

Properties of 0-degree domain walls in cubic ferromagnet in transverse magnetic field

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The paper investigates the influence of an external magnetic field on the structure and properties of a 0° domain wall in an infinite cubic ferromagnet with uniaxial anisotropy induced along the [001] axis. In the case when the field is perpendicular to the plane of spin rotation (transverse fields), its presence results in the magnetization vector leaving the domain wall plane and its rotation relative to the crystallographic axes. Numerical and approximated analytical solutions have been found that correspond to magnetic inhomogeneities with such a structure.

В работе исследуется влияние внешнего магнитного поля на структуру и свойства 0° доменной границы в неограниченном кубическом ферромагнетике с наведенной вдоль оси [001] одноосной анизотропией. В случае, когда поле перпендикулярно плоскости вращения спинов (поперечные поля), его наличие приводит к выходу вектора намагниченности из плоскости доменной границы и к ее повороту относительно кристаллографических осей. Найдены численные и приближенные аналитические решения, соответствующие магнитным неоднородностям с такой структурой.

The mechanism of magnetic momentum non-coherent rotation is known to make the principal contribution to magnetization reversal of real magnets [1, 2]. In view of this, acquiring topicality is the study of structure, origination conditions and further evolution of the magnetization reversal nuclei which are formed on defects of the crystal and which play a dominating role in the processes under study. Investigations show that the most suitable model presentation of such formations are the magnetic inhomogeneities corresponding to 0-degree domain walls (0°DW) of the Bloch type [3]. In spite of the good agreement of the results obtained with the known theoretical calculations [4, 5] and with the experimental data [5, 6], the model presented in [3] is of a limited nature. One of the significant drawbacks of the model is its limited application. In particular, when studying the magnetization reversal processes, the model works efficiently only for those orientations of the magnetic field **H** which do not change the Bloch distribution of magnetic momentums in the DW, *i.e.* it works for those fields **H** that are parallel to the plane of 0°DW (longitudinal fields). Obviously, for further continuation of the research in this field it is necessary to investigate an influence of transverse magnetic fields on the structure and properties of 0°DW.

We will consider a finite sample of cubic ferromagnet in the form of a slab which features uniaxial anisotropy induced along the [001] axis ((001) slab). The energy of magnetic inhomogeneities (thermodynamic potential) of such magnet will be taken with due account of the exchange interaction (characterized by parameter A), induced uniaxial (K_u) and cubic (K_1) anisotropies, Zeeman interaction, as well as magnetostatic energy of volume charges localized in the DW, in other words as [3]:

$$E = DL_x \int_R \left\{ A[(\frac{\partial\theta}{\partial y})^2 + \sin^2\theta(\frac{\partial\varphi}{\partial y})^2] + \right.$$
$$+K_u\sin^2\theta + K_1[\sin^4\theta\sin^2(\varphi+\psi)\cos^2(\varphi+\psi) +$$

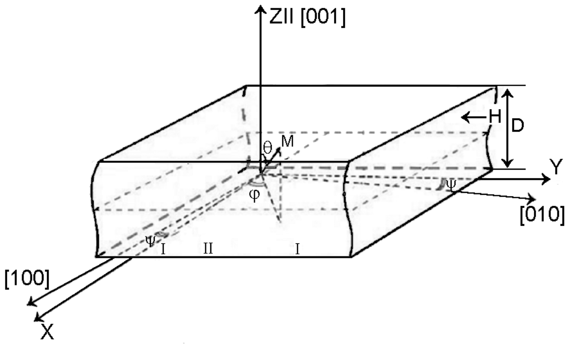

Fig. 1. Geometry of the problem.

$$+ \sin^2\theta\cos^2\theta] + \tag{1}$$
$$+2\pi M_s^2(\sin\theta\sin\varphi - \sin\theta_m\sin\varphi_m)^2 - \mathbf{HM}\Big\}dy,$$

where θ and φ are polar and azimuth angles of magnetization vector $\mathbf{M}$, θ_m and φ_m are values of these angles in the domains (with $y \to \pm\infty$), M_s is magnetization of saturation, L_x is the slab size along OX axis. The coordinate system in this case has been selected (Fig. 1) so that OZ ∥ [001], while OY axis is perpendicular to the DW plane and makes angle ψ with axis [010]. The slab is regarded as sufficiently thick and thus the influence of demagnetizing fields of surface charges is neglected.

Euler-Lagrange equations corresponding to the minimum conditions of (1) are reduced to the equations of the form:

$$\frac{\delta E}{\delta\theta} = 0, \quad \frac{\delta E}{\delta\varphi} = 0, \quad \lim_{y \to \pm\infty} \frac{\delta E}{\delta\psi} = 0 \tag{2}$$

if the following inequality holds, this inequality determining the stability of solutions

$$\delta^2 E(\theta,\varphi,\psi) > 0. \tag{3}$$

It follows from an analysis of equations (2) that in zero field at $0 < q < 1$ ($q = K_u/|K_1|$) there exist solutions of the kind [7]

$$\mathrm{tg}\theta_0 = a\,\mathrm{ch}\xi,\ \varphi_0 = 0,\pi;\ \psi_0 = \pi n/4,\ n \in Z$$
$$\xi = by/\Delta_1, a = \sqrt{q/(1-q)}, \tag{4}$$
$$b = \sqrt{1-q},\ \Delta_1 = \sqrt{A/k_1}$$

corresponding to which is 0°DW with M_0 ∥ [100], [010] in the domains (0°DW(I)). Besides, in the area $K_u < 0$ ($-1 < q < 0$) there takes place a similar solution

$$\mathrm{ctg}\theta_0 = a\,\mathrm{ch}\xi,\ \varphi_0 = 0,\pi; \psi_0 = \pi n/4, n \in Z \tag{5}$$
$$\xi = by/\Delta_1, a = \sqrt{-q/(1+q)},\ b = \sqrt{1+q}$$

corresponding to which is 0°DW with $\mathbf{M}_0$ ∥ [001] in the domains (0°DW(II)). These inhomogeneities are a certain perturbation of the homogeneous state which corresponds to the metastable phase. They are characterized by a bell-shaped form of the distribution of magnetic momentums in the DW. From the point of view of their structure and origination conditions they are nuclei of a new phase which allows using them for modeling the nucleation processes under spin reorientation phase transitions of the first kind in defect-containing finite magnets [7].

When the field is present ($H \neq 0$), solution of equations (2) of the kind (4) or (5) in the case of random orientation of $\mathbf{H}$, cannot be obtained through the known functions. However, in the case when $\mathbf{H}$ lies in the DW plane equations (2) are considerably simplified ($\theta = \theta(y)$, $\varphi_0 = 0$, π; $\psi_0 = \pi n/2$, $n \in Z$). Nevertheless, it does not seem possible to express $\theta(y)$ through the known functions. At the same time, it follows from an analysis of the phase portrait of these equations that cumulative curves exist on the phase plane (θ', θ), these curves being separatrixes in the form of closed loops with self-crossing. As was shown in [3], 0°DWs of the Bloch type correspond to them though the dependence $\theta = \theta(y)$ is different from (4) and (5).

Obviously, the problem becomes more complicated when the magnet is influenced by magnetic field whose direction is perpendicular to the plane of spin rotation in 0°DW (transverse fields). This case is still to be investigated but its analysis can yield nontrivial results. We will study it in greater detail.

As an example, we will consider 0°DW (I) and will regard that $\mathbf{H}$ ∥ [010]. Then the energy density of Zeeman interaction will take the form

$$\varepsilon_H = -\mathbf{HM} = -HM_s\sin\theta\sin(\varphi + \psi). \tag{6}$$

Assuming the contribution of (6) to (1) to be small (*i.e.* for small fields: $\lambda = M_sH/2K_u << |1)$, we will apply the perturbation theory to the system of equations (2) expanding them in power series in terms of parameter λ accurate to the linear terms

$$\theta = \theta_0 + \lambda\theta_1,\ \ \varphi = \lambda\varphi_1,\ \ \psi = \lambda\psi_1, \tag{7}$$

where $\varphi_0 = 0$, $\psi_0 = 0$ while θ_0 is determined from (4). In this case, in a first approximation system (2) changes into

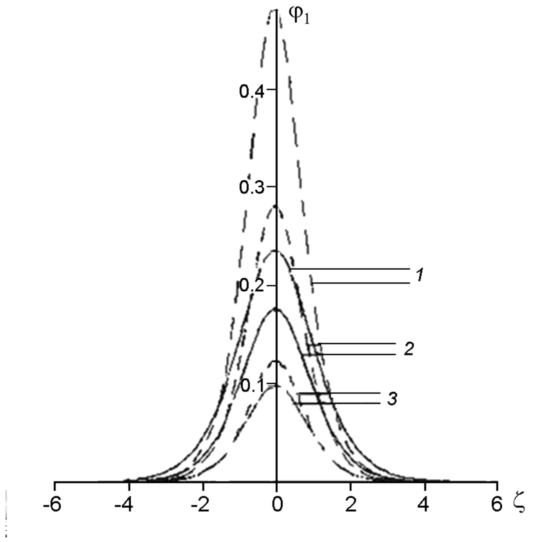

Fig. 2. Graphs of the numerical (solid line) and the approximated (dashed line) solutions for varying values of q with constant $Q = 3$: Curve *1* corresponds to $q = 1.5$, *2* — $q = 1.3$, *3* — $q = 1.1$.

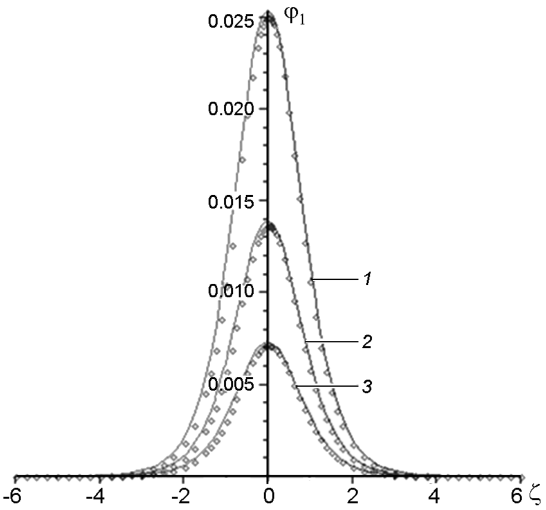

Fig. 3. Graphs of the numerical (solid line) and the analytical (dotted line) solutions $\varphi_1(\zeta)$ for varying values of Q with constant $q = 0.9$: *1* — $Q = 0.2$, *2* — $Q = 0.1$, *3* — $Q = 0.05$.

$$b^2\theta_1'' - \theta_1(q\cos2\theta_0 + \cos4\theta_0 = 0 \quad (8)$$

$$b^2\varphi_1'' + 2b^2\mathrm{ctg}\theta_0\theta_0'\varphi_1' - (\sin^2\theta_0 + Q^{-1})\varphi_1 - \sin^2\theta_0\psi_1 + (\sin\theta_0)^{-1} = 0$$

$$\lim_{\xi\to\pm\infty}(\sin^4\theta_0(\varphi_1 - \psi_1) - \sin\theta_0) = 0.$$

Without going into details of the investigation of the first equation of system (8), it can be stated that its solution does not qualitatively change the dependence $\theta = \theta(y)$ determined in zero order approximation by expression (4). Evidently, such changes of 0°DW structure can be expected when solving the other equations of system (8). In particular, from them (as well as bearing in mind that $\lim_{\xi\to\infty}\sin\theta_0 = 1$) it follows

$$(1 + Q^{-1})\lim_{\xi\to\infty}\varphi_1) = 1 - \psi_1, \quad (9)$$
$$\lim_{\xi\to\infty}\varphi_1 = 1 - \psi_1.$$

Since $Q^{-1} \neq 0$ the solution of system (9) will be

$$\lim_{\xi\to\infty}\varphi_1 = 0, \psi_1 = 1. \quad (10)$$

It follows from here that under the influence of the transverse magnetic field $\mathbf{H} \parallel [010]$ there takes place a departure of magnetization from the DW plane, the angle value of such departure being determined by the second equation in (8). Besides, there takes place a change of 0°DW orientation relative to the crystallographic axes by the angle $\Delta\psi \sim \lambda\psi_1 = HM_s/2K_1$ which is proportional to the field magnitude.

It does not seem possible to find analytically a solution of the second equation of system (8) (this equation determines the qualitative change of 0°DW structure). In view of this a solution is tried by numerical integration. The solution thus obtained has bell-shaped form (Fig. 2), the characteristics (width and amplitude $\varphi_1(0)$) depend substantially on material parameters q and Q.

It is notable that the second equation of system (2) also allows for approximate solutions under certain conditions. In particular, let us consider 0°DW (I) in the vicinity of the upper of its existence area. For this we will assume that $1 - q = \varepsilon > 0$ considering that $\varepsilon << 1$ and, besides, in the initial equation for $\varphi_1(\xi)$ we will switch to the new variable $\zeta = \sqrt{\varepsilon}\xi$. The equation will then take the form

$$\varepsilon\varphi_1'' + 2\varepsilon\mathrm{ctg}\theta_0\theta_0'\varphi_1' - (\sin^2\theta_0 + Q^{-1})\varphi_1 = \sin^2\theta_0 - (\sin\theta_0)^{-1}. \quad (11)$$

Expanding the solution $\varphi_1(\zeta)$ as well as $\theta_0(\zeta)$ in series in small ε and substituting them into (11), after unsophisticated calculations we obtain

$$\varphi_1(\zeta) = \frac{3(1-q)}{2(1+Q^{-1})\mathrm{ch}^2\zeta}. \quad (12)$$

As seen from Fig. 2 the obtained dependency $\varphi_1(\zeta)$ is also bell-shaped which at $\varepsilon = 0.1$ ($q = 0.9$) agrees satisfactorily with the results of numerical calculations. However, even at $\varepsilon \geq 0.2$, the discrepancy between the approximate solution and the numerical one becomes pronounced.

Let us also consider the case of small values of Q: $Q << 1$. Then, disregarding all the terms in the left side of equation (11) that are not proportional to Q^{-1}, we obtain

$$\varphi_1(\zeta) = Q[\sin^2\theta_0 - (\sin\theta_0)^{-1}]. \qquad (13)$$

The solution thus obtained possesses the same graphical pattern (Fig. 3) as those obtained earlier (Fig. 2). However, the discrepancy with the numerical solution becomes pronounced only at $q \geq 0.5$.

It therefore follows from the above calculations that the effect of external magnetic field which is perpendicular to 0°DW plane results in magnetization vector departing from this plane, i.e. 0°DW of the Bloch type transforms into 0°DW with a non-circular trajectory of the magnetization vector. Besides, there takes place a change of 0°DW orientation on the angle which is proportional to the field magnitude. The results obtained allow one to build a more comprehensive model presentation of magnetic inhomogeneities which are formed upon defects for analyzing the processes of magnetization and magnetization reversal of real crystals [3–5].

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Трансформація топології 0-градусних доменних меж у поперечному магнітному полі

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Досліджено вплив зовнішнього магнітного поля на структуру і властивості 0° доменної межі у необмеженому кубічному феромагнетику з наведеної уподовж осі [001] одноосною анізотропією. У разі, коли поле перпендикулярне площині обертання спинів (поперечні поля), його наявність приводить до виходу вектора намагніченості з площини доменної межі і до її повороту щодо кристалографічних осей. Знайдено чисельні і наближені аналітичні рішення, відповідні магнітним неоднорідностям з такою структурою.