DYNAMICALLY BROKEN SYMMETRY IN PERIODICALLY GATED QUANTUM DOTS: CHARGE ACCUMULATION AND DC-CURRENT

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Time-dependent electron transport through a quantum dot and double quantum dot systems in the presence of polychromatic external periodic quantum dot energy-level modulations is studied within the time evolution operator method for a tight-binding Hamiltonian. Analytical relations for the dc-current flowing through the system and the charge accumulated on a quantum dot are obtained for the zero-temperature limit. It is shown that, in the presence of periodic perturbations, the sideband peaks of the transmission are related to the combination of frequencies of the applied modulations. For a double quantum dot system under the influence of polychromatic perturbations, the quantum pump effect is studied in the absence of a source (drain) and static bias voltages. In the presence of a spatial symmetry, the charge is pumped through the system due to a broken generalized parity symmetry.

1. Introduction

Recently, considerable progress has been achieved in fabricating low dimensional systems, and many experimental and theoretical works have been put forward. Especially interesting are quantum systems under the influence of external radio or microwave electromagnetic radiation perturbations, where many interesting effects are observed like photon-assisted tunneling (PAT) [1, 2], turnstile effects and photon-electron quantum pumps [3–5], conductance oscillations [6], and alike [7].

The symmetry of quantum dot (QD) systems (with no source-drain voltage) plays the crucial role, as concerns electron pumping effects. Generally, one can consider symmetries like the time-reversal symmetry, time-reversal parity, and generalized parity [7].

A single electron pump based on asymmetric couplings between a QD and the left and right electrodes was considered in [4]. The couplings were switched on and off alternatively from zero to maximal values (by means of additional electrodes), and this led to the electron pumping. A similar effect can be achieved for dipole driving forces applied to a double QD system (in the large gate voltage regime) or to quantum wires. In this case, one QD site is driven by the external dipole interaction which is out of phase in comparison with the perturbation applied to the second QD site (the QD sites are not driven in homogeneous way), e.g. [7–12]. However, in the presence of a spatial symmetry [and in absence of a source (drain) and static gate voltages], it is also possible to pump electrons, but the symmetry must be broken in a dynamical way. The easiest way to break the time-reversal symmetry is to add a second harmonic to the driving system; i.e. the so-called “harmonic mixing” drive [7, 13, 14], or, in general, the second external perturbation with an arbitrary frequency [15]. In such a case, depending on the parameters of these two time-dependent perturbations, the generalized parity can be broken and a nonvanishing current can flow through the system [7, 13, 16].

There are few studies which address the electron transport through low-dimensional systems in the presence of several polychromatic external perturbations with arbitrary frequencies. Due to numerical problems, most of them concentrate on the case of commensurate frequencies or only bi-harmonic perturbations. It was shown that the external bi-harmonic time-dependent perturbations can be used to control the noise level in quantum systems [9, 17] or, as well, for routing optically induced currents [18, 19]. The shot noise for a single-level quantum dot under the influence of two ac external perturbations (coherent or incoherent) was analyzed in [15]. The coherent destruction of tunneling [20] and the associated dynamical localization in quantum dots under the influence of a time-dependent perturbation with many harmonics were investigated in [21]. Moreover, the dissipative quantum transport in one- or two-dimensional periodic systems that are subjected to electric harmonic mixing perturbations (bi-harmonic) were studied in [22–24]. The nonlinear signal consisted of, e.g., two rectangular-like driving forces which allow one, in turn, to control the overdamped transport in Brownian motor devices [16, 25–27].

In this paper, we will investigate the influence of polychromatic time-dependent energetic perturbations with arbitrary (commensurate and incommensurate) frequencies applied to a QD or a double QD system attached to leads for charge accumulated on the QD and the time-averaged dc-current flowing through the device. For a double QD system, we propose a quantum pump which is based on a scheme which mimics closely a dipole-like perturbation. Thus, our work can be treated as a generalization of the studies of the electron transport through a QD or double QD systems affected by one external perturbation or bi-harmonic electric time-dependent ac-perturbations with arbitrary frequencies. A tight-binding Newns–Anderson Hamiltonian and the evolution operator method are used in our calculations.

The paper is organized as follows. In Sec. 2, the model Hamiltonian and the theoretical description of a single-level quantum dot are presented. The analytic relations for a time-averaged current and a time-averaged charge on the QD are obtained, and the numerical results are depicted and interpreted. In Sec. 3, the current through a double QD system is obtained, and the pumping effect is discussed. The last section, Sec. 4, presents conclusions.

2. Single-Level Quantum Dot

2.1. Theoretical description

In this section, by starting from the second quantization Hamiltonian and using the evolution operator method, we obtain the charge accumulated on a QD and the current flowing through the system under the influence of many external time-dependent perturbations. Our system consists of a single-level quantum dot and two connecting electron electrodes, left (L) and right (R). The total Hamiltonian is then given by $H = H_0 + V$, where

$$H_0 = \sum_{\mathbf{k}\alpha=L,R} \varepsilon_{\mathbf{k}\alpha} c^+_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \varepsilon_d(t) c^+_d c_d \,, \tag{1}$$

$$V = \sum_{\mathbf{k}L} V_{\mathbf{k}L} c^+_{\mathbf{k}L} c_d + \sum_{\mathbf{k}R} V_{\mathbf{k}R} c^+_{\mathbf{k}R} c_d + \text{h.c.} \tag{2}$$

The operators $c_{\mathbf{k}\alpha}(c^+_{\mathbf{k}\alpha})$ and $c_d(c^+_d)$ are the annihilation (creation) operators of the electron in the lead α ($\alpha = L, R$) and at the QD, respectively. The QD is coupled symmetrically to the leads through the tunneling barriers with the transfer-matrix elements $V_{\mathbf{k}L}$ and $V_{\mathbf{k}R}$ (hopping integrals). For the role of the asymmetric lead-"molecule" coupling, see in [28, 29]. The electron-electron Coulomb interaction is neglected in our calculation, cf. [7, 15, 30].

External perturbations are applied to the QD (the QD energy level is driven in time by time-dependent ac-voltages). We consider a harmonic modulation of the external energy level perturbations applied to the QD, i.e.

$$\varepsilon_d(t) = \varepsilon_d + \sum_{i=1}^{n} \Delta_i \cos(\omega_i t + \phi_i), \tag{3}$$

where ω_i, Δ_i, and ϕ_i are the frequency, driving amplitude, and phase of the i-th perturbation.

The current flowing through the system and the charge localized at the QD can be described in terms of the time evolution operator $U(t, t_0)$ given by the equation of motion (in the interaction representation, $\hbar = 1$), i.e.,

$$i\frac{\partial}{\partial t} U(t, t_0) = \tilde{V}(t)\, U(t, t_0)\,, \tag{4}$$

where $\tilde{V}(t) = U_0(t, t_0)\, V(t)\, U^+_0(t, t_0)$ and $U_0(t, t_0) = T \exp\left(i \int_{t_0}^{t} dt' H_0(t')\right)$. The knowledge of the appropriate matrix elements of the evolution operator $U(t, t_0)$ allows us to find the charge accumulated on the QD, $n_d(t)$ (cf. [6, 31, 32]), which is given by

$$n_d(t) = \sum_{\beta} n_\beta(t_0) |U_{d,\beta}(t, t_0)|^2, \tag{5}$$

where $n_\beta(t_0)$ represents the initial filling of the corresponding single-particle states ($\beta = d, \mathbf{k}L, \mathbf{k}R$). The current flowing, e.g., from the left lead can be obtained from the time derivative of the total number of electrons in the left lead, cf. [33]:

$$j_L(t) = -edn_L(t)/dt\,, \tag{6}$$

where $n_L(t)$ can be expressed as follows:

$$n_L(t) = \sum_{\mathbf{k}L} n_{\mathbf{k}L}(t) = \sum_{\mathbf{k}L}\sum_{\beta} n_\beta(t_0)|U_{\mathbf{k}L,\beta}(t,t_0)|^2. \tag{7}$$

Using Eq. 4, the following differential equations for $U_{d,\beta}(t,t_0)$ and $U_{\mathbf{k}L,\beta}(t,t_0)$ matrix elements which are needed to obtain the current and the QD charge can be written in the form

$$i\frac{\partial}{\partial t}U_{d,\beta}(t,t_0) = \sum_{\mathbf{k}\alpha=L,R}\tilde{V}_{d,\alpha}(t)U_{\alpha,d}(t,t_0), \tag{8}$$

$$i\frac{\partial}{\partial t}U_{\mathbf{k}L,\beta}(t,t_0) = \tilde{V}_{\mathbf{k}L,d}(t)U_{d,\beta}(t,t_0), \tag{9}$$

where nonzero elements of the function $\tilde{V}$ are

$$\tilde{V}_{d,\mathbf{k}\alpha} = \tilde{V}^*_{\mathbf{k}\alpha,d} = V^*_{\mathbf{k}\alpha}\exp\left(i\int\limits_{t_0}^{t}(\varepsilon_d(t') - \varepsilon_{\mathbf{k}\alpha})dt'\right). \tag{10}$$

Next, using the *wide band limit* approximation $\Gamma^\alpha(\varepsilon) = 2\pi\sum_{\mathbf{k}\alpha}V_{\mathbf{k}\alpha}V^*_{\mathbf{k}\alpha}\delta(\varepsilon-\varepsilon_{\mathbf{k}\alpha}) = \Gamma^\alpha$ and assuming $\Gamma^L = \Gamma^R = \Gamma$, we can find the following relation for the $U_{d,d}(t)$ matrix element ($t_0 = 0$): $U_{d,d}(t) = \exp(-\Gamma t)$. Similarly for $U_{d,\mathbf{k}\alpha}(t)$ and $U_{\mathbf{k}L,\beta}(t)$, we have

$$U_{d,\mathbf{k}\alpha}(t) = -i\exp(-\Gamma t)\int\limits_0^t dt'\tilde{V}_{d,\mathbf{k}\alpha}(t')\exp(\Gamma t'), \tag{11}$$

$$U_{\mathbf{k}L,\beta}(t) = -i\int\limits_0^t dt'\tilde{V}_{\mathbf{k}L,d}(t')U_{d,\beta}(t'). \tag{12}$$

Note that, as $t \to \infty$, the element $U_{d,d}(t)$ tends to zero, and thus the charge accumulated on the QD does not depend on the initial QD occupation $n_\beta(t_0)$ for large t. Moreover, the current flowing through the system is independent of $n_d(t_0)$, because the element $\frac{d}{dt}\sum_{\mathbf{k}L} n_d(t_0)|U_{\mathbf{k}L,d}(t)|^2$ tends to zero as $t \to \infty$. Finally, the QD charge can be written in the form

$$n_d(t) = \sum_{\mathbf{k}\alpha=L,R} n_{\mathbf{k}\alpha}(t_0)|U_{d,\mathbf{k}\alpha}(t)|^2, \tag{13}$$

and the current through the system reads ($e = 1$)

$$j_L(t) = \Gamma n_d(t) + \mathrm{Im}\left(\sum_{\mathbf{k}L} n_{\mathbf{k}L}(t_0)\tilde{V}_{\mathbf{k}L,d}(t)U_{d,\mathbf{k}L}(t)\right). \tag{14}$$

Equations 13 and 14 are very general relations which should be analyzed using Eq. 10 and Eq. 11. The relation for the current, Eq. 14, has the structure which can be written by means of the transmission T_{LR} and T_{RL}, i.e. $j_L(t) = \sum_{\mathbf{k}L} n_{\mathbf{k}L}(0)T_{LR}(t) - \sum_{\mathbf{k}R} n_{\mathbf{k}R}(0)T_{RL}(t)$, but we note that, in general, $T_{LR}(t) \neq T_{RL}(t)$. In order to obtain the current, one should know the exact form of the $\tilde{V}_{d,\mathbf{k}\alpha}$ function. Using the time-dependence relation for the QD-energy level, Eq. 3, and assuming $\phi_i = 0$, the elements $\tilde{V}_{d,\mathbf{k}\alpha}$, Eq. 10, can be expressed as

$$\tilde{V}_{d,\mathbf{k}\alpha} = V^*_{\mathbf{k}\alpha}e^{i(\varepsilon_d-\varepsilon_{\mathbf{k}\alpha})t}\prod_{i=1}^{n}\sum_{m_i} J_{m_i}\left(\frac{\Delta_i}{\omega_i}\right)e^{im_i\omega_i t}, \tag{15}$$

and the solution of Eq. 11 for the evolution operator elements can be written in the form

$$U_{d,\mathbf{k}\alpha}(t) = -V^*_{\mathbf{k}\alpha}e^{i(\varepsilon_d-\varepsilon_{\mathbf{k}\alpha})t}\times$$

$$\times\sum_{m_1}\dots\sum_{m_n}\frac{J_{m_1}\left(\frac{\Delta_1}{\omega_1}\right)\dots J_{m_n}\left(\frac{\Delta_n}{\omega_n}\right)e^{i\Omega t}}{\varepsilon_d - \varepsilon_{\mathbf{k}\alpha} + \Omega - i\Gamma}, \tag{16}$$

where $m_i \in (-\infty,\infty)$ and is an integer number, $\Omega = m_1\omega_1 + \ldots + m_n\omega_n$, and J_i is the Bessel function. Next, we obtain the dc current through the system, $j_0 = \langle j(t)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} j(t')dt'$ which can be symmetrized, $j_0 = \langle j_L(t)\rangle = (\langle j_L(t)\rangle - \langle j_R(t)\rangle)/2$. Finally, we find

$$j_0 = \frac{\Gamma}{2\pi}\int\limits_{-\infty}^{\infty}(f_R(\varepsilon) - f_L(\varepsilon))\,T(\varepsilon), \tag{17}$$

where $f_{L/R}(\varepsilon)$ is the Fermi function of the L/R electrode, and the transmission reads

$$T(\varepsilon) = \Gamma^2\sum_{m_1}\dots\sum_{m_n}\sum_{m'_1}\dots\sum_{m'_n}\delta_{\Omega-\Omega'}\times$$

$$\times\frac{J_{m_1}\left(\frac{\Delta_1}{\omega_1}\right)\dots J_{m_n}\left(\frac{\Delta_n}{\omega_n}\right)J_{m'_1}\left(\frac{\Delta_1}{\omega_1}\right)\dots J_{m'_n}\left(\frac{\Delta_n}{\omega_n}\right)}{(\varepsilon_d - \varepsilon + \Omega)^2 + \Gamma^2}, \tag{18}$$

where $\Omega' = m_1'\omega_1 + ... + m_n'\omega_n$. Note that, for incommensurate frequencies, the Kronecker delta function $\delta_{\Omega-\Omega'}$ is nonzero only for $m_i' = m_i$. Thus, the transmission can be written in the following short form:

$$T(\varepsilon) = \Gamma^2 \sum_{m_1} \dots \sum_{m_n} \frac{J^2_{m_1}\left(\frac{\Delta_1}{\omega_1}\right) \dots J^2_{m_n}\left(\frac{\Delta_n}{\omega_n}\right)}{(\varepsilon_d - \varepsilon + \Omega)^2 + \Gamma^2}. \quad (19)$$

This equation is valid also in the case where there is a large difference between the frequencies ω_i. Note that the transmission, Eqs. (18) and (19), corroborates with the results obtained by means of the Green's function method for a quantum wire driven by homogeneous external perturbations [30].

The relation for the current, Eq. 17, has the structure of the Landauer formula. We note that the structure in Eq. (17) involves, in the clear contrast to a electric field dipole perturbation [7], no inelastic photon-assisted tunneling events. This is so because, with the time-dependent energy level perturbation used here, the long time-average of $T_{LR}(t)$ and $T_{RL}(t)$ equals $T_{LR}(\epsilon) = T_{RL}(\epsilon)$. For the zero temperature and for incommensurate frequencies of external perturbations, the dc-current can be obtained analytically from the relation

$$j_0 = \Gamma \sum_{m_1} ... \sum_{m_n} J^2_{m_1}\left(\frac{\Delta_1}{\omega_1}\right) ... J^2_{m_n}\left(\frac{\Delta_n}{\omega_n}\right) \times$$

$$\times \left(\arctan \frac{\varepsilon_d - \mu_L - \Omega}{\Gamma} - \arctan \frac{\varepsilon_d - \mu_R - \Omega}{\Gamma} \right). \quad (20)$$

Note that, for $\mu_L = \mu_R$, the dc current is zero.

The corresponding analytic relation for the accumulated quantum dot dc-charge, i.e. $n_0 = \langle n_d(t) \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} n_d(t')dt'$, reads

$$n_0 = \frac{1}{2\pi} \sum_{m_1} \dots \sum_{m_n} J^2_{m_1}\left(\frac{\Delta_1}{\omega_1}\right) \dots J^2_{m_n}\left(\frac{\Delta_n}{\omega_n}\right) \times$$

$$\times \left(\pi - \arctan \frac{\varepsilon_d - \mu_L - \Omega}{\Gamma} - \arctan \frac{\varepsilon_d - \mu_R - \Omega}{\Gamma} \right). \quad (21)$$

It is worth noting that, for $\varepsilon_d \ll \mu$ ($\varepsilon_d \gg \mu$), the charge accumulated on the QD is maximal (minimal). The relations for the dc current, Eq. (20), and for the QD charge, Eq. (21), constitute the main analytic relations of this section.

2.2. QD accumulated charge and dc current

In this section, we analyze the QD charge and the dc current flowing through a quantum dot driven by polychromatic perturbations. All energies are expressed in the units of Γ^0. In order to obtain rather narrow sidebands peaks, we assume $\Gamma = 0.2\Gamma^0$ (taking the unit of energy $\Gamma^0 = 0.05eV$, this corresponds to $\Gamma = 0.01$ eV). For larger Γ, all sideband peaks are wider, and it is difficult to observe many-perturbation effects. The current and the conductance are given in units of $2e\Gamma^0/\hbar$ and $2e^2/\hbar$, respectively. Moreover, we show numerical calculations for two external perturbations case but the generalization for more perturbations is obvious.

In Fig. 1, the QD charge (upper panel) and the dc current flowing through the system (lower panel) are shown for two external perturbations applied to the system ($\omega_1 = 3$, $\omega_2 = 8$, $\Delta_1 = 4$, $\Delta_2 = 8$) – thick lines. The frequencies are commensurate but there is the large difference between them, and the transmissions obtained from Eq. (18) and Eq. (19) are almost the same. Physically, this means that the eight-photon adsorption/emission process based on the third sideband peak should occur to play the role in the transmission, ($8\omega_1 = 3\omega_2$), but this is unlikely process. The broken lines correspond to the single external perturbation case, i.e. $\Delta_2 = 8$ ($\Delta_1 = 0$) – thin broken lines and $\Delta_1 = 4$ ($\Delta_2 = 0$) – thick broken lines, respectively. The chemical potentials are rather small, $\mu_L = -\mu_R = 0.1$, and the current peak for $\varepsilon_d = 0$ (lower panel) is observed for mono- and polychromatic cases (it appears also for the time-independent case).

For only one external perturbation applied to the QD, $\omega = 3$ (or $\omega = 8$), the sidebands peaks are visible for $\varepsilon_d = \pm k\omega$, where k is an integer number. However, in the case where two external perturbations are applied simultaneously to the QD, additional sideband peaks appear. Of course, the single peaks from the first and the second fields are still visible, i.e. for $\varepsilon_d = \omega_1, 2\omega_1$ or ω_2. In Fig. 1, the additional dc current peaks are indicated by points A_1, A_2 (first-order sidebands), and B_1 (second-order sideband). The A_1 (A_2, B_1) sideband peak appears for $\varepsilon_d = \omega_2 - \omega_1$ ($\varepsilon_d = \omega_2 + \omega_1$, $\varepsilon_d = \omega_2 - 2\omega_1$) and is related to the peak for $\varepsilon_d = \omega_2 = 8$ (but not to the main peak for $\varepsilon_d = 0$). It is worth noting that, although the frequencies ω_1 and ω_2 are not equal to 5 or 11, we observe the sideband peaks for these values of ε_d. In general, we observe sideband peaks for $\varepsilon_d = \pm k_1\omega_1 \pm k_2\omega_2$, where k_1 and k_2 are integer numbers. The structure of the dc current curves are reflected also in the charge accumulated on the QD (upper panel). The charge decreases with ε_d, but there are many steps which are related to the

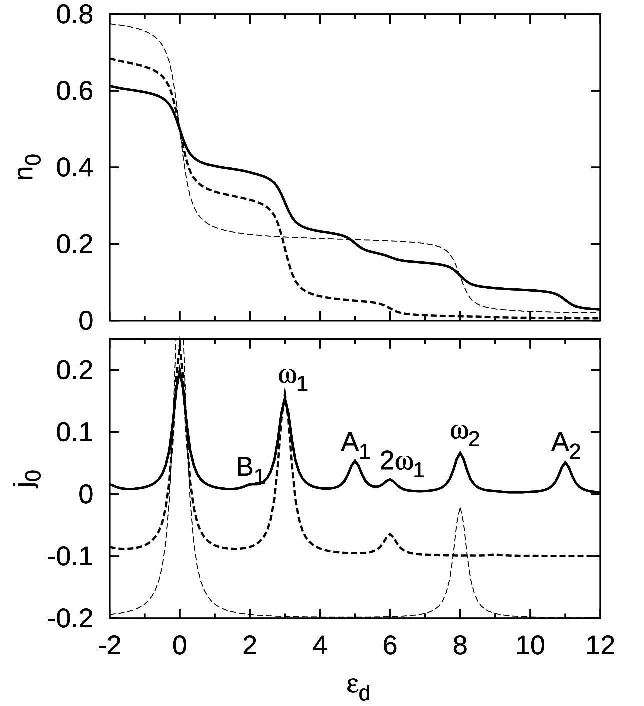

Fig. 1. QD charge (upper panel) and the dc current (lower panel) as a function of ε_d for $\Delta_1 = 4$ and $\Delta_2 = 8$ (thick lines), $\Delta_1 = 4$, $\Delta_2 = 0$ (thick broken lines) and $\Delta_1 = 0$ and $\Delta_2 = 8$ (thin broken lines), respectively. The other parameters are $\omega_1 = 3$, $\omega_2 = 8$, $\mu_L = -\mu_R = 0.1$, $\Gamma = 0.2$. The thick (thin) broken line on the lower panel is shifted by -0.1 (-0.2) for better visualization

current sideband peaks. The additional sidebands, i.e. points A_1, A_2, and B_1 from the lower panel, are visible on the charge curve (Fig. 1, upper panel, thick line).

Next, in Fig. 2, we present the dc current as a function of the driving strengths (amplitudes) of two external perturbations applied to the system. For the second signal, the driving strength decreases with the amplitude of the first perturbation, i.e. $\Delta_2 = 10 - \Delta_1$. The thick (thin) solid line corresponds to $\varepsilon_d = 5$ ($\varepsilon_d = 10$). As one can see for $\varepsilon_d = 10$, the dc current is very small and almost independent of the amplitudes Δ_1 and Δ_2. This conclusion is valid for every ε_d which is not any combination of ω_1 and ω_2. For $\varepsilon_d = 5$ ($\varepsilon_d = \omega_2 - \omega_1$), the current is minimal for $\Delta_1 = 0$ and $\Delta_1 = 10$, but, for the mixed regime of Δ_1 and Δ_2, it is characterized by a local maximum, cf. the thick solid line. This is a very interesting effect, because one can control the current flowing through the system by applying an additional time-dependent perturbation. Note that the maximal value of the current is several times larger than that in the case of one external perturbation, i.e. for $\Delta_1 = 0$ or

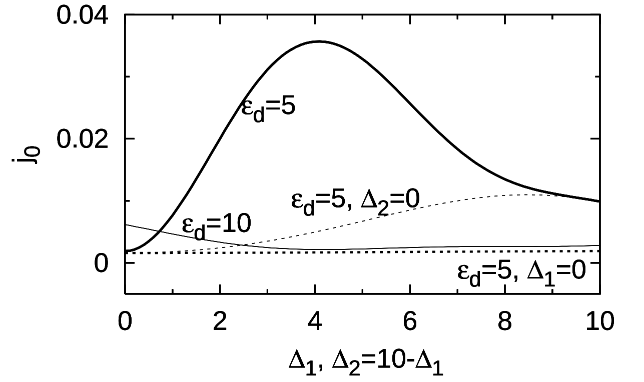

Fig. 2. Dc-current as a function of the amplitude Δ_1 ($\Delta_2 = 10 - \Delta_1$) for $\varepsilon_d = 5$ (thick line) and $\varepsilon_d = 10$ (thin line), $\omega_1 = 3$, $\omega_2 = 8$. The thin (thick) broken line corresponds to one external perturbation case Δ_1, $\omega = 3$ with $\Delta_2 = 0$ (Δ_2, $\omega = 8$ with $\Delta_1 = 0$) and $\varepsilon_d = 5$; $\mu_L = -\mu_R = 0.1$, $\Gamma = 0.2$

$\Delta_1 = 10$. For only one external perturbation applied to the system, the dc current is shown by the broken lines – the thin line for $\Delta_2 = 0$ as a function of Δ_1 ($\omega = 3$) and the thick one for $\Delta_1 = 0$ as a function of Δ_2 ($\omega = 8$), $\varepsilon_d = 5$. One can conclude that one external perturbation slightly changes the current in this case. This confirms that the current maximum, which appears for $\varepsilon_d = 5$ (thick solid curve), is due to a combination of two external perturbations applied to the system. The similar conclusions are valid for other $\varepsilon_d = \pm k_1\omega_1 \pm k_2\omega_2$ ($k_{1,2} \neq 0$).

To analyze the structure of sidebands peaks for polychromatic perturbations, we show the dc current in Fig. 3 as a function of the QD energy level, ε_d, and the external perturbation frequency ω_2 for $\Delta_1 = 0$ (only one perturbation applied to the system with $\Delta_2 = 8$) – upper panel, and for two external perturbations with $\Delta_1 = 3$, $\omega_1 = 3$, $\Delta_2 = 8$ – lower panel. For one, as well as for two time-dependent perturbations applied to the system, we can distinguish between the adiabatic regime for $\omega_2 \leq 1$ and the non-adiabatic one for $\omega_2 > 1$. For $\omega_2 > 1$, one can observe the main current peak for $\varepsilon_d = 0$ and sidebands which are visible for $\varepsilon_d = \pm k\omega_2$. The situation is more complex for two external perturbations applied simultaneously to the QD ($\Delta_1 = 3$, $\omega_1 = 3$, $\Delta_2 = 8$). One observes that each light line from the upper panel possesses satellite lines which are localized at the distance $\pm k\omega_1$ from the original lines. Thus, we observe a very reach structure of the dc current in the presence of polychromatic perturbations.

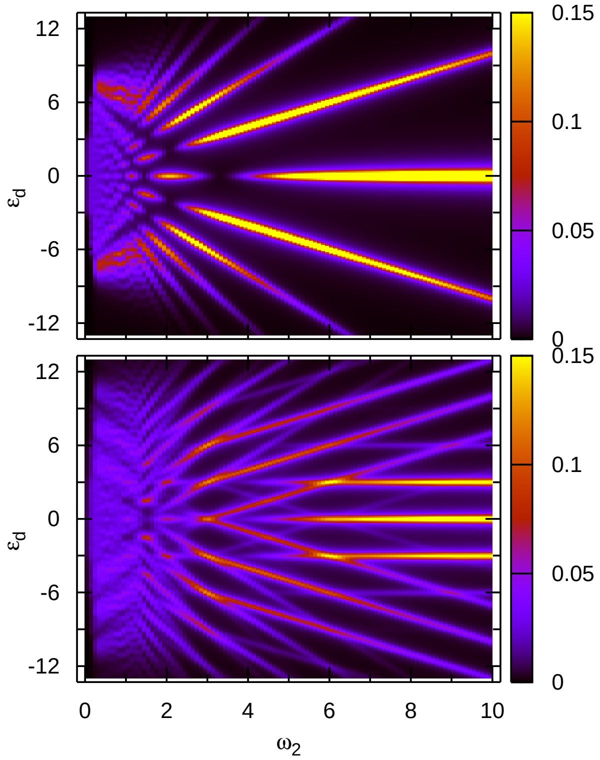

Fig. 3. Dc-current as a function of ε_d and ω_2 for $\Delta_1 = 0$, $\Delta_2 = 8$ (one external perturbation – upper panel) and $\Delta_1 = 3$, $\Delta_2 = 8$ (two external perturbations – lower panel). The other parameters are: $\omega_1 = 3$, $\mu_L = -\mu_R = 0.1$, $\Gamma = 0.2$

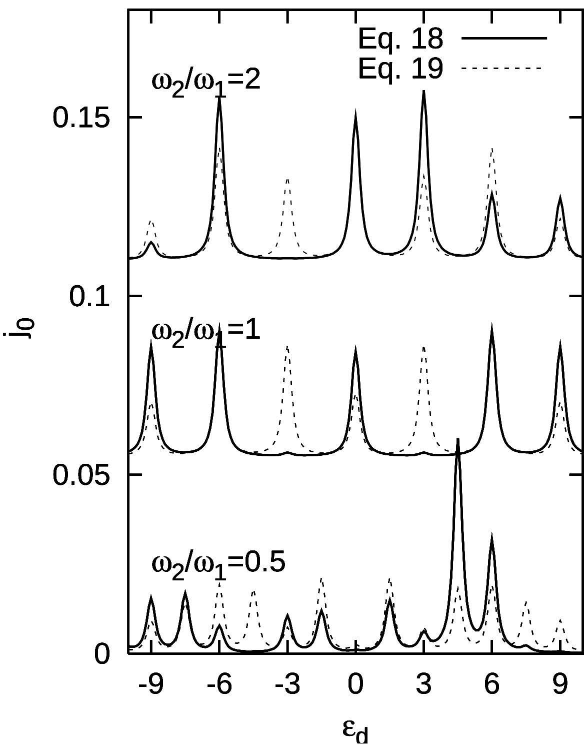

Fig. 4. Dc-current as a function of ε_d obtained for commensurate frequency ratios $\omega_2/\omega_1 = 2, 1$, and 0.5 according to Eq. 17 with the transmission given by Eq. 18 (solid lines) and for a slightly off-commensurate frequency; i.e., $\omega_1 = 3, \omega_2 = 5.999$, see Eq. 19 (broken lines). All parameters are the same as in Fig. 3

2.3. Results: Transmission asymmetry and phase dependence

For one external perturbation applied to a QD, as well as for two external perturbations with the same frequencies, one observes fully symmetric current curves around the value $\varepsilon_d = 0$. However, for two external time-dependent perturbations, the dc current (or the transmission) can be asymmetric, which appears in the case of commensurate frequencies. In Fig. 4, we show the dc current obtained for the frequency ratios $\omega_2/\omega_1 = 2, 1$, and 0.5 using the transmission relations, Eq. (18), (solid lines) and in the case where there is an infinitesimal shift of the frequency ratio from an integer number, Eq. (19) (broken lines); e.g. for $\omega_2/\omega_1 = 2$, we set $\omega_1 = 3$ and $\omega_2 = 5.999$. Note that all broken curves in Fig. 4 are symmetric. The solid lines, however, are asymmetric around $\varepsilon_d = 0$ except for the case $\omega_1 = \omega_2$. Notably, for $\omega_1 = \omega_2$, the system is equivalent to a single, effective external perturbation case with the effective driving amplitude, $\Delta = \Delta_1 + \Delta_2$, and thus the current curves turn out symmetric again. The current obtained for incommensurate frequencies differs from the ones in the case of commensurate frequencies. It is worth noting that the structure of the dc current curves for fixed ω_i depends on the amplitudes of external perturbations, but the positions of peaks remain unchanged.

Next, it is necessary to explain the transmission and current asymmetries for the case of commensurate frequencies. In this case, to obtain the transmission one, should use Eq. (18) which is expressed in terms of four Bessel functions, an energy dependent factor, and the Kronecker delta function. For commensurate frequencies, this delta function produces off-diagonal elements of the Bessel functions which are multiplied by the energy factor. The energy factor depends on the order of the Bessel functions and introduces an asymmetry for positive and negative values of ε_d (this means that the energy factor possesses different values for positive and negative integer orders of the Bessel functions). The off-diagonal elements of the Bessel functions are also responsible for different values of the dc current obtained for $\omega_2/\omega_1 = 1$, according to Eq. (18) and Eq. (19). It is worth noting that, in our calculation, we assume that the QD energy level is driven according to the formula with the cosine function (i.e. the cosine function is an odd function), and the external time-dependent perturbation satisfies the time-reversal symmetry, $a(t) = a(-t)$.

In Fig. 5, we show the transmission for nonzero phase factors of the external perturbations, i.e. ϕ_1 and ϕ_2, cf. Eq. (3). The upper panel shows the results in the case where both phases are changed in the same way, i.e. $\phi_1 = \phi_2$. For $\phi_1 = 0$ (thin solid line), the asymmetry in the transmission is observed, cf. also Fig. 4. If both phases are equal to $\pi/2$ (broken line), the transmission is symmetric (as a function of the energy) - in that case, the external signals are sines. For $\phi_1 = \pi$, the transmission is asymmetric again. The period of the external perturbation for the case of the same phases, $\phi_1 = \phi_2$, is equal to 2π. The situation is somewhat different, when the phase of one external perturbation is constant ($\phi_2 = 0$ - lower panel). Here, the transmission is symmetric only for $\phi_1 = \pi/4$, but this function is asymmetric for $\phi_1 = 0$ and $\pi/2$. Note that the above conclusions are valid for the case of commensurate frequencies. For incommensurate frequencies, the transmission curves are symmetric, cf. Fig. 4. Moreover, as one can see, the transmission curves in the upper and lower panels are the same, only the phases of external perturbations are different. This means that the same effect can be obtained by changing only one phase parameter instead of driving both phases simultaneously.

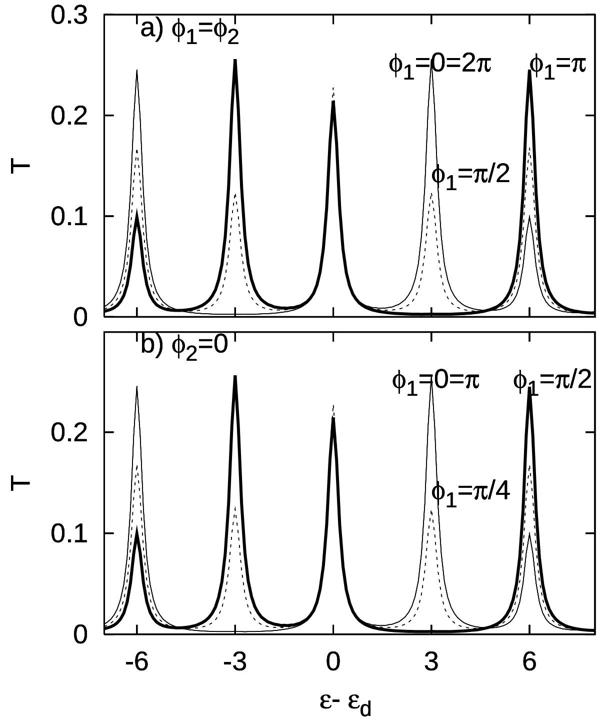

Fig. 5. Transmission versus the energy for two external perturbations applied to the QD ($\omega_1 = 3$, $\omega_2 = 6$, $\Delta_1 = 3$, $\Delta_2 = 8$) with phases: $\phi_1 = \phi_2 = 0, \pi/2, \pi$ (upper panel) and $\phi_1 = 0, \pi/4, \pi/2$ and $\phi_2 = 0$ (lower panel)

3. Double Quantum Dot System and Pumping Effect

The asymmetry effect in the transmission obtained in the previous section can be used to construct a single electron pump [12] based on a two-level fully symmetric system with no source-drain and static bias voltages applied to the system. In this section, we analyze the electron transport through a double quantum dot in the presence of external perturbations. The Hamiltonian of our system can be written, in close analogy to Eq. (1) and Eq. (2), as follows:

$$H_0 = \sum_{\mathbf{k}\alpha=L,R} \varepsilon_{\mathbf{k}\alpha} c^+_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \varepsilon_1(t) c^+_1 c_1 + \varepsilon_2(t) c^+_2 c_2\,, \quad (22)$$

$$V = \sum_{\mathbf{k}L} V_{\mathbf{k}L} c^+_{\mathbf{k}L} c_1 + \sum_{\mathbf{k}R} V_{\mathbf{k}R} c^+_{\mathbf{k}R} c_2 + V_{12} c^+_1 c_2 + \text{h.c.}, \quad (23)$$

where V_{12} is the tunnel coupling (hopping term) between two QD sites. As before, we assume that there are only two external perturbations applied to the system, and one can write the following time dependence of the quantum dot energy levels:

$$\varepsilon_1(t) = \varepsilon_1 + \Delta_1 \cos(\omega_1 t) + \Delta_2 \cos(\omega_2 t), \quad (24)$$

$$\varepsilon_2(t) = \varepsilon_2 + \Delta_1 \cos(\omega_1 t + \phi) + \Delta_2 \cos(\omega_2 t + \phi), \quad (25)$$

where ϕ is the phase difference between the external perturbations applied to the first and second QD sites which play a similar role as dipole forces in the single external harmonic field case, cf. [7, 9, 10]. We stress, however, that here the driving is chosen uniform in the sense that the amplitudes strengths for the driving of the two dots are identical. Note that the role of the phase difference between the first and second external perturbations was considered in [13] for bi-harmonic perturbations. Here, we concentrate on the phase difference between both QD sites. The main reason we omit the phase difference between both external perturbations is that, in the presence of a spatial symmetry ($\varepsilon_1 = \varepsilon_2$, $\mu_L = \mu_R$; i.e., in the absence of a static gate voltage and a source-drain voltage), it is impossible to pump electrons through the system. By introducing the phase difference according to Eq. (24) and Eq. (25), we change the symmetry at the first and second QD sites, and thus electrons can be pumped in the presence of external perturbations and a spatial symmetry. The general formula for the time-dependent current flowing from the left electrode can be written as

$$j_L(t)=\sum_{\mathbf{k}L} n_{\mathbf{k}L}(t_0)\left(\Gamma|U_{1,\mathbf{k}L}(t)|^2+\mathrm{Im}\tilde{V}_{\mathbf{k}L,1}(t)U_{1,\mathbf{k}L}(t)\right)+$$

$$+\sum_{\mathbf{k}R} n_{\mathbf{k}RL}(t_0)\Gamma|U_{1,\mathbf{k}R}(t)|^2, \quad (26)$$

where $V_{\mathbf{k}L,1}(t)$ is defined by Eq. (15), and the evolution operator matrix elements satisfy the following system of differential equations (and similar for $U_{1(2),\mathbf{k}R}$):

$$\frac{\partial}{\partial t}U_{1,\mathbf{k}L}(t)=-iV_{12}e^{i(\varepsilon_1-\varepsilon_2)t}e^{i(f_1-f_2)}U_{2,\mathbf{k}L}(t)-$$

$$-iV_{\mathbf{k}L}e^{i(\varepsilon_1-\varepsilon_{\mathbf{k}L})t}e^{if_1}-\frac{\Gamma}{2}U_{1,\mathbf{k}L}(t), \quad (27)$$

$$\frac{\partial}{\partial t}U_{2,\mathbf{k}L}(t)=-iV_{12}e^{i(\varepsilon_2-\varepsilon_1)t}e^{i(f_2-f_1)}U_{1,\mathbf{k}L}(t)-\frac{\Gamma}{2}U_{2,\mathbf{k}L}(t), \quad (28)$$

where $f_1=\frac{\Delta_1}{\omega_1}\sin(\omega_1 t)+\frac{\Delta_2}{\omega_2}\sin(\omega_2 t)$, $f_2=\frac{\Delta_1}{\omega_1}\sin(\omega_1 t+\phi)+\frac{\Delta_2}{\omega_2}\sin(\omega_2 t+\phi)$. Note that, for $\phi=2\pi k$ and the same on-site energies $\varepsilon_1=\varepsilon_2=\varepsilon_d$, one can obtain the dc current and the transmission analytically. In this case, the current satisfies the Landauer formula, and the time-averaged left-right and right-left transmissions are equal. This result is due to the above-mentioned chosen uniform driving strengths. For incommensurate frequencies of the external perturbations, the formula for the transmission simplifies and can be written as

$$T(\varepsilon)=\Gamma^2\sum_{m_1}\sum_{m_2}V_{12}^2\times$$

$$\times\frac{J_{m_1}^2\left(\frac{\Delta_1}{\omega_1}\right)J_{m_2}^2\left(\frac{\Delta_2}{\omega_2}\right)}{\left((\varepsilon_d-\varepsilon+\Omega)^2-\frac{\Gamma}{2}^2-V_{12}^2\right)^2+\Gamma^2(\varepsilon_d-\varepsilon+\Omega)^2}, \quad (29)$$

where $\Omega=\omega_1 m_1+\omega_2 m_2$. In the case of commensurate frequencies, the formula for the transmission assumes no simple transparent from, but is similar to Eq. (18). Note that, for $\phi=0$ and $\varepsilon_1\neq\varepsilon_2$, it is also possible to analyze the system of differential equations analytically, Eq. (27) and Eq. (28), however the solution is rather complex. Generally, i.e. for $\phi\neq 0$ and $\varepsilon_1\neq\varepsilon_2$, we solve the system of differential equations for $U_{1(2),\mathbf{k}L(R)}$ numerically, then put the solution into the relation for the current, Eq. (26), and average it over the common period of external perturbations. Thus, this procedure can be applied only in the case of commensurate frequencies. In our calculations, we concentrate mainly on the phase difference effect, as it can lead to the electron pumping in the system with no source-drain and static bias voltages. We take the phase difference, ϕ, between QD sites into account, which leads, for $\phi=\pi/2$, to a dipole-like parametrization (the oscillations are out of phase). The similar two-level system under the influence of one external perturbation in the case of a high bias voltage, $\varepsilon_1\ll\varepsilon_2$, was investigated in [34] and, in the presence of a phase difference between both external perturbations and dipole driving forces, in [13].

If only one external perturbation is applied to the double dot system, the phenomenon electron pumping does not occur in the case of the spatial symmetry $\varepsilon_1=\varepsilon_2$, [7]. For a nonzero gate voltage and $\phi=\pi/2$ (i.e. in the presence of the phase difference and the spatial asymmetry), the current can flow through the system [34], but if there is no phase difference between both QD sites, $\phi=0$, the current vanishes again. In order to analyze the role of polychromatic perturbations on the symmetric system, i.e. $\varepsilon_1=\varepsilon_2=0$ (no applied static gate voltages), we depict the dc current in Fig. 6 as a function of the driving amplitude, Δ_2, for two external signals and for the phase difference between QD sites $\phi=0=2\pi$ (thick broken line), $\phi=\pi$ (thin broken line) and $\phi=\pi/2$ (solid line). For $\Delta_2=0$, the current is zero and independent of the phase ϕ; in this case, only one external perturbation is acting, and no symmetry breaking takes place. For two external perturbations and without phase difference between the first and second QD sites, the current also does not flow, being equal to zero for all Δ_2 (thick broken line). However, for $\phi\in(0,2\pi)$, the current flows, although there is no voltages applied to the system. In that case, we realize a sort of a quantum pump which works only in the presence of two time-dependent perturbations, implying that the generalized parity is broken in a dynamical way [7, 13]. Note that, depending on the phase difference between both QD sites, the dc current can be positive or negative, cf. the broken and solid lines.

A remaining question is how the pump current depends on the phase difference between both QD sites. In

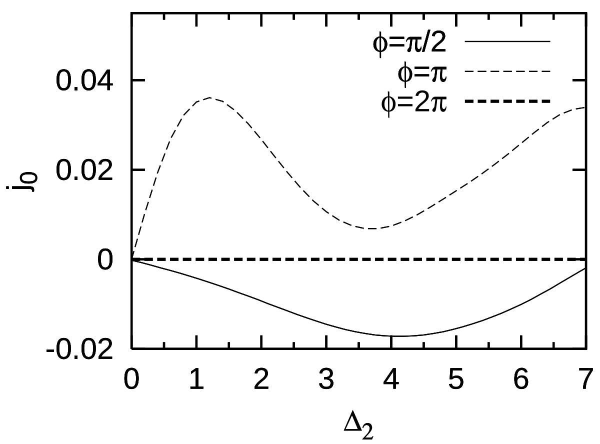

Fig. 6. Dc-current flowing through a double QD symmetric system under the influence of two external perturbations versus the amplitude Δ_2 for the phase difference $\phi = 0 = 2\pi$ (thick broken line), $\phi = \pi$ (thin broken line) and $\phi = \pi/2$ (solid line). The other parameters are $\omega_1 = 3$, $\Delta_1 = 3$, $\omega_2 = 6$, $V_{12} = 1$, $\Gamma = 0.2$, $\varepsilon_1 = \varepsilon_2 = 0$, $\mu_L = \mu_R = 0$

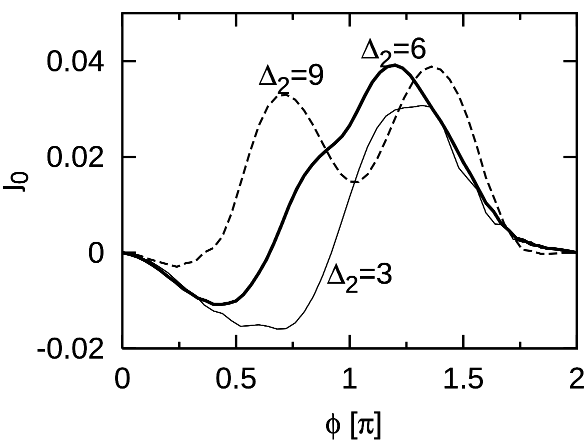

Fig. 7. Dc-current as a function of the phase difference ϕ for two external perturbations $\omega_1 = 3$, $\Delta_1 = 3$, $\omega_2 = 6$ and for $\Delta_2 = 3$ (solid line), 6 (thick solid line), and 9 (broken line). The other parameters are: $V_{12} = 1$, $\Gamma = 0.2$, $\varepsilon_1 = \varepsilon_2 = 0$, $\mu_L = \mu_R = 0$

Fig. 7, we depict the dc current flowing through a double QD symmetric system ($\varepsilon_1 = \varepsilon_2 = 0$) as a function of the (relative) phase ϕ for three values of the driving strength of the second perturbation, i.e. $\Delta_2 = 3$ (solid line), $\Delta_2 = 6$ (thick solid line), and $\Delta_2 = 9$ (broken line). In case of zero phase difference, $\phi = 0, 2\pi$, the current is zero, although two external perturbations are applied to our system, cf. also Fig. 6. For values of ϕ different from $\phi = 0, 2\pi$, a finite pump current flows through the double dot system: depending on the relative phase, it can be either positive or negative.

It is also possible to stop the pumping current all together for specific values of ϕ. Note that, for very large amplitudes of the external perturbation, Δ_2, the current takes on positive values for almost for all ϕ, cf. the broken line.

4. Conclusions

The time-dependent electron transport through a quantum dot and a double quantum dot system in the presence of polychromatic perturbations has been studied within the evolution operator method. The QD has been coupled with two electrodes, and the external time-dependent energy perturbations have been applied to the central region.

The analytic relations for the dc current flowing through the system, Eq. (20), and the charge accumulated on a QD, Eq. (21), have been obtained for incommensurate external perturbations. In addition, the analytic relations for the transmission have been derived for commensurate frequencies Eq. (18) and for incommensurate ones Eq. (19). It has been found that sideband peaks appear for $\varepsilon_d = \sum_{i=1}^{n} \pm k_i\omega_i$, where k_i is an integer number, and n stands for the total number of external perturbations. In the case of two external perturbations, this condition can be written as $\varepsilon_d = \pm k_1\omega_1 \pm k_2\omega_2$; and; e.g. for $\omega_1 = 3$ and $\omega_2 = 8$, additional peaks appear for $\varepsilon_d = 2, 5, \ldots$, cf. Fig. 1. In the presence of external perturbations, one can control the dc current by changing the amplitude strengths of acting perturbations, cf. Fig. 2. In the presence of multiple external perturbations, the current is characterized by satellite peaks, cf. Fig. 3 for two acting energy perturbations.

Moreover, the dc current obtained for commensurate frequencies, e.g. for bi-harmonic perturbations, strongly differs from that obtained for frequencies slightly different from commensurate ones, cf. Fig. 4. The asymmetry in the transmission and the dc current versus the quantum dot energy level ϵ_d for commensurate frequencies case is detected (see Fig. 5). This asymmetry appears only for commensurate external perturbations applied to the system and is related to a phase difference between time-dependent perturbations. For a double QD system, the analytic formula for the transmission has been obtained for incommensurate frequencies, Eq. (29). In addition, an electron quantum pump based on a fully symmetric double QD system has been proposed in the absence of source-drain voltages and static bias voltages, for which the pump current varies as a function of the relative phase shift ϕ, cf. Fig. 6 and Fig. 7.

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ДИНАМІЧНО ПОРУШЕНА СИМЕТРІЯ В ПЕРІОДИЧНО КЕРОВАНИХ КВАНТОВИХ ТОЧКАХ: НАКОПИЧЕННЯ ЗАРЯДУ І ПОСТІЙНИЙ СТРУМ

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Р е з ю м е

Досліджено залежний від часу транспорт електронів через квантову точку і систему двох квантових точок при зовнішній поліхромній періодичній модуляції рівнів енергії в межах методу оператора часової еволюції з гамільтоніаном сильного зв'язку. Одержано аналітичні формули для постійного струму через систему та для заряду, який накопичено на квантовій точці у границі нульової температури. Показано, що в присутності періодичних збурень бокові максимуми передачі залежать від співвідношення зовнішніх модуляцій. Вивчено ефект квантової накачки за відсутності джерела (стока) і статичних напружень зміщення. У випадку просторової симетрії заряд накачується через систему внаслідок порушення симетрії узагальненої парності.