THEORETICAL LIMITATIONS ON ELEMENTS OF THE YUKAWA MATRIX IN THE νMSM MODEL

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The system of equations which couples elements of the Yukawa and active neutrino mass matrices in the νMSM theory (an extension of the Standard Model by three right-handed neutrinos which are singlets of the weak isospin) has been analyzed and solved. On the basis of the solution obtained, more accurate constraints on the model parameters have been determined. The obtained results were also used to study the CP-violating phase in the case where the elements of the active neutrino mass matrix are real-valued. The generation of a baryon asymmetry has been demonstrated to occur in this case as well.

1. Introduction

The Standard Model (SM) for the electroweak and strong interactions [1] is undoubtedly a successful theory that correctly describes processes, where elementary particles with the energies up to about 100 GeV (and, for some processes, up to several TeV) are engaged. The SM has successfully passed an examination by plenty of precision experiments, and it agrees well with the data of cosmological observations. However, there are a number of reasons that give the basis to assert that the SM is not a complete theory. In cosmology, the SM cannot explain such phenomena as the dark energy and the dark matter, as well as the inflationary physics of the Universe evolution. In elementary particle physics, the SM does not explain neutrino oscillations and the baryon asymmetry in the observable sector of elementary particle physics [2].

The SM, which is a renormalized theory and is based on the gauge group $SU(3) \times SU(2) \times SU(1)$, includes three generations of fermions, with left-handed components of fermions forming weak-isospin doublets with respect to the $SU(2)$ group, whereas right-handed components of all fermions, but the neutrino, being weak-isospin singlets. The absence of right-handed neutrino fields in the SM is associated with the fact that, at the moment when the SM was being created, the neutrino was considered to be a zero-mass particle.

However, the phenomenon of neutrino oscillations (transitions between neutrinos with different flavors) experimentally discovered recently [3] testifies that the neutrino mass is nonzero. Therefore, one of the simplest and most promising variants of SM modification is the extension of its fermionic sector by adding several SU(2)-singlet right-handed neutrinos to the theory, which would not interact directly with SM particles [1]. In this case, the introduction of only two right-handed neutrinos gives rise to the appearance of eleven new parameters in the modified theory, which can be used to explain the available experimental data on active neutrino oscillations. On the basis of the so-called see-saw mechanism of neutrino mass generation [4], this model predicts the existence of two massive active neutrinos and one zero-mass neutrino, which does not contradict the available data. However, the extension of SM due to the introduction of only two right-handed neutrinos does not resolve other SM problems. In particular, it does not explain the phenomena of baryon asymmetry and dark matter.

Recently, in works [5, 6], a modification to the Standard Model Lagrangian has been proposed by introducing of three additional SU(2)-singlet right-handed (sterile) neutrinos (with zero electric, weak, and strong charges). It looks natural, taking into account that the number of fermion generations is also equal to three. The masses of right-handed neutrinos are considered to

[1] That is why those neutrinos have been called sterile, and left-handed SM neutrinos are referred to as active.

be less than the characteristic scale of weak interaction ($\lesssim M_Z \approx 100$ GeV, where M_Z is the Z-boson mass). Such a confinement does not introduce new energy scales in comparison with the SM ones and resolves the problem of gauge hierarchy[2], so that the theory remains correct up to the Planck energy scales.

The proposed model was called the Neutrino Minimal Standard Model (νMSM). Owing the fact that the νMSM contains 18 new parameters in comparison with the SM[3], it can explain, in principle, not only neutrino oscillations, but also other observable facts which do not find explanation in the framework of SM.

Until now, explicit expressions for the description of neutrino oscillations, baryon asymmetry generation, and dark matter have been obtained [7]. The majority of those expressions are approximate and serve for estimation purposes. In this work, we have studied the equations of the νMSM which couple the elements of the Yukawa and active neutrino mass matrices in order to determine the restrictions on the model parameters more exactly. The solutions obtained for the indicated equations have been used to study the CP-violating phase in the case where the elements of the active neutrino mass matrix are real-valued.

The paper is organized as follows. In Section 2, the basic relations and some results obtained in the framework of νMSM, which will be needed below, are presented. In Section 3, the analysis of νMSM equations is carried out, and the solution for the ratios between the elements of the second and third columns of the Yukawa matrix is obtained and analyzed in the general case. The results of Section 3 are used in Section 4 to analyze the expression obtained for the CP-violating phase in the framework of our model.

2. Basic Relations of the νMSM Theory

In works [5, 6], in addition to kinetic terms, the SM Lagrangian includes the following terms:

$$\mathcal{L}^{ad} = -F_{\alpha I}\bar{L}_\alpha \tilde{\Phi} N_I - \frac{M_{IJ}}{2}\bar{N}_I^c N_J + \text{h.c.} =$$

$$= -F_{\alpha I}\left(\bar{\nu}_{\alpha L}, \bar{l}_{\alpha L}\right)\begin{pmatrix}\frac{h(\chi)+v}{\sqrt{2}} \\ 0\end{pmatrix}\nu_{IR} - \frac{M_{IJ}}{2}\bar{N}_I^c N_J + \text{h.c.} =$$

$$= -F_{\alpha I}\frac{h(\chi)+v}{\sqrt{2}}\bar{\nu}_{\alpha L}\nu_{IR} - \bar{\nu}_{IR}^c \frac{M_{IJ}}{2}\nu_{JR} + \text{h.c.}, \tag{1}$$

where the subscript $\alpha = e, \mu, \tau$ corresponds to the flavors of active neutrinos; the subscripts I and J change from 1 to 3; L_α is the lepton doublet; N_I are the field functions of right-handed sterile neutrinos; the superscript c means the charge conjugation of the field function; $F_{\alpha I}$ is a new matrix of the Yukawa constants; M_{IJ} is the Majorana mass matrix of right-handed neutrinos; $\Phi = \begin{pmatrix} 0 \\ \frac{h(\chi)+v}{\sqrt{2}}\end{pmatrix}$ is the Higgs field in the unitary gauge; $\tilde{\Phi} = i\sigma_2\Phi^*$; σ_2 is the second Pauli matrix; $h(x)$ is the Higgs field; and the parameter ν is defined by the minimum of the Higgs field potential ($\nu = 247$ GeV).

In the SM, the fermion mass generation is provided by the interaction between fermionic and scalar Higgs fields. The SM has such a structure that, after a spontaneous symmetry violation, the neutrino remains a zero-mass particle. The Dirac mass term ($\sim \bar{\nu}_L\nu_R$) does not emerge due to the absence of right-handed singlet neutrino in the theory, whereas the appearance of the Majorana mass ($\sim \bar{\nu}_L^c\nu_L$) is forbidden by the $SU(2)_L$ invariance. An assumption on the existence of right-handed neutrino leads to the appearance of both Dirac and Majorana mass terms in the Lagrangian,

$$\mathcal{L}^{DM} = M^D\bar{\nu}_L\,\nu_R + M^M\overline{\nu_R^c}\,\nu_R + \text{h.c.}, \tag{2}$$

which can be written down in the general case as follows (see, e.g., works [8, 9]):

$$\mathcal{L}^{DM} = -\left(\overline{(N_L)^c}\,\frac{M^{DM}}{2}\,N_L + \text{h.c.}\right), \tag{3}$$

where

$$N_L = \begin{pmatrix}\nu_L \\ \nu_R^c\end{pmatrix};\ N_L^c = \begin{pmatrix}\nu_L^c \\ \nu_R\end{pmatrix};\ M^{DM} = \begin{pmatrix} M_L & {M_D}^T \\ M_D & M_R\end{pmatrix}.$$

Comparing the form of mass terms in Lagrangian (1) with expression (3), we come to a conclusion that, in this case,

$$M_L = 0, \quad M_D = F^+\frac{\nu}{\sqrt{2}}, \quad M_R = M^*, \tag{4}$$

where M and F are the square matrices of the third order which appear in formula (1).

In works [5, 6], it was demonstrated that the νMSM parameters, the number of which is by 18 more in comparison with those in the SM, can be so fitted that neutrino oscillations and the baryon asymmetry would be

[2] The problem of quantum stability of the Higgs boson mass to radiation corrections given by contributions of heavier particles is meant here.

[3] Namely, these are three Majorana neutrino masses, three Dirac neutrino masses, six mixing angles, and six CP-violating phases.

explained, and the nature of the dark matter would be established simultaneously. For this purpose, there must exist two right-handed neutrinos with large, almost identical, masses (of about or more than 100 MeV) and one right-handed neutrino with a relatively small mass (of about 1 keV). The lightest right-handed neutrino comprises the basis of the dark matter. The two other, mass-degenerate heavy neutrinos make it possible to explain active neutrino oscillations and the baryon asymmetry of the Universe.

In the zeroth-order approximation, the extended Lagrangian $\mathcal{L}_{\nu\text{MSM}}$ is invariant with respect to $U(1)_e \times U(1)_\mu \times U(1)_\tau$ transformations, which provides the individual preservation for e, μ, and τ lepton numbers (just this case is observed in experiments). In addition, in this approximation, two heavy sterile neutrinos are supposed to interact with active ones, whereas the third and the lightest one does not[4]. Those requirements are satisfied by the following forms of matrices M in expression (1): $M_L^{(0)} = 0$,

$$M_R^{(0)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix}; \quad M_D^{(0)+} = \frac{\nu}{\sqrt{2}} \begin{pmatrix} 0 & h_{12} & 0 \\ 0 & h_{22} & 0 \\ 0 & h_{23} & 0 \end{pmatrix}. \tag{5}$$

In the zeroth-order approximation, there exist two massive right-handed neutrinos with identical masses M, the third right-handed neutrino is massless, and the masses of all active neutrinos are equal to zero, which does not agree with the experimental data on neutrino oscillations.

In work [10], to put the theory in agreement with experimental data, small corrections to the matrices M_R and M_D (see Eqs. (5)) were introduced. These corrections violate the $U(1)_e \times U(1)_\mu \times U(1)_\tau$-symmetry, bring about the appearance of a small mass for the third right-handed neutrino, and eliminate the mass degeneration for two other right-handed neutrinos. In turn, this gives rise to the appearance of supersmall masses for the left-handed (active) neutrinos and nonzero angles of their mixing. The indicated corrections can be presented in the following form:

$$M_L^{(1)} = 0; \quad M_R^{(1)} = \begin{pmatrix} m_{11} e^{-i\alpha} & m_{12} & m_{13} \\ m_{12} & m_{22} e^{-i\beta} & 0 \\ m_{13} & 0 & m_{33} e^{-i\gamma} \end{pmatrix};$$

$$M_D^{(1)+} = \frac{\nu}{\sqrt{2}} \begin{pmatrix} h_{11} & 0 & h_{13} \\ h_{21} & 0 & h_{23} \\ h_{31} & 0 & h_{33} \end{pmatrix}, \tag{6}$$

where the matrix elements are considered complex-valued in general, and the relations $|m_{ij}| \ll |M|$ and $|h_{i1}| \ll |h_{i3}| \ll |h_{i2}|$ are adopted.

As is known [8], the mass part of Lagrangian (3) can be diagonalized by changing over from the basis of functions N_L to the basis of functions n_L using the unitary matrix V, namely, $N_L = V n_L$. Then

$$\bar{N}_L = \bar{n}_L V^+; \quad {N_L}^c = (V^+)^T n_L^c; \quad \overline{N}_L{}^c = \overline{(n_L)^c} V^T, \tag{7}$$

where the 6×6-matrix V can be conveniently presented as a product of two matrices, $V = WU$. The matrix W is introduced for the block diagonalization of the matrix M^{DM} [11],

$$W^T M^{DM} W = M^{\text{block}} = \begin{pmatrix} M_{\text{light}} & 0 \\ 0 & M_{\text{heavy}} \end{pmatrix}. \tag{8}$$

The explicit form of the matrix W can be found only approximately, with an accuracy to $M_R^{-1} M_D$-terms,

$$W = \begin{pmatrix} 1 - \frac{1}{2} M_D^+ (M_R M_R^+)^{-1} M_D & M_D^+ (M_R^+)^{-1} \\ -M_R^{-1} M_D & 1 - \frac{1}{2} M_R^{-1} M_D M_D^+ (M_R^+)^{-1} \end{pmatrix}. \tag{9}$$

Since the see-saw mechanism is used, the $M_R^{-1} M_D$ elements are small,

$$M_R^{-1} M_D \equiv \varepsilon \ll 1. \tag{10}$$

Then

$$W = \begin{pmatrix} 1 - \frac{1}{2}\varepsilon^+ \varepsilon & \varepsilon^+ \\ -\varepsilon & 1 - \frac{1}{2}\varepsilon\varepsilon^+ \end{pmatrix}. \tag{11}$$

The matrix W is unitary ($WW^+ = E$) to an accuracy of ε^4-terms. In this approximation, the result of the block diagonalization is as follows:

$$W^T M^{DM} W = \begin{pmatrix} -M_D^T M_R^{-1} M_D & 0 \\ 0 & M_R \end{pmatrix} = \begin{pmatrix} M_{\text{light}} & 0 \\ 0 & M_{\text{heavy}} \end{pmatrix}. \tag{12}$$

[4] In this model, the lightest sterile neutrino plays the role of a dark matter particle. For this reason, it cannot interact with other particles.

We note that the matrix M_{light}, whose characteristic values determine the active neutrino masses, is completely determined in terms of elements of the matrices M_D and M_R.

The matrix U looks like

$$U = \begin{pmatrix} U_1 & 0 \\ 0 & U_2 \end{pmatrix}, \tag{13}$$

where the 3×3-matrices $U_{(1,2)}$ are so selected that the obtained block matrix can be diagonalized:

$$m = \text{diag}(m_1, m_2, \dots m_6) = V^T M V =$$

$$= U^T W^T M W U = U^T M^{\text{block}} U, \tag{14}$$

that is,

$$m = \begin{pmatrix} U_1^T & 0 \\ 0 & U_2^T \end{pmatrix} \begin{pmatrix} M_{\text{light}} & 0 \\ 0 & M_{\text{heavy}} \end{pmatrix} \begin{pmatrix} U_1 & 0 \\ 0 & U_2 \end{pmatrix} =$$

$$= \begin{pmatrix} U_1^T M_{\text{light}} U_1 & 0 \\ 0 & U_2^T M_{\text{heavy}} U_2 \end{pmatrix}. \tag{15}$$

There is the standard parametrization for the matrices $U_{(1,2)}$ [12]:

$$U_{(1,2)} = \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times$$

$$\times \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{16}$$

where $c_{ij} = \cos\theta_{ij}$; $s_{ij} = \sin\theta_{ij}$; θ_{12}, θ_{13}, and θ_{23} are three mixing angles; δ is the Dirac phase; and α_1 and α_2 are the Majorana phases. The angles θ_{ij} can be selected within the interval $0 \leq \theta_{ij} \leq \pi/2$. The phases δ, α_1, and α_1 vary from 0 to 2π. The matrices $U_{(1)}$ and $U_{(2)}$ are characterized by individual independent values of their angles and phases.

Hence, the procedure of mass determination for active and sterile neutrinos is reduced to the diagonalization of matrix (4). This diagonalization can be carried out for the matrices M_{light} and M_{heavy} separately. Since the matrices M_{light} and M_{heavy} are non-Hermitian, the characteristic values of the Hermitian matrices $M^+_{\text{light}} M_{\text{light}}$ and $M^+_{\text{heavy}} M_{\text{heavy}}$ are sought by solving the corresponding equation. The determined characteristic values correspond to the squared characteristic values of the matrices M_{light} and M_{heavy}.

We omit cumbersome mathematical calculations and present the final result obtained in the approximation where the elements in the first column of the Yukawa matrix are neglected. The mass of the lightest active neutrino is

$$m_1 = \frac{[h^+ h]_{11}\, v^2}{2m_{11}} = 0, \tag{17}$$

and the nondiagonalized mass matrix of active neutrinos looks like

$$[M_{\text{light}}]_{\alpha\beta} = \frac{\nu^2}{2M}(\tilde{h}_{\beta 3} h_{\alpha 2} + \tilde{h}_{\alpha 3} h_{\beta 2}), \tag{18}$$

where $\tilde{h}_{\beta 3} = h_{\beta 3} - \frac{m_{33}}{2M} h_{\beta 2}$. The characteristic values of the matrix are

$$m_{2,3} = \frac{v^2}{2M}(F_2 \tilde{F}_3 \pm |h^+ \tilde{h}|_{23}), \tag{19}$$

where $F_2^2 = [h^+ h]_{22}$, $\tilde{F}_3^2 = [\tilde{h}^+ \tilde{h}]_{33}$, and $F_2 \tilde{F}_3 = M(m_2 + m_3)/v^2$.

3. Study of the Yukawa Matrix Parameters for Sterile Neutrinos

The elements of the matrix M_{light} in expression (18) are known, though with considerable errors, from experiments on neutrino oscillations (see, e.g., works [3, 12]). The system of equations (18), which couples elements of the second and third columns of the Yukawa matrix with elements of the active neutrino matrix, has an infinite number of solutions. Really, the substitutions of h_{i2} by $z h_{i2}$ and $\tilde{h}_{i3}$ by $\tilde{h}_{i2}/z$, where z is an arbitrary complex number, does not change the form of expression (18).

Let us analyze the system of equations (18), having rewritten it as follows:

$$M_{ij} = \eta[\tilde{h}_{j3} h_{i2} + \tilde{h}_{i3} h_{j2}], \text{ де } \quad i, j = 1, 2, 3, \ \eta = \frac{v^2}{2M}, \tag{20}$$

where M_{ij} are the elements of the active neutrino matrix M_{light} known from experiment. To find the solutions of Eqs. (20), let us write down the expressions for diagonal,

$$\begin{cases} M_{11} = 2\eta \tilde{h}_{13} h_{12}, \\ M_{22} = 2\eta \tilde{h}_{23} h_{22}, \\ M_{33} = 2\eta \tilde{h}_{33} h_{32}, \end{cases} \tag{21}$$

and non-diagonal,

$$\begin{cases} M_{12} = \eta[\tilde{h}_{23}h_{12} + \tilde{h}_{13}h_{22}], \\ M_{13} = \eta[\tilde{h}_{33}h_{12} + \tilde{h}_{13}h_{32}], \\ M_{23} = \eta[\tilde{h}_{33}h_{22} + \tilde{h}_{23}h_{32}], \end{cases} \quad (22)$$

elements of the active neutrino matrix. From expressions (21), we obtain

$$\tilde{h}_{13} = \frac{M_{11}}{2\eta h_{12}}; \qquad \tilde{h}_{23} = \frac{M_{22}}{2\eta h_{22}}; \qquad \tilde{h}_{33} = \frac{M_{33}}{2\eta h_{32}} \quad (23)$$

and substitute those values into Eqs. (22):

$$\begin{cases} M_{12} = \frac{1}{2}\left(M_{22}\frac{h_{12}}{h_{22}} + M_{11}\frac{h_{22}}{h_{12}}\right), \\ M_{13} = \frac{1}{2}\left(M_{33}\frac{h_{12}}{h_{32}} + M_{11}\frac{h_{32}}{h_{12}}\right), \\ M_{23} = \frac{1}{2}\left(M_{33}\frac{h_{22}}{h_{32}} + M_{22}\frac{h_{32}}{h_{22}}\right). \end{cases} \quad (24)$$

The solution of Eqs. (24) is

$$\begin{cases} A_{12} = \frac{M_{12}}{M_{22}}\left(1 \pm \sqrt{1 - \frac{M_{11}M_{22}}{{M_{12}}^2}}\right), \\ A_{13} = \frac{M_{13}}{M_{33}}\left(1 \pm \sqrt{1 - \frac{M_{11}M_{33}}{{M_{13}}^2}}\right), \\ A_{23} = \frac{M_{23}}{M_{33}}\left(1 \pm \sqrt{1 - \frac{M_{22}M_{33}}{{M_{23}}^2}}\right), \end{cases} \quad (25)$$

where

$$A_{12} \equiv \frac{h_{12}}{h_{22}}, \qquad A_{13} \equiv \frac{h_{12}}{h_{32}}, \qquad A_{23} \equiv \frac{h_{22}}{h_{32}}. \quad (26)$$

Hence, formally, there are eight different solutions, with only four of them being independent. For instance, if we fix the sign before the square roots in expressions for A_{12} and A_{13}, the quantity A_{23} is determined unambiguously due to the relation

$$A_{23} = A_{13}/A_{12}, \quad (27)$$

Experimental restrictions on active neutrino parameters

Central value	99% confidence interval
$\lvert\Delta m^2_{12}\rvert = (8.0 \pm 0.3) \times 10^{-5}\,\text{eV}^2$	$(7.2 - 8.9) \times 10^{-5}\,\text{eV}^2$
$\lvert\Delta m^2_{23}\rvert = (2.5 \pm 0.2) \times 10^{-3}\,\text{eV}^2$	$(2.1 - 3.1) \times 10^{-3}\,\text{eV}^2$
$\tan^2\theta_{12} = 0.45 \pm 0.05$	$30^\circ < \theta_{12} < 38^\circ$
$\sin^2 2\theta_{23} = 1.02 \pm 0.04$	$36^\circ < \theta_{23} < 54^\circ$
$\sin^2 2\theta_{13} = 0 \pm 0.05$	$\theta_{13} < 10^\circ$

in which the elements M_{ij} are expressed in terms of the known parameters of the mixing matrix and the masses of active neutrinos (15) (see, e.g., work [8]):

$$M_{ij} = m_1 U^*_{(1)i1}U^*_{(1)j1} + m_2 U^*_{(1)i2}U^*_{(1)j2} + m_3 U^*_{(1)i3}U^*_{(1)j3}, \quad (28)$$

where $U_{(1)}$ is the the third-order square matrix of neutrino mixing (see formula (16)), and its elements are partially known from experimental data (see Table). It is worth noting that the experimental data on neutrino oscillations determine the differences between the squares of neutrino masses, $\Delta m^2_{12} = m_1^2 - m_2^2$ and $\Delta m^2_{23} = m_2^2 - m_3^2$, rather than the masses of active neutrinos themselves, so that it is impossible to unambiguously determine the masses of the neutrinos in a direct way. Really, if each of the neutrino masses is increased by the same constant, the Δm^2_{12}- and Δm^2_{23}-values do not change. Supposing, that one neutrino is much lighter than the others, the masses of other neutrinos can be determined. In this case, there are two, formally equal, cases, which are referred to as normal and inverse neutrino mass hierarchies.

In the case of normal hierarchy, the masses of active neutrinos are assumed to grow with the increase of their number. The mass of the lightest neutrino is m_1, and that of the heaviest one is m_3, with $m_1 < m_2 < m_3$. Assuming that $m_1 = 0$, we obtain $m_2 = \sqrt{\lvert\Delta m^2_{12}\rvert} \approx 0.009$ eV and $m_3 = \sqrt{\lvert\Delta m^2_{23}\rvert} \approx 0.05$ eV.

In the case of inverse hierarchy, the masses of active neutrinos are assumed to decrease with the growth of their number. The mass of the heaviest neutrino is m_1, and that of the lightest one is m_3, with $m_1 > m_2 > m_3$. Assuming roughly that $m_3 = 0$, we obtain $m_2 = \sqrt{\lvert\Delta m^2_{23}\rvert} \approx 0.05$ eV. Since $\lvert\Delta m^2_{12}\rvert \ll \lvert\Delta m^2_{23}\rvert$, one may take that $m_1 \approx m_2$.

Since the system of equations (20) is written down in the approximation $m_1 = 0$, the phase α_1 is excluded from all expressions (see Eqs. (28) and (16)), and the active neutrino parameters are the following ones: two masses m_2 and m_3; three mixing angles θ_{12}, θ_{13}, and θ_{23}; one Dirac, δ, and one Majorana, α_2, CP-violating phases. Since $M \gg m_{33}$ (see formulas (5) and (6)), we consider below that

$$\tilde{h}_{i3} = h_{i3},\; F_3 = \tilde{F}_3. \quad (29)$$

Solution (25) determines only relations between elements of the second and third columns in the Yukawa matrix. In particular, making use of expressions (23)

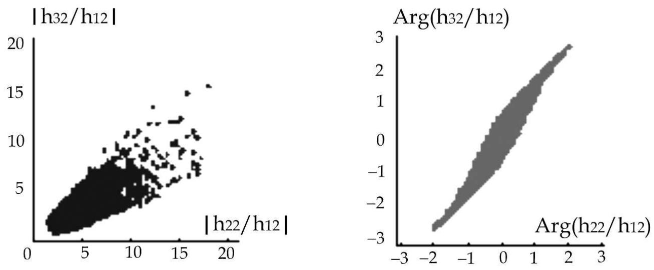

Fig. 1. Ratios between the absolute values (a) and phases (b) of the elements of the second column in the Yukawa matrix. The case of normal hierarchy

and (26), one can demonstrate that

$$\{h_{12}; h_{22}; h_{32}\} = h_{12}\left\{1; \frac{h_{22}}{h_{12}}; \frac{h_{32}}{h_{12}}\right\} =$$

$$= h_{12}\left\{1; {A_{12}}^{-1}; {A_{13}}^{-1}\right\}, \tag{30}$$

$$\{h_{13}; h_{23}; h_{33}\} = h_{13}\left\{1; \frac{h_{23}}{h_{13}}; \frac{h_{33}}{h_{13}}\right\} =$$

$$= h_{13}\left\{1; A_{12}\frac{M_{22}}{M_{11}}; A_{13}\frac{M_{33}}{M_{11}}\right\}, \tag{31}$$

or, in the dimensionless form,

$$\frac{\{h_{12}; h_{22}; h_{32}\}}{F_2} = \frac{e^{i\cdot\arg(h_{12})}\left\{1; {A_{12}}^{-1}; {A_{13}}^{-1}\right\}}{\sqrt{1 + |A_{12}|^{-2} + |A_{13}|^{-2}}}, \tag{32}$$

$$\frac{\{h_{13}; h_{23}; h_{33}\}}{F_3} = \frac{e^{i\cdot\arg(h_{13})}\left\{1; A_{12}\frac{M_{22}}{M_{11}}; A_{13}\frac{M_{33}}{M_{11}}\right\}}{\sqrt{1 + \left|A_{12}\frac{M_{22}}{M_{11}}\right|^2 + \left|A_{13}\frac{M_{33}}{M_{11}}\right|^2}}, \tag{33}$$

where the phases of the elements h_{12} and h_{13} are connected by Eq. (21):

$$\arg(h_{12}) + \arg(h_{13}) = \arg(M_{11}), \tag{34}$$

and the parameters A_{12}, A_{13}, M_{11}, M_{22}, and M_{33} are determined unambiguously in terms of the parameters m_2, m_3, θ_{12}, θ_{13}, θ_{23}, α_2, and δ. The parameters h_{12} and h_{13} can be regarded as arbitrary quantities.

It should be noted that the system of Eqs. (20) is symmetric with respect to the swapping of corresponding elements in the second and third columns of the Yukawa matrix. If there exists a solution for A_{12} and A_{13} at fixed signs before the roots in formula (25) – so that the sign in the expression for A_{23} is also unambiguously fixed – we obtain relations between the elements of the Yukawa matrix in forms (30) and (31). In this case, it can be shown that the simultaneous changes of signs before the roots in the expressions for A_{12}, A_{13}, and A_{23} invoke the swapping of relations between the elements of the second and third columns in the Yukawa matrix.

At every fixed value of the parameters m_2, m_3, θ_{12}, θ_{13}, θ_{23}, α_2, and δ, there exist only two variants of the sign choice in expressions (25) for A_{12}, A_{13}, and A_{23} which do not contradict condition (27). They differ from each other by simultaneous sign changes before the square roots in expressions (25) for A_{12}, A_{13}, and A_{23}. Actually, such a change corresponds to the swapping of the second and third columns, which gives rise to the corresponding variation of active and sterile neutrino mixing angles. This statement is always correct, but for some cases of parameter values, when at least one of the radicands in expression (25) equals zero.

It is convenient to use numerical methods for the further analysis of system (20). For this purpose, we determine the values of $|h_{22}/h_{12}|$, $|h_{32}/h_{12}|$, $|h_{23}/h_{13}|$, and $|h_{33}/h_{13}|$ at every fixed point that falls into the experimentally allowable ranges for the parameters m_2, m_3, θ_{12}, θ_{13}, θ_{23}, α_2, and δ (see Table) and present the results of calculations graphically.

In the framework of the νMSM theory (17), in the case of normal active neutrino mass hierarchy, assuming $m_1 = 0$, and according to the data presented in Table, we obtain that 0.0085 eV $\leq m_2 \leq 0.0094$ eV, 0.046 eV $\leq m_3 \leq 0.056$ eV, $30^\circ \leq \theta_{12} \leq 38^\circ$, $36^\circ \leq \theta_{23} \leq 54^\circ$, $0^\circ \leq \theta_{13} \leq 10^\circ$, and both phases δ and α_2 change from $-\pi$ to π. For the indicated values of neutrino parameters and assuming that $m_2 = 0.009$ eV and $m_3 = 0.05$ eV, we calculated the arrays of pairs ($|h_{22}/h_{12}|$,$|h_{32}/h_{12}|$) and ($|h_{23}/h_{13}|$,$|h_{33}/h_{13}|$) for 10000 random points in the space of parameters θ_{12}, θ_{13}, θ_{23}, α_2, and δ. The results of calculations are depicted in Fig. 1.

In the case of inverse hierarchy, according to Eq. (17), we have $m_1 \to 0$ eV, and, according to Table, we must have $m_3 \to 0$ eV. To make an agreement with the νMSM, the mass states m_3 and m_1 are to be swapped. It can be done by an additional unitary rotation with the help of the matrix

$$\tilde{U} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad U_{(1)} \to U_{(1)}\tilde{U}, \tag{35}$$

where $U_{(1)}$ is the matrix of neutrino mixing in the mass and flavor bases (see Eq. (16)). Then, we may put $m_3 = 0$, choose the central values $m_1 = m_2 = 0.05$ eV for the parameters m_1 and m_2, and leave the mixing angles and phases the same as they were at the normal hierarchy. Those neutrino parameters were used to calculate the corresponding values for ten thousand random points in the space of parameters θ_{12}, θ_{13}, θ_{23}, α_2, and δ. The results of calculations are plotted in Fig. 2.

The analysis of the results obtained demonstrates that, in the case of normal hierarchy (Fig. 1,*a*), the ratios $|h_{32}/h_{12}|$ and $|h_{22}/h_{12}|$ between the elements in the second column fall into the intervals $0.65 \lesssim |h_{32}/h_{12}| \lesssim 24.2$ and $1.4 \lesssim |h_{22}/h_{12}| \lesssim 29.6$. For a random point in the space of parameters θ_{12}, θ_{13}, θ_{23}, α_2, and δ, the values of the ratios $|h_{32}/h_{12}|$ and $|h_{22}/h_{12}|$ more probably lie in the interval, roughly, from 1 to 10, whereas the ratios between the Yukawa constants that exceed 10 are hardly probable.

Note that the value of the ratio between the phases of the Yukawa matrix elements $\mathrm{Arg}\,[h_{32}/h_{12}]$ and $\mathrm{Arg}\,[h_{22}/h_{12}]$ does not extend over the whole allowed range $(-\pi, \pi)$, but is confined to a closed compact region depicted in Fig. 1,*b*.

In the case of inverse hierarchy (Fig. 2,*b*), the element ratios $|h_{32}/h_{12}|$ and $|h_{22}/h_{12}|$ fall within the intervals $0 \leq |h_{32}/h_{12}| \lesssim 3.2$ and $1.1 \lesssim |h_{22}/h_{12}| \lesssim 4.3$. The extremely large values of those ratios are also hardly probable at that. The fact that the ratio $|h_{32}/h_{12}|$ can be equal to zero is explained by the circumstance that $|h_{32}/h_{12}| = A_{12}^{-1} \sim M_{12}$, whereas M_{12} can be equal to zero in the allowed variation ranges for angles and phases, provided that $m_2 = m_3$. Therefore, in contrast to the case of normal hierarchy, the elements $|h_{i2}|$ can be of different orders of magnitude.

Analogously to the case of normal hierarchy, the ratios between the phases of the Yukawa matrix elements $\mathrm{Arg}\,[h_{32}/h_{12}]$ and $\mathrm{Arg}\,[h_{22}/h_{12}]$ are scattered only over a closed compact region, which is depicted in Fig. 2,*b*.

In both cases of normal and inverse hierarchies, the graphically presented values for the relation between the absolute values of the ratios between the elements in the third column of the Yukawa matrix ($|h_{33}/h_{13}|$ versus $|h_{23}/h_{13}|$) are similar to those for the relation presented in Figs. 1,*a* and 2,*a*; and the corresponding relations between the phases ($\mathrm{Arg}\,[h_{33}/h_{13}]$ versus $\mathrm{Arg}\,[h_{23}/h_{13}]$) in the range $(0, 2\pi)$ are similar to those exhibited in Figs. 1,*b* and 2,*b*. If the number of points used for calculations grows, the indicated graphic dependences become identical.

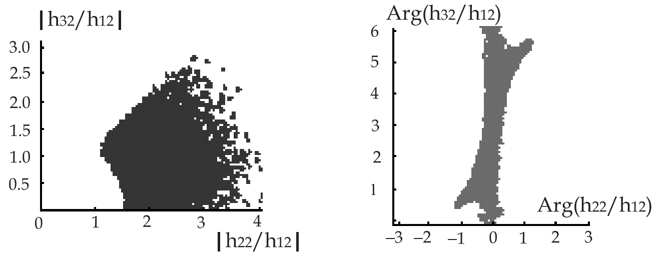

Fig. 2. The same as in Fig. 1, but for the case of inverse hierarchy

4. Analysis of CP-violating Phase in the νMSM Theory

As was shown by A.D. Sakharov [13], for the baryon asymmetry to be generated at a certain stage of the Universe evolution, the following three conditions have to be satisfied simultaneously:

1) baryonic charge violation,

2) C- and CP-violation,

3) violation of thermodynamic equilibrium.

As is known from the field theory, the starting Lagrangian transforms under CP-transformation into another Lagrangian with complex-conjugate coupling constants. Provided that some of those constants include unremovable phases, the CP-invariance of the theory becomes broken. In the SM, neutrinos are zero-mass particles; therefore, a unique source of CP-violation in weak interactions is the single complex element in the Cabibbo–Kobayashi–Maskawa matrix, the latter describing the mixing of quarks belonging to different generations. In the νMSM theory, owing to the existence of neutrinos with nonzero masses, different neutrino generations mix with one another, so that one more probable source of CP-violation emerges.

The solutions obtained in the previous section for the ratios between the elements of the second and third columns in Yukawa matrix (25) can be used to analyze the expression for the CP-violating phase derived in the framework of the νMSM [14]. As was already indicated, in the νMSM, there is an opportunity to generate a baryon asymmetry owing to CP-violating oscillations of light active neutrinos into sterile ones. In this case, the total leptonic number in the system changes, which results in the appearance of a lepton asymmetry, generating, in turn, a baryon asymmetry at sphaleronic transitions [15].

To analyze the CP-violating phase, let us use the following expression obtained in work [14]:

$$\delta_{\rm CP}=\frac{1}{F^6}\left[{\rm Im}(h^+h)_{23}\sum_\alpha(|h_{\alpha2}|^4-|h_{\alpha3}|^4)-\right.$$

$$\left.-\,(F_2^2-F_3^2)\sum_\alpha(|h_{\alpha2}|^2+|h_{\alpha3}|^2){\rm Im}[h^*_{\alpha2}h_{\alpha3}]\right]. \quad (36)$$

Let us consider every component of this expression:

$$F^6=\left(|h_{12}|^2+|h_{22}|^2+|h_{32}|^2\right)^3=$$

$$=|h_{12}|^6\left(1+\left|\frac{1}{A_{12}}\right|^2+\left|\frac{1}{A_{13}}\right|^2\right)^3, \quad (37)$$

$${\rm Im}[h^+h]_{23}={\rm Im}\left[h^*_{12}h_{13}+h^*_{22}h_{23}+h^*_{32}h_{33}\right]=$$

$$={\rm Im}\left[\frac{h^*_{12}}{h_{12}}h_{12}h_{13}+\frac{h^*_{22}}{h_{22}}h_{22}h_{23}+\frac{h^*_{32}}{h_{32}}h_{32}h_{33}\right]=$$

$$=\left(\frac{\nu^2}{M}\right)^{-1}{\rm Im}\left[\frac{h^*_{12}}{h_{12}}\cdot M_{11}+\frac{h^*_{22}}{h_{22}}M_{22}+\frac{h^*_{32}}{h_{32}}M_{33}\right]=$$

$$=\left(\frac{\nu^2}{M}\right)^{-1}{\rm Im}\left[\frac{h^*_{12}}{h_{12}}\left(M_{11}+\frac{A_{12}}{A^*_{12}}M_{22}+\frac{A_{13}}{A^*_{13}}M_{33}\right)\right], \quad (38)$$

$$\sum_\alpha\left(|h_{\alpha2}|^4-|h_{\alpha3}|^4\right)=|h_{12}|^4\left(1+\left|\frac{1}{A_{12}}\right|^4+\left|\frac{1}{A_{13}}\right|^4-\right.$$

$$\left.-\left|\frac{h_{13}}{h_{12}}\right|^4\left\{1+\left|A_{12}\frac{M_{22}}{M_{11}}\right|^4+\left|A_{13}\frac{M_{33}}{M_{11}}\right|^4\right\}\right), \quad (39)$$

$$F_2^2-F_3^2=|h_{12}|^2\left[1+\left|\frac{1}{A_{12}}\right|^2+\left|\frac{1}{A_{13}}\right|^2-\left|\frac{h_{13}}{h_{12}}\right|^2\times\right.$$

$$\left.\times\left\{1+\left|A_{12}\frac{M_{22}}{M_{11}}\right|^2+\left|A_{13}\frac{M_{33}}{M_{11}}\right|^2\right\}\right], \quad (40)$$

$$\sum_\alpha\left(|h_{\alpha2}|^2+|h_{\alpha3}|^2\right){\rm Im}[h^*_{\alpha2}h_{\alpha3}]=$$

$$=|h_{12}|^2\frac{M}{\nu^2}\left\{\left(1+\left|\frac{h_{13}}{h_{12}}\right|^2\right){\rm Im}\left[\frac{h^*_{12}}{h_{12}}M_{11}\right]+\right.$$

$$+\left(\frac{1}{|A_{12}|^2}+\left|\frac{h_{13}}{h_{12}}\right|^2\left|\frac{M_{22}}{M_{11}}A_{12}\right|^2\right){\rm Im}\left[\frac{h^*_{12}}{h_{12}}\frac{A_{12}}{A^*_{12}}M_{22}\right]+$$

$$\left.+\left(\frac{1}{|A_{13}|^2}+\left|\frac{h_{13}}{h_{12}}\right|^2\left|\frac{M_{33}}{M_{11}}A_{13}\right|^2\right){\rm Im}\left[\frac{h^*_{12}}{h_{12}}\frac{A_{13}}{A^*_{13}}M_{33}\right]\right\}, \quad (41)$$

Here, we took into account that $h_{32}=\frac{h_{12}}{A_{13}}$, $h_{22}=\frac{h_{12}}{A_{12}}$, $h_{23}=h_{13}\frac{h_{23}}{h_{13}}=h_{13}\frac{M_{22}}{M_{11}}A_{12}$, and $h_{33}=h_{13}\frac{h_{33}}{h_{13}}=h_{13}\frac{M_{33}}{M_{11}}A_{13}$.

Substituting formulas (37)–(41) into Eq. (36), we obtain the final expression for the CP-phase:

$$\delta_{\rm CP}(\xi,\varepsilon)=|M_{11}|^{-1}C^{-3}[\varepsilon\left({\rm Im}\left[e^{-2i\xi}A\right]B-CD\right)+$$

$$+\varepsilon^3(C_1D-CD_1)+\varepsilon^5\left(C_1D_1-B_1{\rm Im}\left[e^{-2i\xi}A\right]\right)], \quad (42)$$

where, besides the dependence on the masses and the parameters of the active neutrino mixing matrix, the dependence on the following parameters of the Yukawa matrix was singled out:

$$\xi=\arg[h_{12}],\quad \varepsilon=\left|\frac{h_{13}}{h_{12}}\right|=\epsilon\sqrt{C/C1}, \quad (43)$$

and the following notations were used:

$$\epsilon=F_3/F_2;\quad A=M_{11}+\frac{A_{12}}{A^*_{12}}M_{22}+\frac{A_{13}}{A^*_{13}}M_{33},$$

$$B=1+|A_{12}|^{-4}+|A_{13}|^{-4};\; C=1+|A_{12}|^{-2}+|A_{13}|^{-2},$$

$$B_1=1+\left|A_{12}\frac{M_{22}}{M_{11}}\right|^4+\left|A_{13}\frac{M_{33}}{M_{11}}\right|^4,$$

$$C_1=1+\left|A_{12}\frac{M_{22}}{M_{11}}\right|^2+\left|A_{13}\frac{M_{33}}{M_{11}}\right|^2,$$

$$D={\rm Im}\left[e^{-2i\xi}M_{11}\right]+|A_{12}|^{-2}{\rm Im}\left[e^{-2i\xi}\frac{A_{12}}{A^*_{12}}M_{22}\right]+$$

$$+|A_{13}|^{-2}\mathrm{Im}\left[e^{-2i\xi}\frac{A_{13}}{A_{13}^*}M_{33}\right],$$

$$D_1 = \mathrm{Im}\left[e^{-2i\xi}M_{11}\right] + \left|\frac{M_{22}}{M_{11}}A_{12}\right|^2 \mathrm{Im}\left[e^{-2i\xi}\frac{A_{12}}{A_{12}^*}M_{22}\right]+$$

$$+\left|\frac{M_{33}}{M_{11}}A_{13}\right|^2 \mathrm{Im}\left[e^{-2i\xi}\frac{A_{13}}{A_{13}^*}M_{33}\right].$$

Numerical calculations using the obtained expression (42) confirm general properties of CP-phase (36) reported in work [14]:
1) the sign of the CP-violating phase and, hence, the sign of the baryon asymmetry cannot be determined knowing only the elements of the active neutrino matrix;
2) if $\epsilon \to 0$, then $\delta_{\mathrm{CP}} \sim \epsilon$, and δ_{CP} also tends to zero;
3) the CP-violating phase can differ from zero[5], if $\epsilon = 1$;
4) the CP-violating phase can be different from zero in the cases where $\theta_{13} = 0$ and $\theta_{23} = \pi/4$;
5) in the inverse hierarchy case, the CP-violating phase can be different from zero in the case where $m_1 = m_2$, $\theta_{13} = 0$, and $\theta_{23} = \pi/4$.

Expression (42) includes parameters that vary in the known range[6]. This allows the limits that confine the value of CP-violating phase to be estimated. In particular, $|\delta_{\mathrm{CP}}| \lesssim 0.27$ for the normal hierarchy, and $|\delta_{\mathrm{CP}}| \lesssim 0.08$ for the inverse one.

Consider whether the baryon asymmetry is generated in a particular case where the active neutrino mixing matrix $U_{(1)}$ is real-valued. It can be done analytically. Since the elements of the active neutrino mass matrix M_{ij} are given explicitly in terms of the parameters of the mixing matrix (28), the matrix M_{ij} is also real-valued. The form of the matrix $M_{\mathrm{light}} = U_{(1)}^* m U_{(1)}^+$ can be found in the general case where the mixing matrix is real, irrespective of the restrictions dictated by the unitary matrix condition. One can get convinced that the following minors are positive:

$$\left[\begin{array}{l} M_{11}M_{22} - M_{12}^2 = m_2 m_3 (U_{12}U_{22} - U_{13}U_{23})^2 \geq 0, \\ M_{11}M_{33} - M_{13}^2 = m_2 m_3 (U_{12}U_{33} - U_{13}U_{32})^2 \geq 0, \\ M_{22}M_{33} - M_{23}^2 = m_2 m_3 (U_{22}U_{33} - U_{23}U_{32})^2 \geq 0, \end{array}\right. \quad (44)$$

where U_{ij} are the elements of the real-valued matrix U. This means that the radicands in Eq. (25) are negative (or equal zero in those particular cases where expression (44) equals zero).

Hence, the ratio between the elements of the Yukawa matrix (30) and (31) can be complex, which gives rise to the generation of a nonzero CP-phase even in the case where the active neutrino mixing matrix is real-valued. This result was confirmed by numerical calculations making use of the explicit expression (42).

5. Conclusions

Provided that the νMSM is valid and right-handed (sterile) neutrinos do exist – which can simultaneously explain active neutrino oscillations, ensure the baryon asymmetry generation, and elucidate the dark matter structure – rather severe restrictions are imposed on the νMSM parameters which can be experimentally tested[7]. For the time being, the numerous data of observations by means of XMM-Newton, Chandra, INTEGRAL, and Suzaku satellites did not discover evidences for the existence of right-handed neutrinos in the range allowed by instrument capabilities (see work [7] and the references therein). However, new researches are planned – e.g., the Xenia project [16] – which must test the whole range of model parameters that is allowed theoretically; in particular, these are the mixing angle of sterile neutrinos with active ones and the mass of the lightest right-handed neutrino. If the indicated experiments confirm the existence of sterile neutrinos, the exact solutions of νMSM equations will be useful for the analysis and treatment of the results observed.

In this work, we have analyzed the system of equations (18) which couples the elements of the Yukawa matrix in the νMSM with those of the active neutrino mass matrix (28). The element values for the latter are known to within a certain accuracy and specified in experiments on neutrino oscillations. General relations between the elements of the second and third columns in the Yukawa matrix have been obtained as functions of the parameters of the active neutrino mass matrix (30), (31). The same analysis for specific values of active neutrino mixing angles has been carried out in works [5–7, 10, 14].

The obtained solutions (25) were used to numerically determine the value ranges for the ratios between the

[5] Hereafter, the statement "the CP-violating phase can differ from zero" means that the mixing angles and phases can be chosen in such a way that $\delta_{CP} = 0$.

[6] There exists an experimentally allowed range for mixing angles (see Table); the phases vary from 0 to 2π; and $\epsilon \leq 1$.

[7] The matter concerns the search for the lightest right-handed neutrino decay into an active neutrino and a photon. This process is suppressed very much and can take place owing to right-handed neutrino oscillations into an active left-handed neutrino which interacts with SM particles.

absolute values and phases of the Yukawa matrix elements in the cases of the normal and inverse hierarchies, provided that the parameters of the active neutrino mass matrix fall into the experimentally allowed range (see Table). It has been demonstrated that, in the framework of the νMSM, even if the active neutrino mass matrix is real-valued, there can exist a nonzero CP-violating phase that is responsible for the baryon asymmetry generation.

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ТЕОРЕТИЧНІ ОБМЕЖЕННЯ НА ЕЛЕМЕНТИ МАТРИЦІ ЮКАВИ В МОДЕЛІ νMSM

В.М. Горкавенко, С.Й. Вільчинський

Р е з ю м е

У роботі проаналізовано та розв'язано систему рівнянь Стандартної моделі (νMSM) розширеної за рахунок додавання трьох правих нейтрино (синглети слабкого ізоспіну), що пов'язують елементи матриці Юкави з елементами масової матриці активних нейтрино, з метою подальшого отримання більш точного обмеження на значення параметрів моделі. На основі отриманих розв'язків проведено дослідження CP-порушуючої фази для випадку, коли елементи масової матриці активних нейтрино є дійсними. Показано, що в цьому випадку також може відбуватися генерація баріонної асиметрії.