

Light scattering cross-section. Let us consider the nematic LC-matrix containing impurity particles of spherical or cylindrical form in the field of plane monochromatic light wave of frequency ω incident on LC along the undisturbed director $\mathbf{n}_0$. In the Rayleigh-Gans approximation the differential light scattering cross-section on the director field inhomogeneities caused by impurities can be written in the following form [19]

$$\frac{d\sigma}{d\Omega} = \frac{\omega^4}{16\pi^2 c^4} \left|\mathbf{i}\,\hat{\varepsilon}(\mathbf{q})\,\mathbf{f}\right|^2, \tag{20}$$

where

$$\hat{\varepsilon}(\mathbf{q}) = \int\limits_V \left(\hat{\varepsilon}(\mathbf{r}) - \hat{\varepsilon}^0\right) \exp(-i\mathbf{q}\mathbf{r})\, dV.$$

Here $\mathbf{i}$, $\mathbf{f}$ are the unit polarization vectors of incident and scattered light waves with wave vectors $\mathbf{k}$, $\mathbf{k}'$ correspondingly, $\mathbf{q} = \mathbf{k}' - \mathbf{k}$ is the scattering vector, $\hat{\varepsilon}^0$ is the permittivity tensor of the homogeneous nematic, $\hat{\varepsilon}(\mathbf{r})$ is the permittivity tensor of the impurity nematic. Integration in (20) is carrying out over all the volume excluding the the own volume of impurities which we consider to be small and neglect its contribution.

Putting the concentration of impurity particles small so that the areas of the director deformation created by different particles do not overlap, we can write

$$\hat{\varepsilon}(\mathbf{r}) = \hat{\varepsilon}^0 + \sum_{m=1}^{N} \delta\hat{\varepsilon}(\mathbf{r} - \mathbf{r}_m, \Psi_m)\Theta(d - |\mathbf{r} - \mathbf{r}_m|), \tag{21}$$

where $\mathbf{r}_m$ is the radius-vector of the m impurity particle and summation is carrying out over all N particles, $\delta\hat{\varepsilon}(\mathbf{r} - \mathbf{r}_m, \Psi_m)$ is a change of the permittivity tensor of the homogeneous nematic caused by the director field deformation due to the m particle (it was calculated in previous sections), Ψ_m describes the axis orientation of the m cylindrical particle, $\Theta(x)$ is the Heaviside function, d is an average distance between particles.

Let we have the filled nematic with spherical impurity particles. Substituting (21) into (20) and using representation of the pair correlation function $g(\mathbf{r})$ for positions of the particles centers of mass [20]

$$\frac{1}{N} \sum_{\substack{m,m' \\ m\neq m'}} \exp[-i\mathbf{q}(\mathbf{r}_m - \mathbf{r}_{m'})] = \frac{N}{V} \int\limits_V g(\mathbf{r}) \exp(-i\mathbf{q}\mathbf{r})\, dV = \frac{N}{V} g(\mathbf{q}), \tag{22}$$

expression for the differential light scattering cross-section takes the form

$$\frac{d\sigma}{d\Omega} = \frac{N k^4}{16\pi^2} \left|\mathbf{i}\,\delta\hat{\varepsilon}(\mathbf{q})\,\mathbf{f}\right|^2 S(\mathbf{q}), \tag{23}$$

where $k = \dfrac{\omega}{c}$, $\delta\hat{\varepsilon}(\mathbf{q})$ is a Fourier transform of the tensor $\delta\hat{\varepsilon}(\mathbf{r})$, $S(\mathbf{q}) = 1 + \dfrac{N}{V} g(\mathbf{q})$ is a structure factor calculated in the Percus-Yevick approximation [21].

Substituting in (23) expression for $\delta\hat{\varepsilon}(\mathbf{q})$ calculated with the help of formula (9)one can show that only the scattering cross-sections of the $e - e$ and $o - o$ types do not equal zero. Obtained numerically dependence of the dimensionless differential light scattering cross-section of the $e - e$ type with change of polarization under external electric field $\sigma' = \dfrac{1}{NR^2}\dfrac{d\sigma}{d\Omega}$ versus the scattering angle ϑ is shown on Fig.1 for different values of parameter x_E. At calculation we put parameter $kR = 1$ and flexoelectric parameter $\nu_1 = 100$ that corresponds to the values of flexoelectric coefficients e_1 and e_3 from the range $(0.1 - 2.5) \cdot 10^{-4}$ дин$^{1/2}$ [22]. Dependence of the light scattering cross-section on the scattering angle is described by nonmonotonic function and at that the scattering on the large angles ($\vartheta \sim 1$)dominates.

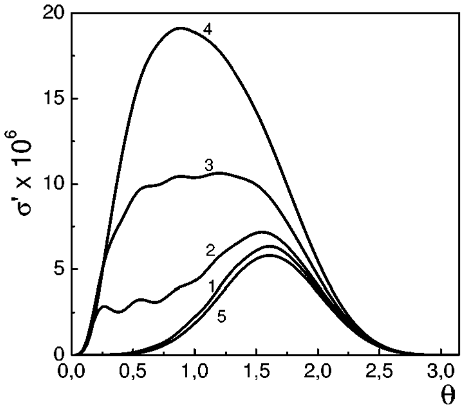

Fig.1. Light scattering cross-section σ' $e-e$ versus the scattering angle ϑ in the case of the spherical impurity particles at fixed value of flexoelectric parameter $\nu_1 = 100$ and different values of external electric field: $x_E = 0$ – 1, 0.1 – 2, 0.2 – 3, 0.3 – 4, 5 (in the absence of flexopolarization).

Despite of flexopolarization the light scattering cross-section does not depend on the kind (homeotropic or circular) of the director anchoring with the particle surface. At the absence of flexopolarization the increase of external electric field (parameter x_E) leads to the decrease of the light scattering cross-section (curve 5) [14]. In presence of flexopolarization the scattering cross-section increases with decrease of flexoelectric parameter ν_1 and increase of external electric field for all values of parameter kR. In last case the maximums of scattering are shifted to the side of the smaller angles.

If the light scattering of the $o-o$ type without change of the polarization direction takes place the light scattering cross-section does not depend on the type of the director anchoring with the particle surface similar to that in the previous case. At all values of kR this type of scattering depends on flexoelectric parameter ν_1 and external electric field in the same way as considered above scattering of the $e-e$ type. But as our calculations show the differential cross-section of the $o-o$ scattering is of the four orders smaller than the cross-section of the $e-e$ scattering.

We consider the case of cylindrical particles which axes are perpendicular to the undisturbed director ($\mathbf{l} \perp \mathbf{n}_0$). Substituting (21) into (20) and carrying out some simple calculations one can get

$$\frac{d\sigma}{d\Omega} = \frac{Nk^4}{16\pi^2}\left(\langle|\mathbf{i}\,\delta\hat{\varepsilon}(\mathbf{q},\psi)\,\mathbf{f}|^2\rangle + |\langle\mathbf{i}\,\delta\hat{\varepsilon}(\mathbf{q},\psi)\,\mathbf{f}\rangle|^2\,(S(\mathbf{q})-1)\right), \tag{24}$$

where ψ is an angle describing the particle axis orientation in the plane which is perpendicular to $\mathbf{n}_0$, the angle brackets $\langle\ldots\rangle$ denote an averaging over orientations of the cylindrical particles.

In the case of the $e-e$ light scattering the calculated dependence of the σ' on the scattering angle ϑ is presented in the Fig.2a for values of parameters $kR = 1$ and $x_E = 0.1$. At calculations we put $L/R = 10$. Scattering on the great angles dominate at all values of the parameter kR. In the absence of flexoelectricity, as in the case of spherical particles, the scattering cross-section does not depend on the type of the director anchoring with the cylindrical particle side surface and monotonically decreases with increase of external electric field. But in the presence of flexoelectricity the type of the director anchoring with the cylindrical particle side surface significantly influences the character of the angular dependence of the light scattering cross-section which at that is qualitatively similar for all types of scattering. As it is known, the light scattering cross-section value is determined by the director inhomogeneous distribution area. As one can see from formula (14) the presence of flexopolarization leads to the increase of the director field disturbance area created by cylindrical particle at the director circular anchoring with the particle surface and, on the contrary, to the decrease of that area at the homeotropic anchoring. Therefore, in the case of the director circular anchoring with the cylindrical particle side surface the scattering cross-section value is greater in the presence of flexopolarization and at that it monotonically increases with increase of electric field for all values of the parameter kR. In the case of the homeotropic director anchoring with

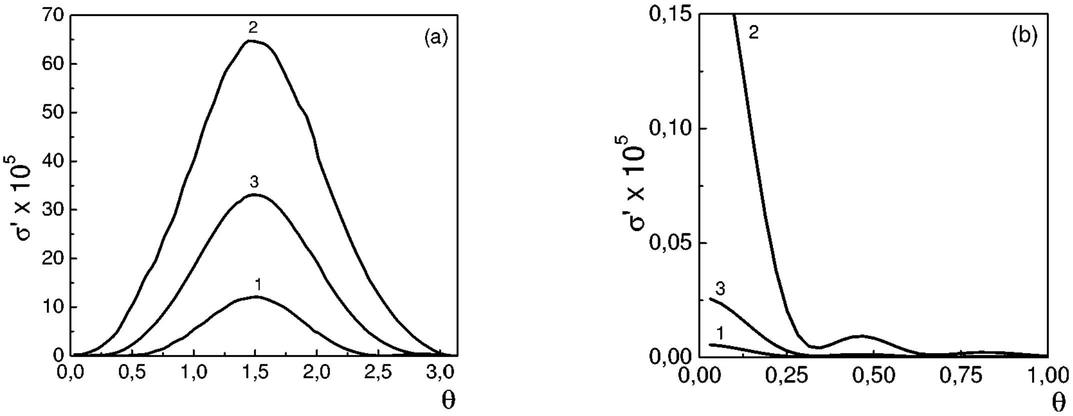

Fig.2. Cross-section σ' of scattering of the $e-e$ (a) and $o-o$ (b) types versus the scattering angle ϑ in the case of the cylindrical impurity particles(the particle axis perpendicular to $\mathbf{n}_0$) at fixed values of external electric field ($x_E = 0.1$) and flexoparameter ($\nu_1 = 100$): 1 – the director homeotropic anchoring, 2 – the circular anchoring, 3 – The homeotropic and circular anchoring in the absence of flexopolarization.

the particle surface the scattering cross-section is smaller in the presence of flexoelectricity but increases with increase of external electric field.

Results of numerical calculation of the σ' versus ϑ are shown in Fig.2b in the case of the scattering of the $o-o$ type. As one can see, for all values of the parameter kR the small angle scattering ($\vartheta < 0.25$, which is determined by the director disturbance at great distances to the particle center, dominates. According to expressions (9) and (14)the director disturbance around cylindrical particle decreases with distance to particle more slow than in the case of spherical particle of the same volume which determines infinitesimality of the cross-section value of the $o-o$ scattering for spherical particles. Differential light scattering cross-section of the $o-o$ type is of the 2-3 orders smaller than the cross-section of the $e-e$ scattering.

The light scattering cross-section of the $e-o$ type with change of polarization has dependence on the angle scattering similar to the small angle scattering of the $o-o$ type and is equal to him by the order of value. As calculations show the $o-e$ scattering repeats the character of the angle dependence of the $e-e$ scattering but is smaller by the one order of value.

As one can see, in the case of the cylindrical particles the scattering cross-section does not equal zero for all four types of scattering at that the $e-e$ scattering appears to be the most strong. Note, that the light scattering cross-section on the unit volume of the scattering particle is approximately of the same value for both the spherical and cylindrical impurity particles despite of the presence of flexoelectricity.

In the case of the cylindrical impurity particles with axes parallel to the undisturbed director the differential light scattering cross-section is determined by expression (23). As was shown above, at the director circular anchoring with the particle side surface the presence of flexoelectricity does not influence the scattering properties of the filled nematics. The last were studied before in papers [13, 14]. Numerical dependence of the scattering cross-section σ' of the $o-o$ type versus the scattering angle ϑ is presented in Fig.3 in the case of the homeotropic anchoring for different values of external electric field obtained at following values of parameters $kR = 1$, $\nu_1 = 50$, $\nu_3 = 100$. In contrast to the case of the particle axis perpendicular to the undisturbed director the small angle scattering cross-section monotonically decreases with increase of external electric field for all values of parameter kR despite of the presence of flexopolarization.

Under (23), as calculations show, the cross-section of the $e-e$ scattering also does not equal zero. This type of scattering depends on the scattering angle similar to that of the small angle scattering of the $o-o$ type and is equal to the last by the order of value.

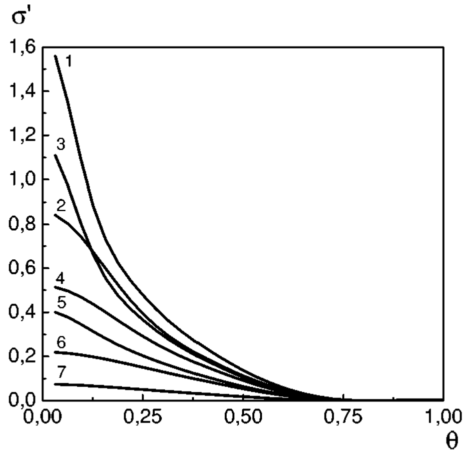

Fig.3. Cross-section σ' of scattering of the $o-o$ type versus the scattering angle ϑ in the case of the cylindrical impurity particles(the particle axis parallel to $\mathbf{n}_0$) at different values of external electric field: $x_E = 0$ – 1; 0.01 – 2, 3; 0.05 – 4, 5; 0.1 – 6, 7; 2, 4, 6 – in the absence of flexopolarization; 3, 5, 7 – in the presence of flexopolarization($\nu_1 = 50, \nu_3 = 100$).

Using experimental data on light scattering and obtained here dependence of the differential scattering cross-section σ on the scattering angle ϑ one can estimate the flexoelectric coefficients e_1, e_3 НЖКК, the director anchoring energy with the particle surface W, and also the nematic elastic constants K и K_{24}.

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Релеєвське розсіяння світла на статичних неоднорідностях директора в наповненому флексоелектричному нематичному кристалі

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Знайдені рівноважні конфігурації директора в околі сферичної і циліндричної домішкових частинок в флексоелектричному нематичному рідкому кристалі в присутності зовнішнього електричного поля. Розглянуті слабі гомеотропне і циркулярне зчеплення директора з поверхнею частинок. Отримані вирази для директора використані для чисельного розрахунку в наближенні Релея-Ганса кутового розподілу перерізу розсіяння світла на неоднорідностях структури директора. Розглянуті всі можливі типи розсіяння і досліджена залежність перерізу від величини флексополяризації, зовнішнього поля та характеру зчеплення директора. Зокрема, показано, що наявність флексополяризації в рідкому кристалі приводить до суттєвої зміни перерізу розсіяння.