

Hysteresis of light induced Freedericksz transition in nematic cell with finite anchoring energy

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Influence of the finiteness of nematic director anchoring energy on hysteresis of light-induced Fredeericksz transition under the light beam with bounded cross size was considered. Values of Freedericksz transition threshold and the director deviation angle jumps with increasing and decreasing of incident light intensity were numerically obtained in dependence on the director anchoring energy magnitude and the light beam cross size. Possible regions of the light beam widths and anchoring energy values at which the Freedericksz transition is accompanied by hysteresis were determined.

Рассмотрено влияние конечности энергии сцепления директора нематика на гистерезис светоиндуцированного перехода Фредерикса в световом пучке с ограниченным поперечным размером. Численно получены значения порогов перехода Фредерикса и скачков угла отклонения директора при возрастании и убывании интенсивности падающего света в зависимости от величины энергии сцепления директора и поперечного размера светового пучка. Определены области допустимых ширин световых пучков и значений энергии сцепления, при которых переход Фредерикса сопровождается гистерезисом.

1. Introduction

It is of wide knowledge that in the cells of nematic liquid crystals (NLCs) the external light fields can cause the threshold reorientation of director – the light induced Freedericksz transition (LIFT) [1, 2], which has been widely used recently in a variety of opto-electronic devices. This phenomenon and the prospects of its applications have been discussed by a number of researchers [3-15].

In spite of the fact that the light induced director reorientation in NLCs is a bulk effect, its main characteristics, such as threshold field and the degree of director reorientation depend essentially on the strength and type of interaction between the nematic and the cell surface. So, the influence of the cell surface may be as significant as to cause Freedericksz transition which can be either spontaneous, or stimulated by changing surface conditions [16-18]. One of the most important parameters affecting the director is its anchoring energy with the surface. It has been shown in [19] that anchoring energy can be different at different deviations of the director from its easy axis both in azimuthal and polar directions. According to the experiments, the values of polar anchoring energy are in the range $10^{-3} - 1\,\mathrm{erg/cm^2}$, and the values of azimuthal anchoring energy – $10^{-5} - 10^{-1}$ erg/cm^2 [20, 21], which depends essentially on the direction of director's easy axis and the temperature. In particular, the polar and azimuthal anchoring energies for cell surfaces treated by polyimide with planar orientation of director's easy axis have been reported to equal $(0.6 - 2.2) \cdot 10^{-2}$ erg/cm^2 and $(1.5 - 5.0) \cdot 10^{-2}$ erg/cm^2 accordingly [22]. The methods of cell surface treatment yielding the desired values of anchoring energy are described in ref. [23-36]. The molecular structure of various polymer surfactants used to treat cell surface and to polarize liquid crystal molecules has also been found to influence the energy and the results have been discussed in [37-40] and [41] respectively.

In certain conditions, the threshold of director reorientation I_{th} found at increasing incident light intensity in NLCs differs from the threshold value I'_{th} at decreasing incident light intensity, i.e. LIFT is accompanied with hysteresis [1, 42, 43]. In this case, once the light intensity reaches its threshold values the director jumps from homogeneous to inhomogeneous state and visa versa, LIFT being the phase transition of the first order. LIFT hysteresis was observed experimentally in additional external magnetic [44] and quasistatic electrical fields in homeotropically [45] and planarly [46] oriented NLC cells. The optical multistability of the NLC cell in the field of linearly polarized radiation was reported in [8, 47]. The LIFT of the first order was experimentally studied in the nematic doped by dendrimer [48].

The parameters of LIFT hysteresis in case of unlimited light beams depend on how large the anchoring energy is between the nematic and the cell surface [42, 49]. However, if the beam cross section is smaller than the cell width, one can assume that the thresholds of director reorientation and the width of hysteresis loop should depend more strongly on the values of anchoring energy between the director and the cell surface than for the light beams of unlimited width. Studies of this kind reported for the case of unlimited light beams were aimed at analyzing the dependence of LIFT threshold on the finite values of the nematic anchoring energy [1, 50, 51]. However, they all were performed without taking into account LIFT hysteresis, which was also studied in the beam of finite cross section, but only for the cell with infinitely rigid boundary conditions on the surface [52].

To sum up, the influence of the value of energy anchoring the nematic and the cell surface on the LIFT hysteresis has been studied in infinitely wide light beams as well as in light beams with finite cross section. The latter study was limited for the cells with infinitely rigid boundary conditions. In our work we shall analyze how the finiteness of the energy anchoring the nematic and the cell surface influences the LIFT hysteresis parameters in the field of light beams with limited width. For simplicity we shall consider the case of beams limited by one dimension.

2. The equations describing NLC director

Let us assume that we have a flat parallel NLC cell plane-parallel restricted by the planes $z = 0$ and $z = L$ with initial homeotropic director orientation. The monochromatic light wave that is polarized linearly along Ox is incident normally on the cell in the direction of Oz axis. To avoid ambiguity we shall assume that the incident beam is limited along Oy axis and its intensity distributed throughout its cross section is described by the function

$$I(y) = I_0 \Theta(a - |y|), \tag{1}$$

where $\Theta(t) = 1$, if $t \geqslant 0$ and $\Theta(t) = 0$, if $t < 0$, $2a$ is the width of the light beam.

The free energy of NLC cell in the field of incident light wave can be described as follows:

$$
\begin{aligned}
&F = F_{el} + F_E + F_S, \\
&F_{el} = \frac{1}{2}\int\limits_V \left\{K_1 (\operatorname{div}\mathbf{n})^2 + K_2 (\mathbf{n}\cdot\operatorname{rot}\mathbf{n})^2 + K_3 [\mathbf{n}\times\operatorname{rot}\mathbf{n}]^2\right\} dV, \\
&F_E = -\frac{1}{16\pi}\int\limits_V \varepsilon_{ij} E_i E_j^* \, dV, \\
&F_S = -\frac{W}{2}\int\limits_S (\mathbf{n}\cdot\mathbf{e})^2 \, dS.
\end{aligned} \tag{2}
$$

In this set of equations F_{el} is the Frank elastic energy, F_E is the contribution of incident light electric field into the free energy of NLC [50], F_S is the nematic surface free energy taken as the Rapini potential [53], $\mathbf{n}$ is the director, K_1, K_2, K_3 are the nematic elastic constants, $\varepsilon_{ij} = \varepsilon_\perp \delta_{ij} + \varepsilon_a n_i n_j$ is the tensor of the nematic dielectric permittivity at the incident light frequency, $\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp > 0$, $\varepsilon_\parallel$, $\varepsilon_\perp$ are the respective parallel and perpendicular to the director tensor components of the homogeneous nematic dielectric permittivity, W is the energy anchoring the nematic with the cell surface, $\mathbf{e}$ is the unit vector of the director's easy orientation axis on the cell surfaces ($\mathbf{e} \parallel Oz$).

We shall restrict ourselves to the analysis of only planar distributions of the director [50]. Therefore, since the system is homogeneous along the axis Ox we shall derive the expression describing the director in the bulk NLC in the following way

$$
\mathbf{n} = \mathbf{e}_x \cdot \sin\varphi(y,z) + \mathbf{e}_z \cdot \cos\varphi(y,z), \tag{3}
$$

where $\mathbf{e}_x$, $\mathbf{e}_z$ are the unit vectors of Descartes coordinate system, φ is the angle of the director deviation from the initial homogeneous direction along Oz axis.

Minimizing the free energy (2) by the angle φ and solving Maxwell's equations for the electric field of the incident light wave in geometrical optics approximation we shall obtain the following stationary equation describing the director

$$
\begin{aligned}
&(1 - k\sin^2\varphi)\frac{\partial^2\varphi}{\partial z^2} + m\frac{\partial^2\varphi}{\partial y^2} - k\sin\varphi\cos\varphi\left(\frac{\partial\varphi}{\partial z}\right)^2 + \\
&+\frac{\pi^2}{L^2}\frac{I_0}{I_{Fr}}\frac{\varepsilon_\parallel^2 \sin\varphi\cos\varphi}{(\varepsilon_\perp + \varepsilon_a\cos^2\varphi)^{3/2}(\varepsilon_\perp + \varepsilon_a\cos^2\varphi_s)^{1/2}}\,\Theta(a - |y|) = 0\,,
\end{aligned} \tag{4}
$$

and its boundary conditions

$$
\begin{aligned}
&\left[(1 - k\sin^2\varphi)\frac{\partial\varphi}{\partial z} + \frac{\varepsilon}{2L}\sin 2\varphi\right]_{z=L} = 0, \\
&\left[(1 - k\sin^2\varphi)\frac{\partial\varphi}{\partial z} - \frac{\varepsilon}{2L}\sin 2\varphi\right]_{z=0} = 0.
\end{aligned} \tag{5}
$$

In this set of equations $k = \dfrac{K_3 - K_1}{K_3}$, $m = \dfrac{K_2}{K_3}$, $\varepsilon = \dfrac{WL}{K_3}$, $I_{Fr} = \dfrac{8\pi^3\varepsilon_\parallel K_3}{\varepsilon_a\varepsilon_\perp L^2}$ is the value of LIFT threshold in the field of homogeneous light beam of infinite width in the case of infinitely rigid anchoring of director with the cell surface [1], $\varphi_s = \varphi(y; z = 0, L)$ is the deviation angle of the director on the cell surfaces.

We shall assume the deviation angles of the director to be small near the threshold of orientational instablity. Let us denote the maximal possible deviation angle of the director inside the cell in the center of the light beam by φ_m, i.e. $\varphi_m = \varphi(y = 0, z = L/2)$. It is evident that φ_m values depend on the values of I_0. Let us analyze the inverse function $I_0(\varphi_m)$, which is unambiguous, and because of the small φ_m angle, we shall present it by the following series

$$I_0 = I_{Fr}\left(\rho + \sigma\varphi_m^2 + \tau\varphi_m^4 + o(\varphi_m^4)\right), \tag{6}$$

where ρ, σ, τ are the expansion coefficients. When expanding (6) we have taken into account that $I_0(-\varphi_m) = I_0(\varphi_m)$, since the sign of the deviation angle φ is determined by the director fluctuations and has no effect on the threshold intensity.

Equation (4) can also be solved by serial expansion over small φ_m:

$$\varphi(y,z) = A(y,z)\varphi_m + B(y,z)\varphi_m^3 + C(y,z)\varphi_m^5 + o(\varphi_m^5), \tag{7}$$

where A, B, C are unknown expansion coefficients. Series (7) contains only the odd powers of φ_m, since the change of sign for φ_m should result in the change of the sign for φ.

According to the maximal angle φ_m defined above, the expansion coefficients (7) should satisfy the following conditions in the middle of the cell in the center of light beam

$$A(y=0, z=L/2) = 1, \quad B(y=0, z=L/2) = C(y=0, z=L/2) = 0, \tag{8}$$

and because of the problem symmetry (by y and z coordinates) they should also satisfy the conditions

$$\begin{aligned} &A(y,z) = A(y, L-z), \quad B(y,z) = B(y, L-z), \quad C(y,z) = C(y, L-z), \\ &A(-y,z) = A(y,z), \qquad B(-y,z) = B(y,z), \qquad C(-y,z) = C(y,z). \end{aligned} \tag{9}$$

By substituting expansions (6), (7) into expressions (4), (5) and assuming the coefficients to equal zero at φ_m of corresponding power, we shall obtain the following linear differential equations with the boundary conditions to determine the unknown functions $A(y,z)$, $B(y,z)$ and $C(y,z)$ with the accuracy of the small terms of the order of φ_m^5:

$$A''_{zz} + mA''_{yy} + \frac{\pi^2}{L^2}\rho A\,\Theta(a - |y|) = 0, \tag{10}$$

$$\left[A'_z - \frac{\varepsilon}{L}A\right]_{z=0} = 0, \tag{11}$$

$$\begin{aligned} &B''_{zz} + mB''_{yy} + \frac{\pi^2}{L^2}\rho B\,\Theta(a - |y|) = \\ &= kA^2A''_{zz} + kA'^2_zA - \frac{\pi^2}{L^2}\left(\rho\alpha A^3 + \sigma A + \rho\beta_1 AA_S^2\right)\Theta(a - |y|), \end{aligned} \tag{12}$$

$$\left[B'_z - \frac{\varepsilon}{L}B\right]_{z=0} = \left[kA^2A'_z - \frac{2}{3}\frac{\varepsilon}{L}A^3\right]_{z=0} \tag{13}$$

$$\begin{aligned} &C''_{zz} + mC''_{yy} + \frac{\pi^2}{L^2}\rho C\,\Theta(a - |y|) = kA^2B''_{zz} + 2kABA''_{zz} - \\ &-\frac{k}{3}A^4A''_{zz} + kA'^2_zB - \frac{2}{3}kA'^2_zA^3 + 2kA'_zB'_zA - \\ &-\frac{\pi^2}{L^2}\left(3\rho\alpha A^2B + \rho\beta A^5 + \sigma B + \sigma\alpha A^3 + \tau A + \right. \\ &\left. + 2\rho\beta_1 AA_SB_S + \rho\beta_1 BA_S^2 + \rho\beta_1\alpha A^3A_S^2 + \rho\beta_2 AA_S^4 + \sigma\beta_1 AA_S^2\right)\Theta(a - |y|), \end{aligned} \tag{14}$$

$$\left[C_z' - \frac{\varepsilon}{L}C\right]_{z=0} = \left[kABA_z' + kA^2B_z' - \frac{k}{3}A^4A_z' - 2\frac{\varepsilon}{L}A^2B + \frac{2}{15}\frac{\varepsilon}{L}A^5\right]_{z=0}. \quad (15)$$

Here $\alpha = \frac{3}{2}\frac{\varepsilon_a}{\varepsilon_\parallel} - \frac{2}{3}$, $\beta = \frac{15}{8}\frac{\varepsilon_a^2}{\varepsilon_\parallel^2} - \frac{3}{2}\frac{\varepsilon_a}{\varepsilon_\parallel} + \frac{2}{15}$, $\beta_1 = \frac{1}{2}\frac{\varepsilon_a}{\varepsilon_\parallel}$, $\beta_2 = \frac{3}{8}\frac{\varepsilon_a^2}{\varepsilon_\parallel^2} - \frac{1}{6}\frac{\varepsilon_a}{\varepsilon_\parallel}$, $A_S = A(y, z = 0)$, $B_S = B(y, z = 0)$, by prime marks of A, B, C functions we denote partial derivatives with respect to corresponding arguments.

By using the method of separation of variables we derive the solutions of equation (10) for the region inside the light beam, $|y| \leqslant a$, and the region outside, $|y| > a$, that satisfy symmetry conditions (9) and boundary condition (11). Meeting the requirement that these solutions and their first-order derivatives should be continuous at the boundaries of these regions ($y = \pm a$) and that the solution should satisfy condition (8), we obtain

$$A(y,z) = \begin{cases} \cos(q_1 y)\cos\left[\mu_1\left(\frac{\pi}{2} - \frac{\pi z}{L}\right)\right], & \text{if} \quad |y| \leqslant a, \\ \cos(q_1 a)\exp\big(-\tilde{q}_1(|y| - a)\big)\cos\left[\mu_1\left(\frac{\pi}{2} - \frac{\pi z}{L}\right)\right], & \text{if} \quad |y| > a. \end{cases} \quad (16)$$

where $q_1 = \frac{\pi}{L}\sqrt{\frac{\rho - \mu_1^2}{m}}$, $\tilde{q}_1 = \frac{\pi\mu_1}{\sqrt{mL}}$, μ_1 is the smallest solution of the equation $\pi\mu_1 \operatorname{tg}(\pi\mu_1/2) = \varepsilon$, and expansion coefficient ρ (6) is determined from the equation $q_1 \operatorname{tg}(q_1 a) = \tilde{q}_1$.

Now we substitute the expression found for $A(y,z)$ (16) into system (12), (13). Restricting the solution of the system by $y \to \pm\infty$ and satisfying conditions (8), (9), we obtain the explicit expression for $B(y,z)$ function as well as the unknown expansion coefficient σ (6) in the following form

$$\sigma = \frac{2mL^2}{\pi^2}\left\{\int_0^a G(y)\big[\tilde{q}_1 \sin q_1(a-y) + q_1\cos q_1(a-y)\big]\,dy + \right. \\ \left. + q_1\int_a^\infty G(y)\exp\big[-\tilde{q}_1(y-a)\big]\,dy\right\} \Big/ \big(\tilde{q}_1 a\sin(q_1 a) + \sin(q_1 a) + q_1 a\cos(q_1 a)\big). \quad (17)$$

In the expression

$$G(y) = \frac{2\pi\mu_1}{mL\big(\pi\mu_1 + \sin(\pi\mu_1)\big)}\int_0^L \Big(kA^2A_{zz}'' + kAA_z'^2 - V_{zz}'' - mV_{yy}'' - \\ - \frac{\pi^2}{L^2}\rho\big(\alpha A^3 + \beta_1 AA_S^2 + V\big)\Theta(a - |y|)\Big)\cos\left[\mu_1\left(\frac{\pi}{2} - \frac{\pi z}{L}\right)\right]\,dz, \quad (18)$$

the function $V = V(y,z)$ has the following form

$$V(y,z) = \frac{\varepsilon}{\pi}\Big(k - \frac{2}{3}\Big)\cos^3\Big(\frac{\pi\mu_1}{2}\Big)\sin\Big(\frac{\pi z}{L}\Big)\,g(y), \quad (19)$$

where

$$g(y) = \begin{cases} \cos^3(q_1 y), & \text{if} \quad |y| \leqslant a, \\ \cos^3(q_1 a)\exp[-3\tilde{q}_1(|y| - a)], & \text{if} \quad |y| > a. \end{cases} \quad (20)$$

Solving systems (14), (15) in the similar way by taking into account the explicit forms of functions $A(y,z)$ and $B(y,z)$ we obtain the value of τ, the expression for which will not be considered here because of space limitation.

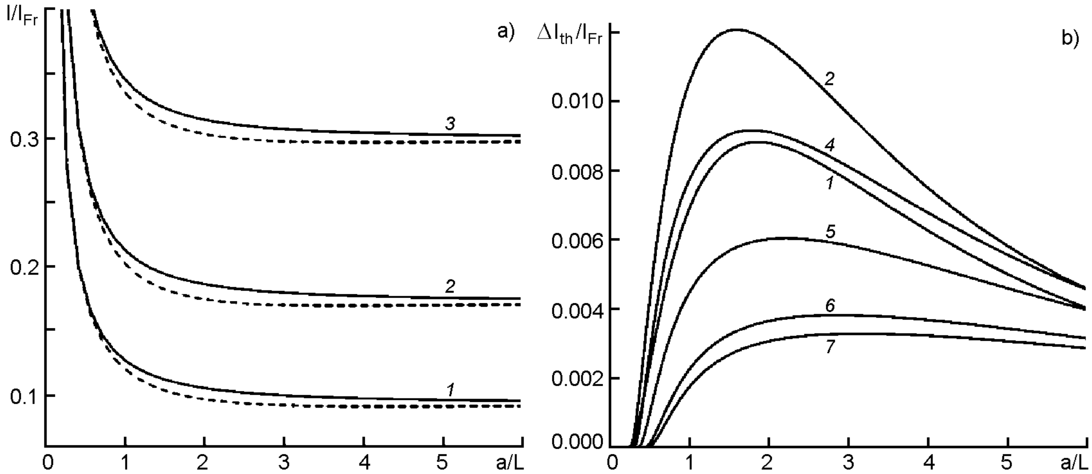

Fig. 1. The dependences of the LIFT threshold I_{th}/I_{Fr} (a) with the increase (solid lines), and decrease (dashed lines) of intensity for hysteresis loop width $\Delta I_{th}/I_{Fr}$ (b) on a/L ratio. ε=0.5 (*1*), 1 (*2*), 2 (*3*), 3 (*4*), 5 (*5*), 10 (*6*), 20 (*7*).

3. Freedericksz transition in light beams

Taking into account the addends to the order of φ_m^4 including in serial expansion (6), we find the solution of biquadratic equation obtained for φ_m

$$\varphi_m^2 = \left[-\sigma \pm \sqrt{\sigma^2 + 4\tau\left(I_0/I_{Fr} - \rho\right)}\right] \Big/ (2\tau), \tag{21}$$

were ρ, σ, τ parameters found above depend on the cross section of incident light beam, the magnitude of the anchoring energy binding the nematic with the cell surface, the cell thickness and NLC parameters.

On condition that $\sigma > 0$, when the intensity I_0 of the incident light reaches the threshold of orientation instability $I_{th} = \rho I_{Fr}$ the system continuously turns from homogeneous into inhomogeneous state, with the hysteresis being absent. This was the case considered both for the cells with infinite and finite anchoring energies [50, 51]. We shall analyze the case when $\sigma < 0$.

In case of infinitely wide light beams [42] when LIFT reaches the threshold $I_0 = I_{th}$ the system, according to equation (21), jumps from homogeneous ($\varphi_m = 0$) into inhomogeneous state with $\varphi_m = \sqrt{-\sigma/\tau}$. When the intensity I_0 of incident light decreases from the region of large I_{th} values the system jumps back from inhomogeneous state with $\varphi_m = \sqrt{-\sigma/(2\tau)}$ to initial homogeneous state at smaller threshold values $I_0 = I'_{th} < I_{th}$. The magnitude of the threshold for this inverse transition is derived, similarly to the way reported in [42], from the condition of non-negativeness of radical expression in (21) and consequently is equal to $I'_{th} = I_{Fr}\left(\rho - \sigma^2/(4\tau)\right)$. The difference between the threshold values of direct and inverse transitions represents the width of hysteresis loop $\Delta I_{th} = I_{th} - I'_{th} = I_{Fr}\,\sigma^2/(4\tau) > 0$. The expressions describing the parameters ρ, σ, τ have been obtained in the previous section and in general case only their numerical values can be found.

Fig. 1a shows the dependence of threshold values of LIFT at the increasing I_{th}/I_{Fr} and decreasing I'_{th}/I_{Fr} in the incident light intensity on the dimensionless half-width of light beam a/L for several values of anchoring energy ε, calculated for NLC parameters $k = 0.6$, $m = 0.3$, $\varepsilon_\parallel = 3.06$, $\varepsilon_\perp = 2.37$ which are close to the typical ones reported in [21, 54]. It is seen, that for all values of ε the value of both thresholds decreases monotoncally with the increase of a/L ratio. In the extreme case of infinitely wide light beam ($a/L \to \infty$) the threshold value I_{th} becomes constant and depends only on the anchoring energy ε with the increase in intensity $I_{th} = I_{Fr}(\mu/\pi)^2$ [51], where μ is the smallest positive root of equation $\mathrm{tg}\,\mu = 2\varepsilon\mu/(\mu^2 - \varepsilon^2)$. As the anchoring energy ε grows the threshold LIFT values increase monotonously with the increase I_{th} and decrease I'_{th} in the incident light intensity, and in the extreme case of absolutely rigid anchoring $\varepsilon \to \infty$ they become constant [52].

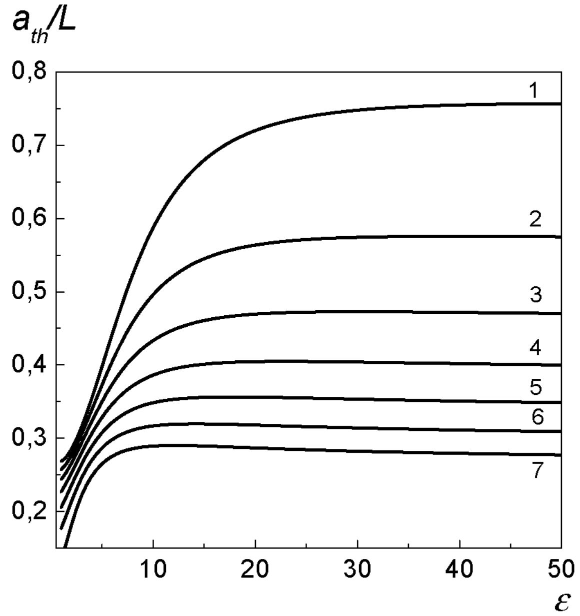

Fig. 2. The dependences of the critical light beam half-width a_{th}/L on the value of anchoring energy ε at $m = K_2/K_3 = 0.3$. $k = (K_3 - K_1)/K_3$=0.550 (*1*), 0.575 (*2*), 0.6 (*3*), 0.625 (*4*), 0.65 (*5*), 0.675 (*6*), 0.7 (*7*).

The dependence of hysteresis loop width ΔI_{th} on a/L ratio is nonmonotonic for all values of ε (see Fig. 1b). As the beam cross section becomes larger, ΔI_{th} first increases from 0 to a certain maximum value (corresponding to the light beam with the half-width of about the cell's thickness), and then decreases until in the extreme case of infinitely wide beam it becomes constant. As the anchoring energy grows the maxima on $\Delta I_{th}(a/L)$ curves shift to the region of larger values of a/L. For example, in the NLC cell (with $k = 0.6$, $m = 0.3$, $\varepsilon_{\|} = 3.06$, $\varepsilon_{\perp} = 2.37$ – the parameters' values which have been given above [21, 54]) of thickness $L = 250\,\mu m$ the maximum width of LIFT hysteresis loop at $\varepsilon = 1$ (curve *2* on Fig. 1) corresponding to incident light beam of half-width $a = 1.45L$ is equal to $1.64\,W/cm^2$, the LIFT threshold value being equal to $26.58\,W/cm^2$ when the intensity increases. It should be noted that in the experiments reported in [44] the threshold value was $130 - 170\,W/cm^2$ in the 5CB cell of $380\,\mu m$ thickness exposed to the field of argon laser beam ($\lambda = 514.5\,nm$) with the cross section of $0.9\,mm$ ($a/L = 1.18$). In the experiments reported [45] the $254\,\mu m$ thick cell from nematic mixture of ROTN-200 was radiated by light beam (of argon laser as well) with diameter $0.3\,mm$ ($a/L = 0.6$). The obtained threshold value was about $280\,W/cm^2$. In both reported experiments [44, 45] the hysteresis loop width did not exceed $4\,W/cm^2$, the anchoring of the director with the cell surface being assumed to be infinitely rigid.

As seen from Fig. 1b, for each given value of ε there is a finite critical value of the light beam half-width $a_{th}(k, m, \varepsilon)$ (i.e. the value of a corresponds to $\sigma = 0$). However, in the light beams with $a < a_{th}$ the LIFT is not accompanied with hysteresis, while in the beams with $a > a_{th}$ it is. Fig. 2 shows the critical values of the light beam half-width a_{th} versus the anchoring energy ε for different values of parameter k at the fixed values of m. As seen for Fig. 2, the critical values of a_{th} gradually grow with the growth of ε for $k \lesssim 0.6$. At $k \gtrsim 0.6$ this dependence becomes less monotonous and reaches practically constant value at $\varepsilon \gtrsim 30$. As the parameter k is growing the critical values of a_{th} gradually decrease. The calculations show that the critical values of light beam half-width a_{th} increase as the parameter m increases. Thus, for all the values of anchoring energy ε the increase of k parameter and decrease of m extend the range of LIFT hysteresis occurrence by the widths of incident light beam.

The dependences of the hysteresis loop width ΔI_{th} calculated versus anchoring energy ε for different a/L ratios are given in Fig. 3. As seen from the figure, the hysteresis loop is maximal at the anchoring energy for $a/L \lesssim 10$. At $a/L \gtrsim 10$ the hysteresis loop width practically does not depend

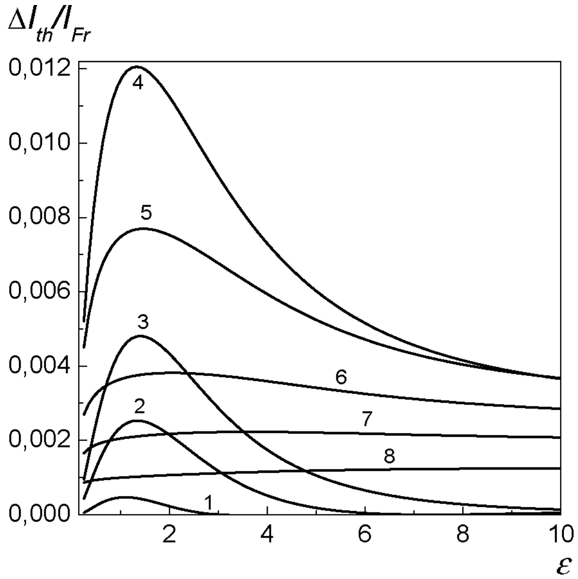

Fig. 3. The dependences of the hysteresis loop width $\Delta I_{th}/I_{Fr}$ on the value of anchoring energy ε. a/L=0.3 (*1*), 0.4 (*2*), 0.5 (*3*), 2 (*4*), 4 (*5*), 7 (*6*), 10 (*7*), 15 (*8*). $k = 0.6$. $m = 0.3$.

on the anchoring energy. As Fig. 3 shows, there are the following critical values of the beam half-width $a_{th}^{(min)}$, $a_{th}^{(max)}$ which depend on the NLC cell parameters: at $a > a_{th}^{(max)}$ the hysteresis is present, while at $a < a_{th}^{(min)}$ it is absent at any values of the anchoring energy. For the NLC parameters given above the calculated $a_{th}^{(min)} = 0.24L$, $a_{th}^{(max)} = 0.47L$. For every given value $a_{th}^{(min)} < a < a_{th}^{(max)}$ (see Fig. 3) there exists a finite grow (critical) value of the anchoring energy $\varepsilon_{th}(k, m, a/L)$ (the value ε of corresponding to $\sigma = 0$), so that at $\varepsilon < \varepsilon_{th}$ the LIFT is accompanied with hysteresis ($\sigma < 0$), and at $\varepsilon > \varepsilon_{th}$ the LIFT is possible only without hysteresis ($\sigma > 0$). As the beam width grows the critical value of ε_{th} gradually increases and in the beams with $a > a_{th}^{(max)}$ approaches infinity. This behavior of the hysteresis loop width dependence on the anchoring energy values is accounted for by counteraction of two factors – the beam width and the anchoring energy whose small values require respective large and small threshold values of the director orientation instability.

The results of the study can be used to select NLC cell parameters as well as the parameters of the incident light beam to provide the optimal conditions of investigating LIFT hysteresis. For instance, in the MBBA cell whose surfaces are covered by Langmuir-Blodgett monolayers containing fatty acids the polar anchoring energy for homeotropic orientation of the director's easy axis can be equal to $4 \cdot 10^{-3}$ erg/cm^2 [38]. MBBA parameters at the temperature 26°C are $k = 0.48$, $m = 0.5$ [55, 56]. As the calculations show, in this 30 μm cell of the NLC the LIFT hysteresis can be observed at the wavelength $\lambda = 632.8$ nm ($\varepsilon_{\perp} = 2.37$, $\varepsilon_{\parallel} = 3.06$ [54]) in the light beams with the half-width exceeding 0.015 mm. The maximal hysteresis loop $5.78\, W/cm^2$ is obtained in the light beam with 0.13 mm half-width. With the increase in incident light intensity the value of LIFT threshold is equal to 3080 W/cm^2.

Fig. 4 shows the jumps in maximal deviation angles $\Delta\varphi_m$ of the director when the light intensity reaches its threshold values I_{th} and I'_{th} given as the function of the anchoring energy ε. The dependences $\Delta\varphi_m(\varepsilon)$ correlate with the dependence of the hysteresis loop width on ε.

4. Conclusions

Thus, the finiteness of the energy anchoring the NLC with the cell surface influences essentially not only the threshold values of LIFT (hysteresis loop width), but also the permissible values of the incident light beam cross section. The dependence of hysteresis loop on the value of the anchoring energy is nonmonotonic in the light beams with $a/L \lesssim 10$, the maximal hysteresis loop width being reached at

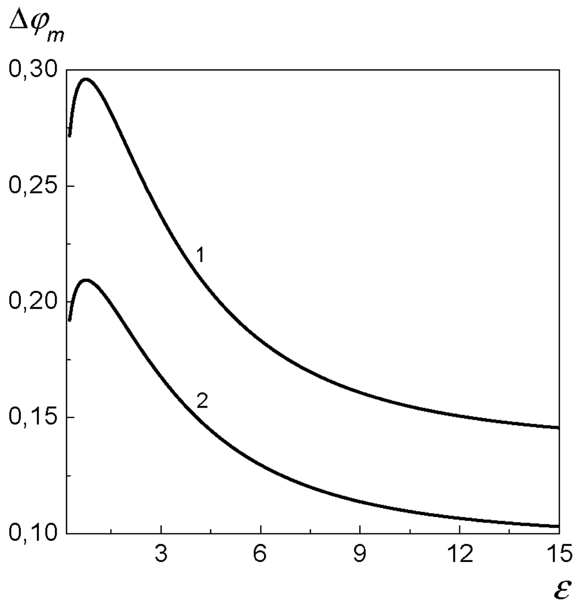

Fig. 4. The jumps $\Delta\varphi_m$ of the maximal deviation angles of the director with (*1*) the increase and (*2*) the decrease of intensity as the function of ε for the light beam with $a/L = 1$, $k = 0.6$, $m = 0.3$.

the anchoring energy values $\varepsilon \sim 1$. In the light beams with $a/L \gtrsim 10$ the hysteresis loop width grows weakly with the increase of the anchoring energy. As the calculations show, the hysteresis loop has the maximum for the light beam width in all anchoring energy values. The range of widths at which the LIFT is accompanied with hysteresis increases with the growth of parameter k and decrease in the values of m.

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Гістерезис світлоіндукованого переходу Фредерікса в нематичній комірці з скінченною енергією зчеплення

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Розглянуто вплив скінченності енергії зчеплення директора нематика на гістерезис світлоіндукованого переходу Фредерікса в світловому пучку з обмеженим поперечним розміром. Чисельно отримані значення порогів переходу Фредерікса і стрибків кута відхилення директора при зростанні і спаданні інтенсивності падаючого світла в залежності від величини енергії зчеплення директора і поперечного розміру світлового пучка. Визначені області допустимих ширин світлових пучків і значень енергії зчеплення при яких перехід Фредерікса супроводжується гістерезисом.