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Magnon damping in spin nematic state near antiferromagnetic critical point

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Relaxation of elementary excitations with small wave vectors (magnons) in the nematic phase of magnet with spin $S = 1$ near antiferromagnetic SU(3) critical point has been studied. In the vicinity of the critical point the dispersion law minimum appears near the border of Brillouin zone. The elementary excitations near this minimum (rotons) have the gapped dispersion law, their damping is high, and they essentially influence on the relaxation of magnons. The roton contribution to the damping of magnons is strong both at the critical point and close to it.

Исследованы процессы релаксации элементарных возбуждений с малыми волновыми векторами (магнонов) в нематической фазе магнетика со спином S=1 вблизи SU(3) критической точки перехода в антиферромагнитное состояние. В окрестности критической точки в законе дисперсии появляется минимум вблизи границы зоны Бриллюэна. Соответствующие элементарные возбуждения вблизи этого минимума (ротоны) имеют щелевой закон дисперсии, их затухание велико и они оказывают существенное влияние на затухание магнонов. Вклад ротонов в затухание магнонов оказывается большим как в самой точке перехода, так и вблизи нее.

In the modern solid state electronics it is used mostly traditional magnetic materials, both soft magnets and hard magnets, with nonzero spontaneous magnetization. The properties of these materials are pretty good studied. There is another type of magnetic materials such as antiferromagnets, for which magnetization equals zero. Their properties are more variable. In particular, they have much higher frequency of magnetic resonance and giant limit of motion velocity of domain boundaries.

In recent years, researchers express considerable interest in spin nematics, which are similar to antiferromagnets [1].

It is well know that $S = 1$ isotropic magnet manifests spin nematic state with magnetization equals to zero and having nonzero quadrupolar order parameter [2]. This system can be described by the following Hamiltonian, see for details [2–9]:

$$\hat{H} = -\frac{J}{2}\sum_{n,m}\hat{\mathbf{S}}_{\mathbf{n}}\hat{\mathbf{S}}_{\mathbf{m}}(\Delta + \hat{\mathbf{S}}_{\mathbf{n}}\hat{\mathbf{S}}_{\mathbf{m}}). \qquad (1)$$

Here the parameters $J > 0$ and $J\Delta$ determine biquadratic and bilinear exchange interaction between the nearest neighbors. $\mathbf{S}_{\mathbf{n}}$ is spin operator at site $\mathbf{n}$. It is know from [2, 3, 6], that for $0 < \Delta < 1$ it is realized nematic phase with zero average spin and nontrivial spin quadrupolar average, with quantum critical points at $\Delta = 1$ and $\Delta = 0$, corresponding to transitions to ferromagnetic and antiferromagnetic states, respectively. For spin nematic within its entire stability region the damping of the elementary spin excitations with small wave vectors (magnons) caused by their interaction with each other is low, and magnons are well defined by Goldstone excitations [4]. In the limit of small wave vectors they have a linear dispersion law, while the damping is quadratic in their frequency. At the ferromagnetic critical point magnon spectrum softens and becomes quadratic, however Goldstone's behavior is preserved [5].

Near the ferromagnetic critical point the energy of elementary excitations grows with the growing of their wave vector, and close to Brillouin zone boundary spectrum has a maximum. Thus for this region the magnon-magnon interaction is a sole source of their damping. In contrast, near the antiferromagnetic critical point elementary excitations with low wave vectors have a linear dispersion law, but spectrum has a sharp minimum for wave vectors close to the border of Brillouin zone. This minimum resembles that for liquid helium and it can be called as roton minimum. Therefore, nearly the antiferromagnetic critical points there are two types of elementary excitations with low energy: magnons with small wave vectors and rotons with wave vectors close to the edge of Brillouin zone. Gap in the spectrum of rotons closely to point of the transition to the antiferromagnetic state is small. Exactly at the antiferromagnetic critical point the dispersion relation of rotons becomes linear, $\varepsilon(\mathbf{k}) \propto \Delta k = |\mathbf{k} - \mathbf{k}_0|$, where $\mathbf{k}_0$ is the edge of Brillouin zone. As it has been shown, the roton damping in the critical point is high and proportional to $(1/\Delta k)\ln(1/\Delta k)$ [5]. In contrast, the standard magnon damping caused by magnon-magnon interaction has no peculiarities at this point [4, 5].

In this paper rotons contribution to the damping of magnons in nematic phase of a magnet with $S = 1$ is studied. As will be shown here, rotons give an important contribution to the magnon damping such that magnons become highly damped excitations as well.

To describe the magnetic excitations it is convenient to use the phenomenological approach based on a complete set of generalized state of the group SU(3) [2, 7]. These state are conveniently written as

$$|\mathbf{u},\mathbf{v}\rangle = \sum_{j=x,y,z} \left(u_j + iv_j\right)|\psi_j\rangle, \qquad (2)$$

where $\mathbf{u}$ and $\mathbf{v}$ are the real orthogonal vectors subject to the constraints

$$\mathbf{u}^2 + \mathbf{v}^2 = 1, \quad (\mathbf{u},\mathbf{v}) = 0, \qquad (3)$$

$|\psi_x\rangle = (|-1\rangle - |+1\rangle)/\sqrt{2}$,

$|\psi_y\rangle = i(|-1\rangle + |+1\rangle)/\sqrt{2}$, and $|\psi_z\rangle = |0\rangle$, where $|\pm 1\rangle$ and $|0\rangle$ are the conventional quantum states with the given z-projection of spin, $S_z|\sigma\rangle = \sigma|\sigma\rangle$, $\sigma = 1,\ 0,\ -1$. The total spin $S = 1$ is determined by the vector of an average spin value $\langle\mathbf{S}\rangle$ and quadrupole averages which are expressed in term of $\mathbf{u}$ and $\mathbf{v}$ vectors as following [7, 8]

$$\langle\mathbf{S}\rangle = 2[\mathbf{uv}], \qquad (4)$$
$$\langle S_iS_k + S_kS_i\rangle = 2(\delta_{ik} - u_iu_k - v_iv_k).$$

The expected value of Hamiltonian (1) of the states (2) determines energy of the system in the molecular field approximation consistently talking into account quantum properties of the system of spins $S = 1$. In terms of vectors $\mathbf{u}$ and $\mathbf{v}$ specified at different lattice sites, this energy has the form of a sum over the nearest neighbor pairs

$$W = -\frac{J}{2}\sum_{nm}\{[(\mathbf{u_n}\mathbf{u_m}) + (\mathbf{v_n}\mathbf{v_m})]^2 + [(\mathbf{u_n}\mathbf{v_m}) - (\mathbf{v_n}\mathbf{u_m})]^2 - 4(1-\Delta)[(\mathbf{u_n}\mathbf{u_m})(\mathbf{v_n}\mathbf{v_m}) - (\mathbf{u_n}\mathbf{v_m})(\mathbf{v_n}\mathbf{u_m})]\}. \qquad (5)$$

Spin nematic state with $\langle\mathbf{S}\rangle = 0$ is stable at $0<\Delta<1$. For one of the possible implementations of the uniform nematic phase, which is characterized by $\mathbf{V}_0 = 0$ and and vector director $\mathbf{u}_0^2 = 1$, quadrupole averages are $\langle(\mathbf{Su_0})^2\rangle = 0$, $\langle(\mathbf{S}\times\mathbf{u_0})^2\rangle = 1$ [4, 5].

In the framework of approach of the average field theory on the SU(3)-coherent states given by (2), the dynamic of the magnet with Hamiltonian (1) is described by Lagrangian of the form [2]

$$L = -2\hbar\sum_n \mathbf{v}_n \partial\mathbf{u}_n/\partial t - W(\mathbf{u},\mathbf{u}), \qquad (6)$$

where $W(\mathbf{u},\mathbf{v})$ is the above energy of the system (5).

Consider fluctuation of the variables $\mathbf{u}$ and $\mathbf{v}$ describing the spin nematic state. Assuming for definiteness that $\mathbf{u}_0 \| \mathbf{e}_z$, one can write vectors $\mathbf{u}$ and ν in terms of Cartesian coordinates longitudinal and transversal of the deviations from the ground state. The longitudinal components $u_{\mathbf{n},z}$ and $\nu_{\mathbf{n},z}$ are dependent variables and quadratic in the transversal components of $u_{\mathbf{n},x}$, $u_{\mathbf{n},y}$ and $\nu_{\mathbf{n},x}$, $\nu_{\mathbf{n},y}$, which are chosen as independent variables. Thus, in the spin nematic there are two branches of transversal oscillations of vector $\mathbf{u}$ polarized in the mutually perpendicular directions. Then one can expand Lagrangian in powers of these variables. For our purposes it is sufficient to take into account bilinear and biquadratic summands in $u_{\mathbf{n},\mathbf{x}}$, $u_{\mathbf{n},\mathbf{y}}$ and $\nu_{\mathbf{n},x}$, $\nu_{\mathbf{n},y}$ and write $L = L_2 + L_4$. For description of the nonequilibrium thermodynamics of the system it is convenient to pass to Hamilton formalism. This can be done on the basis of Lagrangian L_2 (see for details [5]). From the form L_2 it follows that the variables $u_{\mathbf{n},x}$, $u_{\mathbf{n},y}$ can be chosen as the coordinates while $-2\hbar\nu_{\mathbf{n},x}$ and $-2\hbar\nu_{\mathbf{n},y}$ play the role of the corresponding canonical momentums. Then Hamiltonian takes the form of a sum of bilinear and biquadratic summands over the coordinates and momentums,

$$H = H_2 + H_4, \tag{7}$$

where H_2 is quadratic part of the energy W_2 written through the coordinates and momentums, and H_4 is just $-L_4$ with the generalized velocities expressed in terms of the coordinates and momentums.

The transition to the quantum Hamiltonian is done as in [5], by the presentation of H_2 and H_4 in terms of creation and annihilation operators of elementary excitations polarized in directions of x and y, respectively. Quadratic part H_2 takes diagonal form with the dispersion law $zJ\sqrt{(1 - c_k)(1 + c_k - 2\Delta \cdot c_k)}$, where $c_k(1/z)\sum_a \exp(i\mathbf{ka})$, z is number of nearest neighbors, $\mathbf{a}$ is set of nearest neighbors vectors of cubic lattice. The above dispersion law is valid within the whole stability region for the nematic phase and for arbitrary wave vectors till the border of Brillouin zone.

Properties of the elementary excitations are dependent on value of wave vector and value of Δ. For small wave vectors $ak <<|1$, $k = |\mathbf{k}|$ Hamiltonian H_2 of is ones for the two-component ideal magnon gas

$$H_2^{(m)} = \sum_k \varepsilon_k (a_k^* a_k + b_k^* b_k), \tag{8}$$

where $a_k{}^* a_k$ and $b_k{}^* b_k$ are operators of creation and annihilation x and y polarized of magnons respectively with the dispersion law

$$\varepsilon_k = Jak\sqrt{2z(1 - \Delta) + (2\Delta - 1)(ak)^2}. \tag{9}$$

This dispersion law is gapless for all the values of Δ and has minimum at $k = 0$. Magnons, according to [5] have the all properties of Goldstone excitations. In long-wave approximation they have linear dispersion law, $\varepsilon_k = \hbar ck$, $c = J(a/\hbar)\sqrt{2z(1 - \Delta)}$, and the relaxation rate γ_k is quadratic per their frequency.

Near the border of Brillouin zone a behavior of dispersion law is different. For wave vectors $\tilde{k} = \pi/a - k$, $ak <<1$ the dispersion law is written as

$$E_{\tilde{k}}^2 = 2zJ^2[2z\Delta + (1 - 3\Delta)(ak)^2]. \tag{10}$$

At $\Delta \geq 1/3$ the spectrum is monotonous and has maximum for wave vectors on the border of Brillouin zone. At $\Delta < 1/3$ the roton minimum appears. The dept of this minimum is determined by the value of parameter Δ, and it is small at $\Delta \to 0$. Such excitations can be called rotons. Thus near antiferromagnetic critical point one can introduce two types of low-energy elementary excitations: magnons and rotons. Hamiltonian non-interacting rotons can be written in the form

$$H_2^{(r)} = \sum_k E_k (A_k^* A_k + B_k^* B_k), \tag{11}$$

where $A_k{}^* A_k$ and $B_k{}^* B_k$ are creation and annihilation operators for rotons with polarizations along x and y, respectively. At $\Delta = 0$ rotons have linear dispersion law, $E_k = \hbar ck$, and their damping is high, $\gamma_k \propto T(T/E_k)\ln(T/E_k)$ [5]. At temperatures $(T/J)^2 >> \Delta$ the numbers of rotons are non small and they can essentially influence on the relaxation of magnons.

The interaction Hamiltonian of the magnons and rotons can be written as the sum of terms containing products of four operators. An analysis of the energy and momentum conservation laws shows that for small wave vectors of the magnons and rotons only the processes of scattering are allowed, and the interaction Hamiltonian can be presented in the form

$$H = \frac{1}{N}\sum_{1,2,3,4} \Delta(\mathbf{1} + \tilde{\mathbf{2}} - \tilde{\mathbf{3}} - \mathbf{4}) \times \tag{12}$$
$$\times[\Phi(a_1^* A_2^* A_3 a_4 + b_1^* B_2^* B_3 b_4) + \Phi_1 a_1^* A_2^* B_3 b_4 +$$
$$+ \Phi_2 a_1^* B_2^* A_3 b^4 + h.c.],$$

where $\mathbf{1} \equiv \mathbf{k}_1$, $\mathbf{4} \equiv \mathbf{k}_4$ and $\tilde{\mathbf{2}} \equiv \mathbf{k}_2$, $\tilde{\mathbf{3}} \equiv \mathbf{k}_3$ are linear momentums of magnons and rotons, respectively. The first term describes scattering processes of magnons and rotons without change of their polarization, whereas the second term describes scattering processes of equally polarized magnons and rotons with change of the polarization of the both scattering quasiparticles. Finally, third term describes processes of scattering different polarized magnons and rotons with change of the polarization for both scattering particles. Φ, Φ_1 and Φ_2 are the amplitudes of the corresponding processes, which have the following forms

$$\Phi = \Phi_0 \frac{E_2E_3 - E_0^2 + (E_2 - E_3)(\varepsilon_1 - \varepsilon_4) + \varepsilon_1\varepsilon_4}{\sqrt{\varepsilon_1 E_2 E_3 \varepsilon_4}},$$

$$\Phi_1 = \Phi_0 \frac{E_2E_3 - E_0^2 - \varepsilon_1E_3 - \varepsilon_4E_2 + \varepsilon_1\varepsilon_4 + \Delta(\varepsilon_1E_2 + \varepsilon_4E_3)}{\sqrt{\varepsilon_1 E_2 E_3 \varepsilon_4}},$$

$$\Phi_2 = 2\Phi_0 \frac{E_2E_3 - E_0^2 + \varepsilon_1E_2 + \varepsilon_4E_3 + \varepsilon_1\varepsilon_4 - \Delta(\varepsilon_1E_3 + \varepsilon_4E_2)}{\sqrt{\varepsilon_1 E_2 E_3 \varepsilon_4}}, \quad (13)$$

where $\Phi_0 = zJ/16$, and $E_0 = 2zJ\Delta^{1/2}$ is the energy gap for the roton spectrum.

A decrement of the magnon damping can be calculated as the imaginary part of the mass operator for the one-particle Green's functions [10] and can be present as

$$\gamma_k(T) = \pi\eta(\varepsilon_k)\sum_{p,q} \frac{\Phi_g^2 \delta(\varepsilon_k + E_p - E_q - \varepsilon_{|\mathbf{k+p-q}|})}{\eta(E_p)\eta(E_q)\eta(\varepsilon_{|\mathbf{k+p-q}|})}. \quad (14)$$

Here the notation $\eta(\varepsilon_k) = \sinh(\varepsilon_k/2T)$ is used to short equations, T is the temperature in energy units, summation is performed over the first Brillouin zone, and $\Phi_g^2 = \Phi^2 + \Phi_1^2 + \Phi_3^2$.

In the system of coordinate with the polar axis along vectors $\mathbf{k}$ delta function can be written as

$$\delta\left(\varepsilon_k + E_p - E_q - \varepsilon_{|\mathbf{k+p-q}|}\right) = \frac{2|\mathbf{k+p-q}|}{hcq|\mathbf{p+k}|} \frac{\delta(x - x_0)}{\sqrt{(z_1 - z)(z - z_2)}}, \quad (15)$$

where $x = \varphi_p - \varphi_q$, and φ_p, φ_q are azimuthal angles of the vectors $\mathbf{p}$ and $\mathbf{q}$, $z = \mathbf{kq}/kq$, x_0 is a root of the equation $\varepsilon_k + E_p - E_q - \varepsilon_{|\mathbf{k+p+q}|} = 0$, and $z_{1,2}$ are roots of the equation $\partial\left(\varepsilon_k + E_p - E_q - \varepsilon_{|\mathbf{k+p+q}|}\right)/\partial x = 0$. The roots $z_{1,2}$ are real if the condition $q_2 < q < q_1$ is fulfilled. For small-momentum limit of the magnons $\frac{\Delta^{1/2}}{a} >> k \to 0$ this condition takes the form

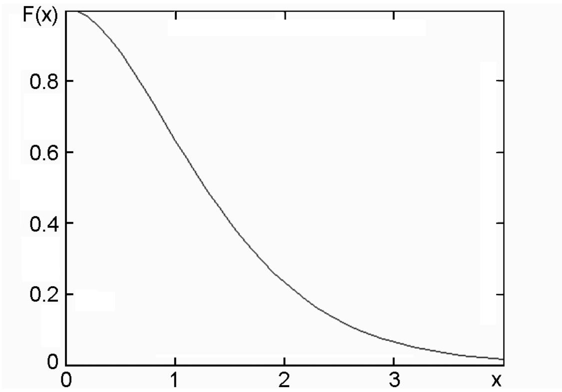

Fig. Plot of the function F versus the parameter $x = E_0/2T$.

$$q_{1,2} = p \pm k\frac{\sqrt{p^2 + b_0^2}}{b_0} \cdot \frac{\sqrt{p^2 + b_0^2} - py}{b_0}, \quad (16)$$

where $b_0 = (\sqrt{2z}/a)\Delta^{1/2}$, and $y = \mathbf{kp}/kp$. Thus, the momentums of the thermal rotons differ by the small quantity of the magnon momentum. The amplitudes on the mass surface of the processes take the form

$$\Phi = \Phi_1 = \frac{1}{2}\Phi_2 = \Phi_0 \frac{p}{k} \frac{p}{\sqrt{p^2 + b_0^2}}. \quad (17)$$

Thus the amplitudes diverge at $k \to 0$, but the calculation shows that the damping is finite because at $k \to 0$ the phase volume of the process tends to zero as k^2.

Passing in (14) from summation to integration and analytical calculation of the integrals leads to the simple formula for the magnon damping

$$\gamma_k(T) = \frac{9}{32\pi^3} \cdot \frac{\Phi_0^2 a^6}{hcb_0^2} \int_0^\infty \frac{p^6 dp}{\sinh^2(E_p/2T)}. \quad (18)$$

Integral in (18) can be calculated numerically for arbitrary temperatures, and the magnon damping can be written as following

$$\gamma_k(T) = \frac{3\pi^3 T}{2^{10} 7z}\left(\frac{T}{E_0}\right)^2 \left(\frac{T}{J}\right)^4 F\left(\frac{E_0}{2T}\right), \quad (19)$$

where $F(E_0/2T)$ is a function of the temperature, which has the form

$$F(x) = \frac{I(x}{I(0)}, \qquad I(x) = \int_x^\infty \frac{(\xi^2 - x^2)^{5/2}}{\sinh^2 \xi} d\xi. \quad (20)$$

The result of a numerical calculation of the function $F(x)$ is presented in Fig.

Thus the roton contribution to the damping of magnons with small wave vectors $ak << \Delta^{1/2}$ has finite value at $k \to 0$ and it increases when approaching the antiferromagnetic critical point, where E_0 is decreasing.

At the critical point $\Delta = 0$ the dispersion relation for both magnons and rotons becomes gapless. To find the magnons damping it is necessary to put $E_p \to \varepsilon_p$ and $E_q \to \varepsilon_q$ in the relation (14). Then one arrives to the standard integral known for the case four Goldstone quasiparticles scattering, common to that in [5], but with essentially different amplitude, which are divergent at $k \to 0$ on the mass surface

$$\Phi = \Phi_0 \frac{3pq - p^2 - q^2}{\sqrt{kpq|\mathbf{k+p-q}|}}, \qquad (21)$$

$$\Phi_1 = \Phi_0 \frac{2pq - p^2}{\sqrt{kpq|\mathbf{k+p-q}|}},$$

$$\Phi_2 = 2\Phi_0 \frac{2pq - q^2}{\sqrt{kpq|\mathbf{k+p-q}|}}.$$

The analytical expression for the damping can be found for limit cases of small and large magnon wave vectors, $hck << T$ and $hck >> T$. Let us consider the most interesting case at $k \to 0$. A calculation done for $hck << T$ gives the following expression

$$\gamma_k(T) = \frac{3}{2^6 \pi^3 z} \zeta(5) \left(\frac{T}{J}\right)^4 T \ln\left(\frac{T}{hck}\right), \; hck << \mathrm{T}. \qquad (22)$$

Well, exactly at the SU(3) antiferromagnetic critical point the magnon damping at $hck < T$ diverges logarithmically.

Thus, in contrast with the ferromagnetic critical point [5], where magnons are well-defined by Goldstone excitations till the value $\Delta = 1$, Goldstone behavior of magnons near the antiferromagnetic critical point is broken because of roton contribution to the magnon relaxation. Such behavior of the magnon damping appears due to abnormal behavior of the amplitudes of the magnon-roton scattering, which diverge at $k \to 0$.

Let's discuss this problem in more details. At first blush the amplitudes for the magnon-magnon scattering (13) are similar to those found previously for ferromagnets [11], antiferromagnets [12], or spin nematics [4]. These amplitudes are proportional to the values of momentums of the quasiparticles. For the standard case, all momentums are small, and according to Adler's principle all of them become zero on the mass surface {4,1,12]. In contrast, for magnon-roton scattering the momentum of rotons are not small. Thus, if the magnons momentums tend to zero, the numerators of (13) are finite while denominators tend to zero. As a result of this feature, the abnormal behavior of the magnon relaxation appears that described above.

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Загасання магнонів у спиновому нематику поблизу антиферомагнітної критичної точки

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Досліджено процеси релаксації елементарних збуджень з малими хвильовими векторами (магнонів) у нематичній фазі магнетика із спином S=1 поблизу SU(3) критичної точки переходу в антиферомагнітний стан. В околі критичної точки в законі дисперсії з'являється мінімум поблизу межі зони Бріллюена. Відповідні елементарні збудження поблизу цього мінімуму (ротони) мають щілинний закон дисперсії, їх загасання велике, і вони роблять істотний вплив на загасання магнонів. Внесок ротонів у загасання магнонів виявляється великим як у самій точці переходу, так і поблизу неї.