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High-gradient fields in magnets with giant anisotropy

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The gradient of strong stray fields generated by various systems of permanent magnets with giant magnetic anisotropy has been calculated. It is shown that the gradient values near singular points are characterized by the dependence $\nabla H \approx AM_S(1/r)$, where A is a constant for this system of magnets, M_S is the saturation magnetization of the magnet material, r is the distance from the singular point. The field gradient in those areas may reach about 10^6 to 10^8 Oe/cm. The indicated gradient level is comparable with maximum values achieved in superconducting solenoids supplied with the conical tips produced of soft magnetic material with high M_S. It is established that the volume forces with the specific density of $f \approx 4M_S^2/r$ arise near singular points in the magnet material being in high-gradient field. The mechanical stress in a magnet caused by these forces is characterized by the dependence $\sigma \approx 4\pi M_S^2 \ln(a/X_{min})$ and may reach 2–3 kg/mm^2.

Рассчитан градиент сильных полей рассеяния, генерируемых различными системами из постоянных магнитов с гигантской магнитной анизотропией. Показано, что величина градиента вблизи сингулярных точек характеризуется зависимостью $\nabla H \approx AM_S(1/r)$, где A — некоторая постоянная для данной системы магнитов. Вблизи сингулярных точек $|\nabla H|$ может достигать значений $|\nabla H| \approx 10^6$–10^8 Э/см. Указанные значения градиента поля сравнимы с предельными его величинами, которые достигаются в сверхпроводящих магнитах с коническими наконечниками, изготовленными из материалов с высокой индукцией. Установлено, что в высокоградиентном поле в материале магнита вблизи сингулярных точек возникают объёмные силы с удельной плотностью $f \approx 4M_S^2/r$. Механические напряжения в магните, связанные с этими силами, характеризуются зависимостью $\sigma \approx 4\pi M_S^2 \ln(a/X_{min})$ и могут достигать значений 2–3 кг/мм^2.

The systems of permanent magnets generating strong magnetic stray fields, which are the fields with strength values H exceeding the saturation induction B_S of the magnet substance, $H > B_S = 4\pi M_S$, were described in several works [1–5]. Existence of such fields was proved by calculations and tested experimentally [3, 4]. To provide the strong stray fields, the magnet material uniaxial anisotropy field $H_K >> 4\pi M_S$, and coercive force $H_C \sim 4\pi M_S$ are necessary. In [5], the frame of magnet systems generating the strong fields was defined; such systems were optimized as well. The strong fields were revealed [3, 4] in the rare earth (RE) based permanent magnet systems. These relatively new functional materials are used widely in various fields of modern engineering [6]. The peculiar property of these materials is magnetic anisotropy giant fields amounting to hundreds thousands of Oersteds [7]. The application areas of those magnet systems are being widened steadily as the knowledge on their properties becomes more profound. Revealing the strong

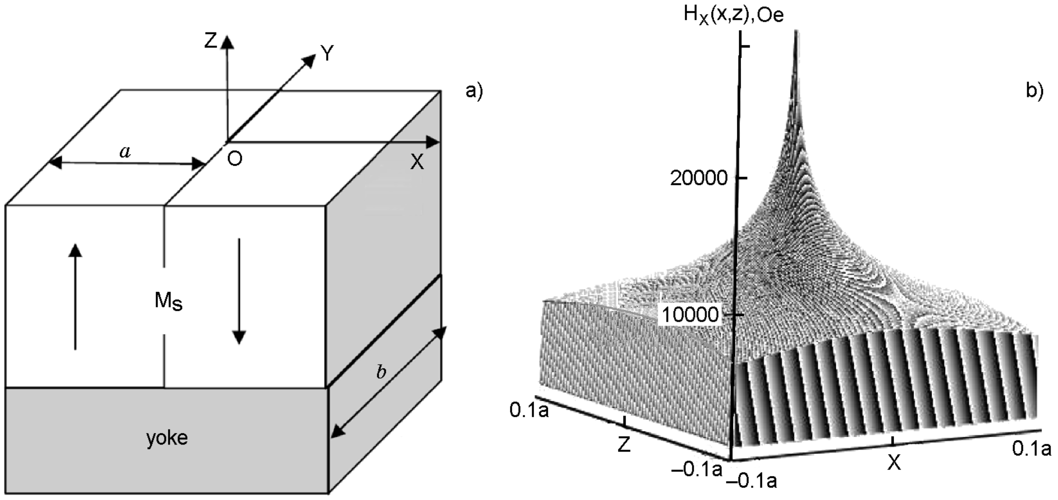

Fig. 1. System of 2 magnets and yoke (a); plot of stray-field tangential $H_X(x,z)$ component for the points located on the plane XOY in the region $-0.1a < x < 0.1a$; $-0.1a < z < 0.1a$ (b). For plotting, $b \to \infty$; $M_S = 750$ Gs were assumed.

magnetic field effect in these magnets extends substantially the functional capabilities thereof: for example, those can be used in magnetic recording techniques as magnetic heads for recording onto high-coercive storage medium [8], in biology [9], for development of ESR microscope [10], etc.

The strong stray fields in materials with giant anisotropy in the most cases are not only strong but very inhomogeneous. In [3, 4], the field gradient ∇H for a simple system of two magnets was evaluated. Since any systematic investigations on the field gradient were carried out neither in these works, nor in others [5, 8, 10, 11], this study is carried out. In particular, it is of interest to evaluate mechanical stress occurring in the magnet material with high gradient field. Further, the magnetic field gradient, as shown in [9], defines its influence on biological objects. The extent of magnetic field effect thereon is defined by the product $H\nabla H$. The same characteristic is important in the case of an inhomogeneous field application for separation of substances with different magnetic susceptibility. Thus, the gradient is an important physical characteristic of magnetic field together with the strength one. The high-gradient fields can be considered as a tool for influence onto different test objects in physics, engineering, and biology. Therefore, it is important to know the field gradient limiting values reached near singular points in different magnet systems. Thus, studying the strong magnetic field gradients is an important scientific and practical task.

Let us consider the gradient of the field created by the simplest system of two magnets (Fig. 1a). The field strength vector over the system surface includes two components: $H_x(x,z)$ and $H_z(x,z)$, the third component being $H_y(x,z) \approx 0$ (Fig. 1b). The vertical component of the stray field $H_z(x,z)$ is described by the expression:

$$H_z(x,z) = 2M_S\left\{\mathrm{arctg}\left(\frac{a+x}{z}\right) - \mathrm{arctg}\left(\frac{a-x}{z}\right) - 2\mathrm{arctg}\left(\frac{x}{z}\right)\right\}, \quad (1)$$

where a is characteristic magnet dimension.

The horizontal component $H_x(x,z)$ is expressed as

$$H_x(x,z) = M_S[\ln(a^2 + z^2 + 2ax + x^2) - 2\ln(x^2 + z^2) + \ln(a^2 + z^2 - 2ax + x^2)]. \quad (2)$$

The surface in Fig. 1b characterizes the field component $H_x(x,z)$.

The tensor of field gradient is determined from the condition $d\mathbf{H} = \mathbf{T_H}d\mathbf{r}$, where

$$\mathbf{T_H} = \begin{vmatrix} \dfrac{\partial H_x}{\partial x} & \dfrac{\partial H_x}{\partial z} \\ \dfrac{\partial H_z}{\partial x} & \dfrac{\partial H_z}{\partial z} \end{vmatrix}. \quad (3)$$

As the magnetic field H is a potential one and satisfies the Laplace equation [12], the field gradient tensor T_H is symmetric,

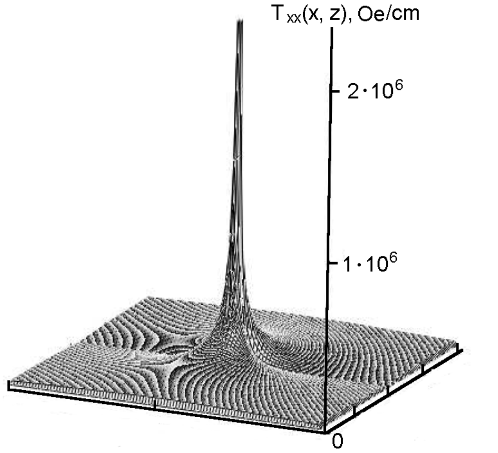

Fig. 2. Plot of field gradient tensor $T_{xx}(x,z)$ component for the points located in the region $-0.1a < x < 0.1a$; $-0.1a < z < 0.1a$. For $T_{xx}(x,z)$ calculations, the value M_S = 1000 Gs was assumed.

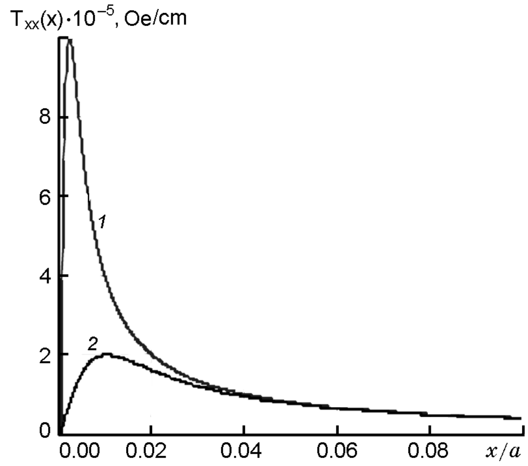

Fig. 3. Field gradient tensor T_{xx} component *vs*(x/a) for different distances from the plane *XOY*: $z = 0.002a$ (*1*) and $z = 0.01a$ (*2*).

and the following conditions are fulfilled for its components:

$$\frac{\partial H_z}{\partial z} = -\frac{\partial H_x}{\partial x}, \qquad \frac{\partial H_z}{\partial x} = \frac{\partial H_x}{\partial z}. \tag{4}$$

From Equations (1–4) for the system of two magnets, the following expressions can be obtained for the T_H tensor components ($T_H{}^x$ that is presented below):

$$\frac{\partial H_x(x,z)}{\partial x} = 4M_S\frac{x}{x^2 + z^2} - 2M_S\frac{a + x}{(a + x)^2 + z^2} - 2M_S\frac{x - a}{(a - x)^2 + z^2}$$

$$\frac{\partial H_x(x,z)}{\partial z} = 4M_S\frac{z}{x^2 + z^2} - 2M_S\frac{z}{(a + x)^2 + z^2} - 2M_S\frac{z}{(x - a)^2 + z^2} \tag{5}$$

$$\frac{\partial H_z(x,z)}{\partial x} = 4M_S\frac{z}{x^2 + z^2} - 2M_S\frac{z}{(a + x)^2 + z^2} - 2M_S\frac{z}{(x - a)^2 + z^2}$$

$$\frac{\partial H_z(x,z)}{\partial z} = -4M_S\frac{x}{x^2 + z^2} + 2M_S\frac{a + x}{(a + x)^2 + z^2} + 2M_S\frac{x - a}{(x - a)^2 + z^2}.$$

It follows from Eq.(5) that on the surface of the magnets ($z = 0$) in the points with coordinates $x = 0$ and $x = \pm a$, all the components of the field gradient tensor approach infinity. It is seen from Eq.(2) that these points are singular also for the stray field component H_X, i.e. the field gradient reaches its largest values in the same points as the field H_X does. The surface in Fig. 2 shows the calculated values of the gradient component $\partial H_x/\partial x$. The singular points are all those positioned at the system edges shown in Figs. 1a, 1b at $x = \pm a$, $z = 0$, and on the *OY* axis. In the vicinity of these points, the field gradient component takes on large values. At the points lying out of *OY* axis, the field gradient has finite values. This is seen from Fig. 3.

Knowing the field gradient tensor components, it is possible to calculate the derivative $\mathbf{T_n}$ of the field vector $\mathbf{H}$ in an arbitrary direction $\mathbf{n} = \cos\varphi\mathbf{i} + \sin\varphi\mathbf{k}$. The $\mathbf{T_n}$ vector is equal to scalar product $T_n = (\mathbf{T_H},\mathbf{n})$. Substituting the tensor components from Eqs.(5) into this expression, we get $|\mathbf{T_n}|$ is independent on the direction of $\mathbf{n}$ vector drawn from the point *O* at small distances r from *OY* axis ($r = [x^2 + z^2]^{0.5} << $ a):

$$|\mathbf{T_n}| \approx \frac{4 \cdot \sqrt{2}\mathbf{M_S}}{\sqrt{x^2 + z^2}} = \frac{4 \cdot \sqrt{2}\mathbf{M_S}}{r}. \tag{6}$$

In the case of $r \sim a$, it is necessary to use directly the Eqs.(5) to calculate the gradient.

For the points positioned on the *OX* or *OZ* axes at small distances from the coordinate origin, the expressions for derivatives in *OX*(*OZ*) directions are the following :

$$|\mathbf{T_X}| = \frac{4 \cdot \sqrt{2}\mathbf{M_S}}{x}; \qquad |\mathbf{T_Z}| = \frac{4 \cdot \sqrt{2}\mathbf{M_S}}{z}. \tag{7}$$

The Eqs. (5)–(7) characterize the gradient of the field generated by the system of two magnets (Fig. 1a). If a single magnet of parallelepiped shape is considered [5], the field gradient module is halved, i.e. at a small distance from its edge ($r << a$) this module is $|\nabla H| \approx 2\sqrt{2}M_S(1/r)$. The similar calculations for the systems of three and four magnets described in [5] give the values $|\nabla H| \approx 3\sqrt{6}M_S(1/r)$ and $|\nabla H| \approx 8M_S(1/r)$, respectively. When the number of magnets in the system increases further, the strength of the strong stray field at $r << a$ is characterized by the dependence $H \approx AM_S \ln(a/r)$, where the pre-logarithm factor A depends on the magnet system type, as shown in [5]. To estimate the limiting gradient values in different systems, the formula $|\nabla H| \approx A\sqrt{2}M_S(1/r)$ can be used. Note that in a cylindrical magnet with radial magnetization, as shown in [11], the coefficient $A = 2\pi$, while in the narrow gap between a pair of such cylindrical magnets, $A = 4\pi$.

Let us estimate the limiting gradient values for the system shown in Fig. 1a. Using Eq. (5)–(7), we get that in the small vicinity of the point O, the gradient can reach high values: $\nabla H = 10^5$ to 10^7 Oe/cm at $r \approx 100$ to 0.5 μm and a sufficiently large magnet characteristic dimension a ($r < 0.01a$).

It should be noted that large gradient values can be created not only with permanent magnets with giant magnetic anisotropy. For example, a high-gradient field arises in a narrow gap between two conic tips positioned in high magnetic field produced by superconducting solenoids. To calculate the field gradient of this system, the value $A = 4.8$ can be used found in [5] for a system of two conic magnets with large anisotropy. It follows therefrom $|\nabla H| \approx 4.8M_S(1/r)$. As the magnetization of conic tips made of iron or Co–Fe alloys exceeds at least twice the M_S of RE-based magnets, the field gradient in this system should be very high. In the system of eight permanent magnets, the field gradient is $|\nabla H| \approx 8\sqrt{2}M_S(1/r)$ [5]. This value is comparable to field gradients of superconducting magnets with conic tips. Thus, to create the fields with high gradients, it is necessary to use the systems of permanent magnets with large saturation induction and giant magnetic anisotropy, for example, as for $SmCo_5$ ($H_K \approx 450$ kOe). In this case, the gradient value may reach $\nabla H \approx 10^5$–10^8 Oe/cm at small distances from the magnet edge.

Let us consider the mechanical stresses connected with ponderomotive forces in magnets with giant anisotropy. The stray fields in the magnet are known to be produced by magnetic "charges", thus, the fields are not external. To calculate the stress caused by the "Coulomb" forces, an elementary volume ΔV should be chosen centered about a certain point of the magnet, and the stray field strength H should be calculated without taking into consideration the stray fields from the "charges" localized in this volume. The stray field from the rest of the magnet will be external with respect to the chosen elementary volume. The magnetic energy of the elementary volume ΔV in the field H, produced by the external neighborhood, is $E_H = -(\mathbf{M},\mathbf{H})\Delta V$, and the force $\mathbf{F}$ acting on the volume ΔV is calculated as energy gradient E_H, i.e. $\mathbf{F}(x,z) = \nabla E_H = -\nabla(\mathbf{M},\mathbf{H})\Delta V$. At $\mathbf{M} = \mathrm{const}$, this expression may be written as follows:

$$\mathbf{F}(x,z) = (T_H,\mathbf{M}) = (T_H,\mathbf{M_S})\Delta V\ , \tag{8}$$

therefrom we get the expression for density of volume (specific) forces [12]:

$$\mathbf{f}(x,z) = \mathbf{F}(x,z)/\Delta V \approx (T_H,\mathbf{M_S}) \quad \text{or } \mathbf{f} \approx (\mathbf{M_S},\nabla)\mathbf{H}, \tag{9}$$

where ∇ is differential operator, $\nabla = (\partial/\partial x)\mathbf{i} + (\partial/\partial z)\mathbf{k}$.

In a system of two magnets (Fig. 1a), the magnetization vector is directed parallel to OZ axis in all the points of the magnet, hence, $\mathbf{M_S} = M_S\mathbf{k}$, where $\mathbf{k}$ is a unit vector. Taking this into consideration, the expression (9) takes the form:

$$\mathbf{f}(x,z) \approx M_Z(\partial H_Z/\partial x)\mathbf{i} + M_Z(\partial H_Z/\partial z)\mathbf{k} = = M_Z(\partial H_X/\partial z)\mathbf{i} + M_Z(\partial H_Z/\partial z)\mathbf{k}. \tag{10}$$

If the expressions for field gradient tensor components (5) are substituted into (10), the module of volume force density will be equal to

$$|\mathbf{f}(x,z)| \approx M_Z[(\partial H_Z/\partial z)^2 + (\partial H_Z/\partial z)^2]^{0.5} \approx \approx 4M_S^2[(x^2 + z^2)^{-0.5}] \approx 4M_S^2/r. \tag{11}$$

Here, the smallest value $r = r_0$ is determined from the relation $\Delta V \approx (r_0)^3$.

Let us evaluate the stress σ occurring in an elementary area $\Delta S = \Delta z \cdot \Delta y$ positioned in the point $M(x,0)$ at the distance x from OY axis in the system of two magnets (Fig. 1a). For estimating calculations, we shall limit ourselves by the only component f_X.

The force acting on the mentioned area element is equal to

$$F = \int_V f(x)dV = \int_{X_{min}}^{x} 4M_S^2 \Delta S \frac{dx}{x}. \qquad (12)$$

The stress occurred are described as

$$\sigma(x) \approx \int_{X_{min}}^{x} \frac{4M_S^2}{x} dx = 4M_S^2 \ln(x/X_{min}). \qquad (13)$$

In Eqs.(12) and (13), X_{min} is the minimum distance from the magnet edge, which is assumed to be commensurable with interatomic distance, i.e. $X_{min} \approx 10^{-7}$ cm.

The limiting stress value can be determined from the relationship $\sigma \approx 4M_S^2 \ln(a/X_{min})$. It follows therefrom that the maximum stress occurs near the magnet axis, and its value is $\sigma \sim 2–3$ kg/mm^2.

As it is known [12], in a ferromagnetic with induction B, positioned in an external homogeneous magnetic field H, the stress $\sigma \sim BH/8\pi$ occurs due to ponderomotive forces. Since the external field strength created by a permanent magnet does not exceed $H \leq 5 \cdot 10^3$ Oe, the stress value does not exceed $\sigma \sim 0.02$ kg/mm^2. In the case of inhomogeneous field, additional stresses occur. Those stresses are substantially higher (approximately, by a factor of $\ln(a/X_{min})$), than in the magnet with small anisotropy, where strong stray fields do not occur. Note that relatively low stress level in the magnet ($\sigma \leq 2–3$ kg/mm^2) at high density of volume forces f near singular points (see Eq.(11)) may be explained by rapid attenuation of these forces as the distance from the magnet singular points increases.

Thus, the study carried out for high-gradient fields generated by the permanent magnet systems with giant anisotropy allows the following conclusions. The field gradient module in the systems mentioned reaches $|\nabla H| \approx 10^6–10^8$ Oe/cm, which is comparable to the fields occurring in superconducting solenoids with conic tips made of soft magnetic material with high induction. The high-gradient field generated near singular points causes occurrence of high volume density forces in the magnet material. For example, in the system of two magnets (Fig. 1a) the forces amount $f \approx 4M_S^2/r$. These result in additional stress of $\sigma \sim 2–3$ kg/mm^2 in the magnet.

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Високоградієнтні поля розсіяння у магнітах з гігантською анізотропією

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Обчислено градієнт сильних полів розсіяння, що генеруються різними системами з постійних магнітів з гігантською магнітною анізотропією. Показано, що величина градієнта поблизу сингулярних точок характеризується залежністю $\nabla H \approx AM_S(1/r)$, де A — певна стала для даної системи магнітів. Градієнт поля може сягати значень порядку 10^6–10^8 Е/см. Вказаний рівень градієнта є порівнянним з граничними його величинами, які досягаються у надпровідних магнітах з конічними наконечниками, виготовленими з магнітом'яких матеріалів з високою індукцією. Встановлено, що у високоградієнтному полі у матеріалі магніту поблизу сингулярних точок виникають об'ємні сили з питомою густиною $f \approx 4M_S^2/r$. Механічні напруги у магніті, пов'язані з цими силами, характеризуються залежністю $\sigma \approx 4\pi M_S^2 \ln(a/X_{min})$ і можуть досягати значень 2–3 кг/мм^2.