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# Quantum size effect and interlayer electron tunneling in metal-semiconductor superlattices

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A new type of quantum size effect in metal-semiconductor superlattices is predicted. Giant oscillations of the transverse tunnel conductivity arise if size quantization of the electron spectrum in the metal layers takes place. This effect is due to the fact that the probability of metal electron tunneling through a semiconductor layer depends sharply on the electron incidence angle. The oscillations have been found to exist even in disordered systems, provided the electrons in metal layers undergo low-angle scattering on imperfections.

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Recently new unusual oscillation effects have been discovered [1–4] on metal-semiconductor Mo/Si superlattices (SL) with a constant thickness of Si layers, $d_s$ , and a variable thickness of the metal ones, $d_m$ . SL in-plane resistivity, $\rho_l$ , as well as superconducting characteristics (the transition temperature, $T_c$ , the transverse critical magnetic field derivative, $dH_{c\perp}/dT|_{T_c}$, and the coupling strength) reveal oscillating periodic dependence on $d_m$ . All oscillations are well pronounced, and in the case of the coupling strength they reach a giant amplitude. Of even greater importance is the fact that the oscillation effects are inherent only to *multi*layers. Three layer samples, Si/Mo/Si, have not revealed any oscillations [3]. This fact alone suggests that the oscillations cannot be explained in simple terms of the conventional quantum size effect [5], though their period in $d_m$ does not conflict with a value predicted by the size-quantization theory for metal single films.

In this note we would like to discuss an anisotropic tunneling through SL semiconductor layers as a possible explanation of the above unusual size effects. To demonstrate the possibility of such effects we shall consider here the transverse SL conductivity which originates from the interlayer tunneling of electrons.

The idea is based on the fact that the probability of the tunneling of metal electrons through a semiconducting interlayer, $W$, differs from zero only for those with a practically normal incidence on an interface metal—semiconductor. Owing to this sharp dependence of $W$ on the incidence angle of a tunneling electron, $\theta$, the probability $W$ experiences sharp outbursts as quantized electron energies in the metal pass through the Fermi level with a $d_m$ variation. This effect reminds in some sense so-called giant resonance oscillations of the ultrasonic absorption in metals [6] with the essential difference that instead of a small electron group singled out by the resonance condition there is one determined by the sharpness of the function $W(\theta)$ mentioned. It is this group that participates in the tunnel current. It is obvious that such an effect can lead to the giant oscillations of the tunnel current.

At first sight, the effect described seems to be irrelevant to the experiments mentioned above because of a rather strong disorder in metal layers. Nevertheless we shall show that for the small group of electrons we are interested in the size quantization results in an enhancement of their lifetime in $d_m/a >> 1$ times ($a$ is the interatomic distance in metal). As will be shown, such an enhancement is sufficient to provide giant oscillations of the tunnel current.

To show this we consider a periodic one-dimensional system comprised of the alternating quantum wells (conducting layers) and the tunnel barriers (semiconductor layers). For simplicity sake we shall assume further that the electron dispersion law is quadratic and isotropic.

As follows from general quantum mechanics considerations, $W$ as a function of the in-plane momentum modulus, $p_\|$ , and the electron energy, $E$ (which is considered to be close to the Fermi energy, $E_F$) can be represented in the form

$$W = A \exp\left\{-\frac{2d_s\sqrt{(\delta p)^2 + 2m_s(E - E_F) + p_\|^2}}{\hbar}\right\}, \quad (1)$$

where $d_s$ is the thickness of semiconductor layers,

$$\delta p = \sqrt{2m_s\Delta}, \quad (2)$$

$\Delta$ is a phenomenological parameter which is of order of the typical energy of the effective tunnel barrier, the constant $m_s \sim m_0$ ($m_0$ is the free electron mass); the form of the pre-exponential factor $A$ is irrelevant to further consideration. The $\Delta$ value cannot exceed one-half the semiconductor gap, $E_g$ , which is, in turn, much less than $E_F$ . In the case of amorphous semiconductors, which is realized in SL Mo/Si, there are strong reasons to expect that energy parameter $\Delta$ is even much less than $E_g$ . It is just the smallness of $\Delta$ that causes the sharp dependence of $W$ on the angle $\theta$ or, what is the same, on $p_\|$ . Formula (1) shows that the probability $W(p_\|)$ reaches its maximum at the normal incidence ($p_\| = 0$) of an electron on the metal-semiconductor interface, abating to exponentially small values within an interval of order of $\delta p << p_F = \sqrt{2mE_F}$ ($m$ is the mass of an electron in the metal). Such a behavior of $W(p_\|)$ is a main point of our consideration.

Another important scale in the momentum space results from the size quantization in the metal layers. In the isotropic case under consideration the size-quantization electron spectrum in a metal layer (in the limit $W = 0$) is a set of terms

$$E_n^0(p_\|) = [(\hbar\pi n/d_m)^2 + p_\|^2]/2m, \quad (3)$$

where $n$ is the term number.

It is clear (see Fig. 1) that at zero temperature only quantized $p_\|$ values (we denote them by $p_n$) which are the roots of the equation

$$E_n^0(p_\|) = E_F \quad (4)$$

make a contribution to SL kinetic characteristics. At finite but rather small $W$ the situation is not

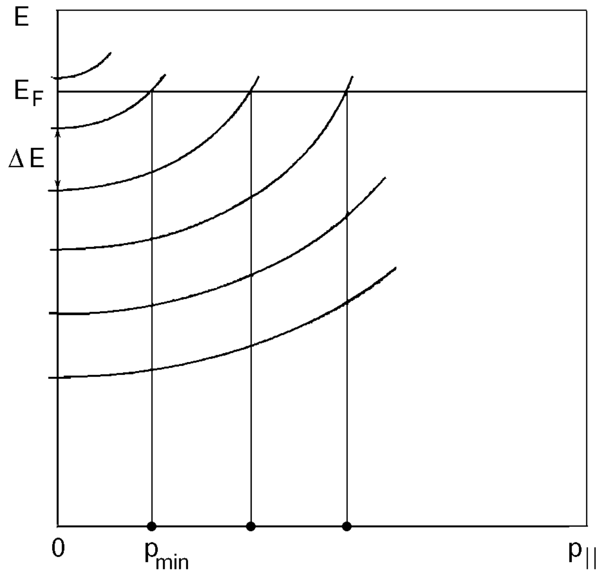


*Fig. 1.* The size-quantization spectrum in a metal layer. The bold dots on the $p_\|$-axis are $p_n$ (quantized values of $p_\|$ at $E = E_F$).

changed essentially because the broadening of size-quantization levels, $\delta E \sim \sqrt{W}\Delta E$, which is produced by the electron tunneling is small as compared with the typical distance

$$\Delta E \sim \hbar v_F/d_m \quad (5)$$

between nearest terms ($v_F$ is the Fermi velocity). Therefore, permitted $p_\|$ values are localized within narrow momentum intervals that are isolated one from another (their lengths are $\propto \sqrt{W}$) centered at $p_n$ .

As follows from (1), only those $p_\|$ values make the main contribution to a tunnel current, $I$, which meet condition $p_\| \lesssim \delta p$. Taking also into account condition (4), one finds that $I$ depends crucially on the ratio between $\delta p$ and the smallest of the quantized $p_n$ values

$$p_{\min} = \left(\left\{\frac{d_m p_F}{\hbar}\right\}\frac{\pi\hbar}{d_m}p_F\right)^{1/2}. \quad (6)$$

Here { . . . } means the fractional part of a number. If $p_{\min} >> \delta p$ the tunnel current is negligibly small, while at $p_{\min} \lesssim \delta p$ it essentially increases, reaching a maximum at $p_{\min} = 0$. One can easily see from (6) that the monotonic change in the metal layer thickness $d_m$ results in oscillations of $p_{\min}$ between the value $p_{\min} = 0$ and its maximum value

$$\Delta p = \left(\frac{2\pi\hbar}{d_m}p_F\right)^{1/2} \sim p_F\left(\frac{a}{d_m}\right)^{1/2}. \quad (7)$$

As is seen from the figure, these oscillations arise because the monotonic change in $d_m$ produces successive passage of size-quantization terms (3) through the Fermi level. Such an oscillatory behavior of $p_{\min}$ results in giant oscillations of the transverse current if the following condition takes place:

$$\Delta p \gtrsim \delta p \tag{8a}$$

or, in terms of $d_m$:

$$d_m \lesssim a(m/m_s)E_F/\Delta \ . \tag{8b}$$

In the opposite case, $\Delta p << \delta p$, the amplitude of $I$ oscillations is bound to be exponentially small in the parameter $\delta p/\Delta p$. The criterion (8) of existence of the giant oscillations is not a rigid restriction. Though the parameter $\Delta p$ is much less than $p_F$ , it considerably exceeds the typical distance, $\hbar/d_m$ , between quantized values $p_n$ (see (4)). Therefore, the requirement (8) can be fulfilled at $d_m >> a$. Certainly, the criterion (8) is not the only one determining appearance of the giant oscillations. Along with it, the common conditions of quantum-size-effect existence must be fulfilled:

$$\hbar/\tau \lesssim \Delta E \sim \hbar v_F/d_m \ , \tag{9}$$

$$T \lesssim \Delta E \ . \tag{10}$$

Here $\tau$ is the time of electron life in a quantized state, $T$ is the temperature. The latter condition is weaker than the previous one, and we can assume, for simplicity, that $T = 0$.

At sufficiently large $\tau$ (this statement will be specified below) the tunnel current may be calculated directly in terms of the multilayer electron spectrum. The latter is a set of minibands

$$E(n, p_\parallel, \mathcal{P}) = E_n^0(p_\parallel) + \delta E(p_\parallel, \mathcal{P}) \tag{11a}$$

which are size-quantization terms (3) broadened due to finiteness of tunneling probability. This broadening is determined by the small correction

$$\delta E = \frac{2\hbar v_{F\perp}(p_\parallel)}{d_m} \sqrt{W} \cos (\mathcal{P} d/\hbar), \tag{11b}$$

where $\mathcal{P}$ is a new quantum number (quasimomentum) enumerating the stationary states in minibands, $v_{F\perp} = \sqrt{2mE_F - p_\parallel^2}/m$ is the modulus of the transverse velocity of Fermi electron with a given $p_\parallel$ , $d = d_s + d_m$ . Taking into account that the average transverse velocity in a miniband stationary state, $v_\perp = \partial E(n, p_\parallel, \mathcal{P})/\partial \mathcal{P}$, has the form

$$v_\perp = 2(d/d_m)\, v_{F\perp} \sqrt{W} \sin (\mathcal{P} d/\hbar) \ , \tag{12}$$

after some calculations carried out in the relaxation time approach we obtain the transverse conductivity, $\sigma_\perp$, in the form

$$\sigma_\perp = \frac{me^2\tau d}{2\pi\hbar^2 d_m^2 m_s^2} [p_{\min}^2(d_m) + (\delta p)^2] \times$$
$$\times \exp \left\{ -\frac{2d_s}{\hbar} \sqrt{p_{\min}^2(d_m) + (\delta p)^2} \right\} . \tag{13}$$

This formula describes the limiting case $\Delta p >> \delta p$. Here we have specified the expression for the pre-exponential $A$ in (1), assuming for definiteness that the semiconductor layer may be considered as a square-topped barrier. As one can see from the expressions (13), (6), the oscillatory dependence $\sigma_\perp$ on $d_m$ is a periodic succession of sharp spikes whose height is of order of the transverse conductivity itself. They arise when $d_m$ lies within rather narrow ranges determined by the relations

$$\left\{ \frac{d_m \, p_F}{\hbar} \right\} \sim (\delta p/\Delta p)^2 << 1 .$$

Just at these $d_m$ values electrons with $p_\parallel = p_{\min}$ tunnel between adjacent metal layers. Outside these ranges the tunneling is weak. The formula (13) also shows that the amplitude of $\sigma_\perp$ oscillations is

$$\sim (\hbar/d_m \, p_F)^2 \exp \left\{ -\frac{2d_s}{\hbar} \delta p \right\} \sigma_0 \ .$$

Here $\sigma_0$ is the conductivity of the metal.

The expression (13) holds true only when the collision broadening $\hbar/\tau$ is much less than the typical miniband width $\delta E \sim \sqrt{W}\Delta E$. It is a very rigid restriction. The situation

$$\delta E << \hbar/\tau \lesssim \Delta E$$

seems to be much more realistic. In such a case the electron scattering completely destroys quantum coherent interference in the multilayer system as a whole, but it does not markedly affect the size quantization in individual metal layers. One can show that in this intermediate situation the formula (13) holds true with an accuracy of corrections $\sim W^{3/2}$. In such a case $\tau = \hbar/\Gamma$, where $\Gamma$ is the imaginary part of the mass operator of the one-electron Green function in metal for normal incident electrons.

In the limiting case

$$\hbar/\tau >> \Delta E \tag{14}$$

collisional broadening destroys not only the minibands but also the terms of size quantization in separate metal layer. It is clear that under such conditions the considered oscillation effects are absent.

As has been mentioned above, the present work was stimulated by the experimental observations [1–4] of the oscillations of kinetic and superconducting parameters on Mo/Si SLs. These investigations have been carried out on rather disordered Mo/Si multilayers with mean free path of electrons which is less than $d_m$ [7,8]. At first sight the observation of the oscillatory behavior is impossible under these conditions. Here we shall show that in a case of soft (low-angle) scattering on the imperfections the lifetime of the size-quantized electron states for the electron group which is responsible for the tunneling ($p_\parallel \lesssim p_{\min}$) can significantly exceed the typical $\tau$ in a metal layer. Such a situation arises when the typical scale of a space inhomogenity in Mo layers, $L$, is more than $a$ (in the experiment cited $L$ was ~ $10a$ for all $d_m$ values). Actually, from the general expression for the inverse lifetime, $\tau^{-1}(n, p_\parallel)$, of an electron in a given size-quantization state, $|n, p_\parallel\rangle$, we obtain

$$\tau^{-1}(n, p_\parallel) \propto \sum_{n',\, p'_\parallel} \overline{|\langle n, p_\parallel|V|n', p'_\parallel\rangle|^2}\, \delta(E_n^0(p_\parallel) - E_{n'}^0(p'_\parallel)) \,. \tag{15}$$

Here $V = V(\mathbf{r})$ is a random potential in the metal layer, $\mathbf{r}$ is the electron radius vector, the line over the matrix element means the averaging over the random realization of $V(\mathbf{r})$. In virtue of the fact that the matrix element in (15) is not small only for momenta transferred $\lesssim \hbar/L$, only transitions with $|p_n - p_{n'}| \lesssim \hbar/L$ should be taken into account. As is clear from our preceding reasoning, the distance between the least $p_n$ and its nearest neighbor is ~ $\Delta p$. This value can exceed $\hbar/L$ despite the fact that $L << d_m$. For this reason $\tau(n, p_\parallel)$ for the electrons participating in the tunnel transport turns out to be $d_m/a >> 1$ times more than the typical $\tau$ value. That is why the giant oscillations can indeed arise in rather disordered multilayers.

In summary, we have considered new quantum oscillation effect arising in metal/semiconductor multilayers due to combination of size quantization in thin metal layers and selective tunneling of electrons through the semiconductor interlayers. It is shown that giant oscillations of $\sigma_\perp$ appear, which result from sharp $W$ dependence on an incidence angle of electron, so that only the electrons belonging to the small group with the least of quantized $p_\parallel$ values contribute to the tunnel current. Another remarkable feature of the quantum oscillations described is that the disorder is not so destructive for the above effect as it is for the conventional quantum size effects. The next step is to show how this phenomenon affects the in-plane transport and superconducting properties in the experimental situation.


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