AVERAGE RESONANCE PARAMETERS OF Ni AND Zn NUCLEI

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The complete sets of average resonance parameters S_0, S_1, R'_0, R'_1, and $S_{1,3/2}$ for nickel and zinc nuclei with a natural isotope composition have been obtained. They were determined by analyzing the average experimental differential cross-sections of low-energy neutron elastic scattering with the help of a method developed by the authors. The analysis of obtained results, recommended parameters, and some literary data has been carried out. The conclusion has been made that the recommended values of the parameter S_1 are underestimated by a factor of two for both nuclei concerned.

1. Introduction

Within the period of researches of average nuclear resonance parameters, five editions of the Atlas of recommended resonance parameters have already been published [1]. In comparison with the previous edition [2], there appeared a lot of new experimental data, which allowed one to put in order the dependences of parameters S_0, S_1, and R'_0 on the atomic weight A and to considerably reduce discrepancies between the parameters of certain nuclei and the results of calculations obtained in the framework of the optical model. However, there remained the unresolved problems concerning the minima of parameters S_0 (at $100 < A < 140$) and S_1 (at $A \leq 70$). In those ranges, the parameters of neighbor nuclei are 5 to 10 times different, which contradicts the ideology of the optical model and constrains its progress.

Isotopes of nickel and zinc are located in that A-range, where the magnitudes of the strength function S_1 for many nuclei reveal considerable discrepancies with the results of calculations carried out in the framework of the optical model. For those nuclei, recommended are the parameters S_0, S_1 and R'_0, R'_1 we used to calculate the weighted average values for nuclei with natural isotope compositions. Their comparison with the data of work [3] has demonstrated that the values of the parameters S_0 and S_1 are different by a factor of 2 to 5. In this connection, we tested the agreement of those parameters with experimental data and determined new parameter sets; it was done with the help of a method developed by us for the analysis of the differential cross-sections of elastic low-energy neutron scattering. This method has been successfully used earlier for the determination of resonance parameters of even isotopes of cadmium and tin [4], as well as some other nuclei.

2. Determination Technique for Average Resonance Parameters

Neutron scattering by nuclei at energies up to about 450 keV mainly occurs at the orbital moments $l = 0$ and 1. In this case, the differential cross-sections of elastic scattering can be expanded in a series of Legendre polynomials as follows:

$$\sigma_{\rm el}(\mu) = \frac{\sigma_{\rm el}}{4\pi}\{1 + \omega_1 P_1(\mu) + \omega_2 P_2(\mu)\}, \quad (1)$$

where $\mu = \cos\theta$, θ is the scattering angle, $\sigma_{\rm el}$ is the integral cross-section of elastic scattering, P_l are the Legendre polynomials, ω_1 and ω_2 are the expansion coefficients of the differential cross-sections. These coefficients are referred to as the angular moments of the scattering indicatrix, and they are equal to $\omega_l = (2l+1)\bar{P}_l$, where $\bar{P}_l$ are the Legendre polynomials averaged over the angles with the weight of a differential scattering cross-section. For even-even nuclei and provided that $\sigma_t \approx \sigma_{\rm el}$, we obtained the following expressions for the expansion coefficients [5]:

$$\omega_1 = \frac{6\pi\lambda^2}{\sigma_{\rm el}}(1 - \eta_{0\rm Re} - \eta_{1\rm Re} + \eta_{0\rm Re}\eta_{1\rm Re} + \eta_{0\rm Im}\eta_{1\rm Im}), \quad (2)$$

$$\omega_2 = \frac{2}{\sigma_{\rm el}}(\sigma_{s1} + \pi\lambda^2 T_{1,3/2}), \quad (3)$$

where $\eta_l = \eta_{l\rm Re} + i\eta_{l\rm Im}$ are the diagonal elements of the average scattering matrix, σ_{s1} are the cross-sections of potential neutron scattering with $l = 1$, and $T_{1,3/2}$ are the penetrability coefficients for $l = 1$ and $j = 3/2$.

In the optical model, the cross-sections $\sigma_{\rm el}$ consist of the corresponding partial cross-sections of compound

and potential neutron scattering, $\sigma_{\rm el} = \sigma_{c0} + \sigma_{c1} + \sigma_{s0} + \sigma_{s1}$ which are expressed in terms of matrix elements η_l. In the resonance theory, the average cross-sections also consist of the corresponding cross-sections of resonance and potential scatterings which are expressed, in turn, in terms of average resonance parameters. In the case of narrow resonances ($\Gamma \ll D$), the partial cross-sections given by the optical model coincide with the corresponding cross-sections obtained in the resonance theory [6]. This allows the matrix elements η_l to be expressed in terms of resonance parameters. Thus, should the quantities $\sigma_{\rm el}$, ω_1, and ω_2 in Eqs. (1)–(3) be expressed in terms of average resonance parameters, the fitting of those quantities to their experimental values can be used to determine the average resonance parameters S_0, S_1, R'_0, R'_1, and $S_{1,3/2}$ (they are fitting parameters). From the relation $S_1 = 1/3(S_{1,1/2} + 2S_{1,3/2})$, the parameter $S_{1,1/2}$ can be found.

For carrying out the calculations, we used the corresponding program for a fitting on the basis of the χ^2-minimization method. Three quantities – $\sigma_{\rm el}$, ω_1, and ω_2 – were fitted simultaneously, and the χ^2-criterion could be monitored for each quantity separately. The technique for the determination of average resonance parameters is explained in work [5] in detail.

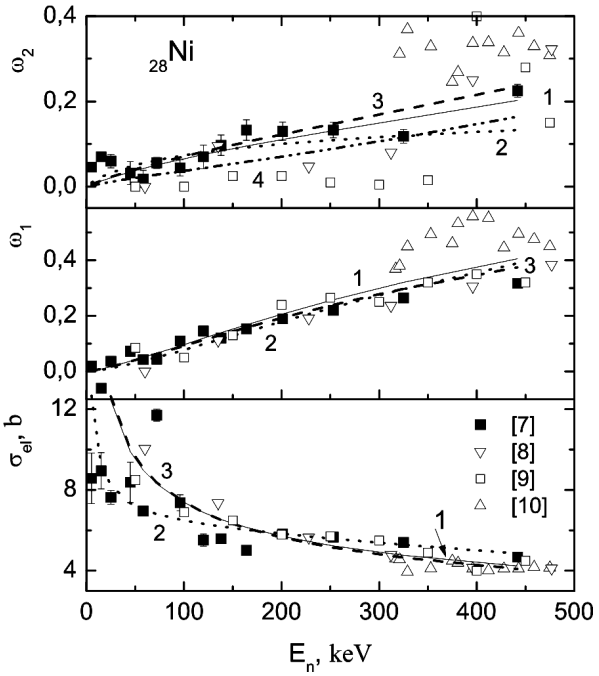

Fig. 1. Energy dependences of the quantities $\sigma_{\rm el}$, ω_1, and ω_2 for a $_{28}$Ni nucleus. Symbols correspond to experimental data, curves are the results of calculations with various sets of resonance parameters (see the text)

3. Determination of Resonance Parameters and Their Analysis

The complete sets of average resonance parameters S_0, S_1, R'_0, R'_1, $S_{1,3/2}$ for nickel and zinc nuclei with the natural composition of isotopes were determined by fitting the quantities $\sigma_{\rm el}$, ω_1, and ω_2 to the corresponding experimental values taken from work [7] (we carried out the additional averaging of data at the beginning of the energy range). The same data were used to make all the fittings described below and to estimate the quality of their description by means of the resonance parameters given by other authors. To check the reliability of the data of work [7] and to make a general evaluation of experimental data in the energy range under investigation, we present our results together with the available experimental data of other authors in the figures given below.

For nickel and zinc isotopes, recommended are the parameters S_0, S_1, and R'_0 [1] which were used to calculate the weighted average values for nuclei with natural isotope compositions. The obtained values were fixed to determine, by fitting, the other parameters from the complete set. In addition, we calculated the quantities $\sigma_{\rm el}$, ω_1, and ω_2 using the parameter sets taken from work [3] which were obtained from the same experimental data of work [7], but another method was applied. In every case, the description quality of experimental data was examined by the χ^2-value and visually in the plots.

$_{28}$Ni. In Fig. 1, the experimental energy dependences of the quantities $\sigma_{\rm el}$, ω_1, and ω_2 obtained in works [7–10] for nickel nuclei are given. To improve the presentation of the $\sigma_{\rm el}$-dependence, the first and third points of work [7] were reduced by a factor of two, and the second one by a factor of four. It should be noted that the resonance structures of the total cross-sections only start to reveal themselves in the studied energy range [11], and the resonances in the cross-sections $\sigma_{\rm el}$ reported in work [7] manifest themselves at energies of about 15 and 65 keV, which testifies to their insufficient averaging. In addition, the authors noted that the cross-sections measured below an energy of about 80 keV are underestimated owing to a substantial resonance self-shielding. The figure demonstrates that there are the appreciable discrepancies between experimental data obtained by various authors; they are especially considerable for the quantities ω_1 and ω_2. The curves in the figure correspond to the results of calculations with the use of different sets of resonance parameters. Curves *2* exhibit the dependences for $\sigma_{\rm el}$, ω_1, and ω_2 calculated with the following

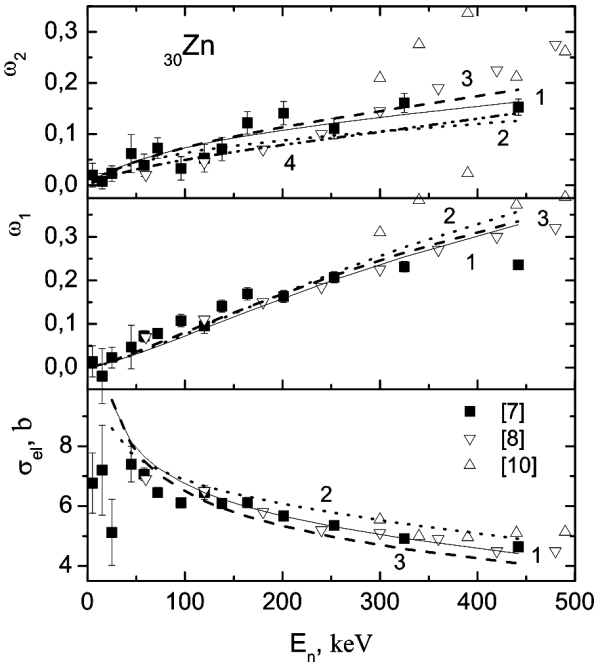

Fig. 2. Energy dependence of the quantities $\sigma_{\rm el}$, ω_1, and ω_2 for a nucleus $_{30}$Zn

set of parameters taken from work [3]: $S_0 = 1.40(30)$, $S_1 = 2.49(52)$, $R'_0 = 5.90(49)$, $R'_1 = -0.11(50)$, and $S_{1,3/2} = 1.44(23)$ (hereafter, the parameters S_l and R'_l are given in units of 10^{-4} and fm, respectively, and the numbers in parentheses indicate the corresponding error). From the figure, one can see that only the experimental data for ω_1 are described satisfactorily, whereas the other data demonstrate considerable discrepancies. In particular, the calculated cross-sections are less than the experimental ones at the beginning of the energy range, which can be explained by a small value of the parameter S_0.

The following average values are recommended for nickel parameters: $S_0 = 3.59$, $S_1 = 0.50$, and $R'_0 = 6.49$ [1]. Using the fitting procedure, we determined the other parameters: $R'_1 = 3.79$ and $S_{1,3/2} = 1.52$. The dependences for the quantities $\sigma_{\rm el}$, ω_1, and ω_2 calculated with this set of parameters are presented in the figure by curves *3*. It is evident that the description of experimental data is satisfactory on the whole, being better than that for the first set. However, from the relation $S_1 = (S_{1,1/2} + 2S_{1,3/2})/3$, it follows that $S_{1,1/2} = -1.54$. According to formula (3), the description of the quantity ω_2 is mainly determined by the parameters R'_1 and $S_{1,3/2}$. The contribution of the parameter R'_1, owing to its smallness, is appreciable only at energies higher than 200 keV. Practically, the parameter $S_{1,3/2}$ is determined unambiguously from the experimental ω_2-values, and the figure testifies that its value is optimal in this case. Therefore, the reason for why the value of the parameter $S_{1,1/2}$ is unphysical is a small magnitude of the parameter S_1. To be convinced of that, we carried out additional calculations. Provided that $S_{1,1/2} = 0$ and $S_{1,3/2} = 1.52$, the value of S_1 cannot be less than 1.01. For $S_1 = 0.50$ and $S_{1,1/2} = 0$, we have $S_{1,3/2} = 0.75$. When the parameters $S_0 = 3.59$, $S_1 = 0.50$, $R'_1 = 6.49$, and $S_{1,3/2} = 0.75$ were fixed, the fitting procedure gave rise to $R'_1 = 4.03$. The calculated values for ω_2 are depicted in the figure by curve *4* (the dependences for $\sigma_{\rm el}$ and ω_1 are close to curves *3*). One can see that they describe neither the data of work [7] nor the data of work [8, 9], the energy dependence of which are abnormal and cannot be described at all using resonance parameters. Therefore, there are reasons to consider the recommended value for the parameter S_1 as underestimated.

The insufficient averaging of the data of work [7] and the underestimation of cross-sections at the beginning of the energy range due to the resonance self-shielding result in a mutual disagreement among the quantities $\sigma_{\rm el}$, ω_1, and ω_2, which revealed itself in considerable χ^2-values at their description, especially in those for the cross-sections. Under such conditions, χ^2 is not a reliable criterion for the quality of a description of experimental data, so that the determination of resonance parameters by the automatic fitting and using those data cannot produce the reliable results as well. This is evidenced by the results of work [3], in which the values of the parameters S_0 and S_1 are several times different from those recommended for this A-region [1], and the description of experimental data is rather doubtful. Therefore, we determined the resonance parameters by stage-by-stage calculations of $\sigma_{\rm el}$, ω_1, and ω_2 with sequential variations of each parameter until the optimal description of experimental data was achieved. The description was estimated both visually (in the plots) and by the χ^2-criterion. As a result, the following values were obtained: $S_0 = 3.80$, $S_1 = 0.95$, $R'_0 = 6.25$, $R'_1 = 3.36$, and $S_{1,3/2} = 1.40$. The calculated values for $\sigma_{\rm el}$, ω_1, and ω_2 are presented by curves *1* in the figure. According to the χ^2-criterion, our description of experiments was better than that with the parameters of work [3]. Our parameters agree well with their dependences on the atomic weight A, except the parameter S_1, the value of which, however, agrees with new data for neighbor nuclei [1, 2].

$_{30}$Zn. In Fig. 2, the experimental energy dependences of $\sigma_{\rm el}$, ω_1, and ω_2 obtained in works [7, 8, 10] are de-

Average resonance parameters for Ni and Zn nuclei

Nucleus	$S_0\times10^4$	$S_1\times10^4$	R'_0, fm	R'_1, fm	$S_{1,1/2}\times10^4$	$S_{1,3/2}\times10^4$
$_{28}$Ni	3.80(35)	0.95(26)	6.25(26)	3.36(75)	0.05(1.0)	1.40(32)
$_{30}$Zn	1.80(25)	0.95(20)	6.50(25)	1.27(90)	0.05(0.9)	1.40(35)

picted. To improve the presentation of cross-sections $\sigma_{\rm el}$ in the figure, the second and third points taken from work [7] were reduced by a factor of two. The authors indicated that the cross-sections measured below of about 50 keV were underestimated owing to the resonance self-shielding. From the figure, one can see that the appreciable discrepancies between the ω_1- and ω_2-values obtained at the end of the energy range in various works are observed. The curves in the figure correspond to the results of calculations with different sets of resonance parameters.

Curves *2* in the figure correspond to the results of calculations with the parameters taken from work [3]: $S_0 = 1.15(9)$, $S_1 = 1.42(22)$, $R'_0 = 6.73(12)$, $R'_1 = 0.75(35)$, and $S_{1,3/2} = 1.23(15)$. Among all the data, only the quantity ω_1 is well described. The other data demonstrate appreciable discrepancies with experiment.

For zinc nuclei, the recommended average parameters are $S_0 = 2.00$ and $S_1 = 0.58$. By fitting, the other parameters were determined: $R'_0 = 6.35$, $R'_1 = 2.11$, and $S_{1,3/2} = 1.34$. The calculated quantities $\sigma_{\rm el}$, ω_1, and ω_2 are presented in the figure by curves *3*. One can see that the cross-sections $\sigma_{\rm el}$ at energies above 150 keV are appreciably smaller than the experimental ones, whereas the quantities ω_1 and ω_2 are described satisfactorily. However, as it was for nickel nuclei, there appears an issue concerning the value of the parameter S_1. From the obtained parameter set, it follows that $S_{1,1/2} = -0.94$. If $S_1 = 0.58$ and $S_{1,1/2} = 0$, the maximal possible value is $S_{1,3/2} = 0.87$. Having fixed the parameters $S_0 = 2.00$, $S_1 = 0.58$, and $S_{1,3/2} = 0.87$, we obtained, by fitting, the following other parameters: $R'_0 = 6.35$ and $R'_1 = 2.18$. The description of the quantities $\sigma_{\rm el}$ and ω_1 remained practically unchanged. The calculated values of ω_2 are shown by curve *4* in the figure. One can see that the description of ω_2 became considerably worse, both visually and according to the χ^2-value. This means that the value $S_{1,3/2} = 0.87$ is too small to provide a satisfactory description of ω_2-values, so that a conclusion can be drawn that the recommended value of the parameter S_1 is underestimated.

We determined a new set of resonance parameters in the same way as it was done for nickel nuclei. As a result, the following parameters were obtained: $S_0 = 1.80$, $S_1 = 0.95$, $R'_0 = 6.50$, $R'_1 = 1.27$, and $S_{1,3/2} = 1.40$. The results of calculations are shown in the figure by curves *1*. One can see that the description of experimental data became appreciably better in comparison with that for the previous sets, both visually and according to the χ^2-value. The obtained parameters S_0 and R'_0 agree well with their dependences on A [1], but the parameter S_1, as it was for nickel, is almost twice as large. The results obtained for both nuclei are listed in Table.

Hence, the values of the parameter S_0 obtained by us for both nuclei confirmed the recommended values, but not those given in work [3]. The values obtained for the parameter S_1 confirmed neither the recommended values nor those reported in work [3]. The recommended value of the parameter R'_0 for nickel nuclei was confirmed. From the recommended values of the parameter R'_0 for isotopes ^{64}Zn and ^{66}Zn (the latter amounts to 77% of a natural mixture), the weighted average value $R'_0 = 5.58$ was obtained, which is much less that the recommended values for this A-region [1]. By fixing this R'_0-value and the recommended values for the parameters S_0 and S_1, we determined, by fitting, the other parameters: $R'_1 = 2.41$ and $S_{1,3/2} = 1.31$. The difficulties associated with the magnitude of the parameter S_1 remained in the obtained parameter set; moreover, it does not describe experimental cross-sections at all, because the calculated cross-sections are much less than the experimental ones. The values of R'_1 and $S_{1,3/2}$ taken from three parameter sets are mutually agreed, in general, within the error limits.

4. Conclusions

In this work, by analyzing the experimental differential cross-sections for low-energy neutron elastic scattering, new complete sets of average resonance parameters S_0, S_1, R'_0, R'_1, and $S_{1,3/2}$ for nickel and zinc nuclei with the natural isotope composition have been determined. In general, the resonance parameters characterize specific nuclei, being determined for nuclear isotopes. However, data for the natural isotope mixture are also very important, in particular, for the specification of parameter dependences on the atomic weight A and for the improvement of theoretical calculations in the framework of the optical model. As a result of the analysis made, the recommended values for the resonance parameter S_1 were found to be underestimated by a factor of about two for both nuclei. At last, it was proved that the available

experimental data can be described better without drastic and unjustified variations of the parameters S_0 and S_1, as it was done in work [3].

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СЕРЕДНІ РЕЗОНАНСНІ ПАРАМЕТРИ ЯДЕР Ni І Zn

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Р е з ю м е

Отримано повні набори середніх резонансних параметрів S_0, S_1, R'_0, R'_1, $S_{1,3/2}$ для ядер нікелю і цинку з природним складом ізотопів. Їх визначено з аналізу середніх експериментальних диференціальних перерізів пружного розсіяння нейтронів низьких енергій розробленим авторами методом. Проведено аналіз отриманих результатів, рекомендованих параметрів та деяких літературних даних, на основі якого зроблено висновок, що рекомендовані для обох ядер параметри S_1 є заниженими у два рази.