THE THEORY OF HIGH-TEMPERATURE SUPERCONDUCTIVITY IN MANY-BAND SYSTEMS. MgB_2

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The main stages of development of the theory of superconducting systems with overlapping energy bands are formulated. The main references of the classical papers of the author of this theory, Prof. V.A. Moskalenko, and his coworkers are listed. The list also includes papers related to high-temperature superconductivity. Some peculiarities of the two-band model which gives qualitatively new results in comparison with the usual one-band model, are enumerated. The application of the two-band model to the description of thermodynamical properties of compound MgB_2 is also discussed. The references covering our research of the kinetic properties of superconductors with overlapping energy bands are provided. In particular, we present the Ginzburg–Landau (GL) equations for the two-band superconductors doped with impurities and the results on the influence of impurities on the energy gap, as well as those concerning the dynamical properties of two-band superconductors.

1. Two-Band High-Temperature Superconductivity

In this work, we partially touch upon that distant heroic time, when the mechanism of such amazing effect as superconductivity was discovered (the year of 1957).This problem had been solved as a result of the long-time and intense theoretical and experimental researches (it had taken almost 50 years), and it had appeared as the most important cryogenics discovery. The significant results in the construction of the theory of superconductivity belong to the great physicist and mathematician N.N. Bogoliubov [1–4], whose centenary we celebrate nowadays. The great contribution to the development of the theory was given by his disciples: S.V. Tyablikov, D.N. Zubarev, D.V. Shirkov, and other eminent scientists. Along with the development of the theory of superconductivity in the isotropic systems, Bogoliubov came up with the idea of the anisotropic properties of superconducting systems accounting for the need to describe real metals. The first work in this direction, where the anisotropy appears in the band structure of the considered system, belongs to V. Moskalenko.

The model of a superconductor with the overlapping of energy bands on the Fermi surface was proposed by V. Moskalenko [5] and some later also by H. Suhl *et al.* [6] about 50 years ago.

This model considers the anisotropy of energy bands and describes the superconducting properties of transition metals that have two or more groups of electrons belonging to different energy bands on the Fermi surface.

The main assumption of the model is the formation of Cooper pairs of electrons inside one energy band and the transition of a pair as the whole entity from one energy band to another one. This results in the appearance of the intraband V_{nn} and the interband $V_{nm}(m \neq n; n,\ m = 1, 2)$ electronic interactions, which leads to the additional attraction of electrons. This, in its turn, favors an increase of the superconducting transition temperature. This is equivalent to the diagonal approximation over band indices, and two order parameters Δ_{11} and Δ_{22} appear in the two-band model.

Having made these assumptions, V. Moskalenko and his co-workers have carried out the investigations of the thermodynamic and electromagnetic properties of many-band superconductors. A few books and a lot of articles on this problem have been published (see, e.g., [7–16] and references therein, as well as [17, 18]).

It had appeared a new scientific trend of studies of the multiband superconductors' properties. Along with the Moldavian theoretical physicists, a lot of scientists in different countries worked in this direction (see, e.g.,

[19–22] and references in [15]). It should be noted that the most valuable contribution to the development of the theory of superconductivity in the systems, where two and more energy bands overlap on the Fermi surface, belongs to the Moldavian scientists.

At the time preceding the discovery of high-T_c superconductivity, the theory of superconductivity with overlapping energy bands had been developed in order to describe the physical characteristics in the systems with heavy fermions [23, 24]. The discovery of high-T_c superconductivity [25] has aroused the attention to this problem again, which is explained by the overlapping of two or more energy bands on the Fermi surface in the high-T_c superconductor materials. Such a conclusion results from the band structure of oxide ceramics [26, 27]. This can be understood as well, having considered a layered structure of these systems.

The Hamiltonian that describes the system with n layers and takes interactions of electrons inside each layer and between layers into account can be transformed without difficulty into a Hamiltonian that describes n band systems [28–30]. This transformation has been done by the diagonalization and the introduction of the symmetric and anti-symmetric states of electrons.

Since the discovery of high-T_c superconductivity, a great number of theoretical studies of the many-band model [5] regarding the high-T_c materials have been made.

An increase in the number of energy bands on the Fermi surface leads to an increase in the total density of electron states and to an additional interband electron-electron interaction which favors the superconducting state. This interaction destroys the validity of the universal BCS relations, and the thermodynamic characteristics depend significantly on the properties of the anisotropic system.

Series of experimental studies have been performed there as well. They discovered the presence of two energy gaps in the energy spectrum of electrons and proved, in its turn, the appearance of singularities caused by the overlapping of energy bands on the Fermi surface in the energy spectrum of electrons (see, e.g., [31]).

Nowadays, the researchers have concluded that the experiment in the oxide metals does not confirm the presence of two energy gaps. This may be a result of many difficulties of the high-T_c superconductivity and, in particular, of a disorder in the system (great concentrations of impurity or oxygen deficiency). In this case, despite the presence of two order parameters, only a single energy gap is found in two-band superconductors at high concentrations of impurity [13]. There has appeared the averaged order parameter $\bar{\Delta} = \dfrac{N_1\Delta_1 + N_2\Delta_2}{N_1 + N_2}$ that determines the averaged energy at $T = 0$ [32]

$$\frac{E_S - E_N}{V} = -\frac{1}{2}\bar{\Delta}^2(N_1 + N_2),$$

where N_i is the density of electron states on the ith fold of the Fermi surface.

At great concentrations of an impurity, the jump of the heat capacity at the critical point $T = T_c$ in a two-band system has the form [13]

$$\frac{C_S - C_N}{C_N} = \frac{8\pi^2}{7\beta_c}\frac{N_1 + N_2}{\zeta(3)}.$$

These expressions coincide formally with the case of the pure one-band model with the only difference that the quantity $\beta_c = (kT_c)^{-1}$ is determined on the two-band background and contains all electron-electron interaction couplings (the intraband and interband ones). In addition to this, the density of electron states is replaced by the total density of both bands. Note that, in the two-band systems with the great concentration of a non-magnetic impurity, the equation of Ginzburg–Landau is reduced to the one-band equation for the wave function $\Delta(\mathbf{x})$ that is the gravity center of wave functions $\Delta_1(\mathbf{x})$ and $\Delta_2(\mathbf{x})$. In this way, we have to take into account that the quantity T_c and the parameter κ of this one-band equation are determined on the two-band basis.

Therefore, the overlapping of energy bands on the Fermi surface contributes essentially to the physical quantities even at a strong disorder in the system.

The conclusion about the absence of two gaps in the electron energy spectrum (because the experiment does not confirm it) or even omitting the contribution of both energy bands that overlap on the Fermi surface to the thermodynamic and electromagnetic characteristics of the oxide metals was earlier stated. More detailed experimental studies should be done both at the high and low densities of charge carriers in order to decrease the disorder of a two-band system.

Note also that Moskalenko's model [5], as well as its generalizations for the arbitrary densities of charge carriers (including the very low densities), is an isotropic one. So, the model assumes the singlet pairing of Cooper pairs. There is no difficulty to construct a theoretical two-band model with the d-pairing of electrons, as it has been done for the two-band superconductors with non-magnetic impurity [33].

The consideration of the overlapping of energy bands leads not only to the quantitative difference of the results

from the case of the one-band superconductor, but, in some cases, to qualitatively new results. For example:

(1) In the two-band superconductors, the high temperatures of the superconducting transition are possible not only in the case of the attractive interaction between electrons, but even in the case of the repulsive one ($\lambda_{nm} < 0$, $n, m = 1 - 2$), but when the relation $\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21} < 0$ is fulfilled.

(2) In the impurity two-band superconductors, for example, the Anderson theorem is violated at $\Delta_1 \neq \Delta_2$, and the dependence of thermodynamic quantities on the concentration of a non-magnetic impurity due to the interband scattering of electrons on the impurity atoms is observed.

(3) In the two-band superconductors, the collective oscillations of the exciton-like Leggett mode caused by fluctuations of the phase of order parameters from different bands are observed. In the three-band systems, and in the two-band ones with account of the Cooper pairs of electrons from different energy bands, that is reduced to the effective three-band model, the number of such oscillatory modes can be two.

(4) Using the two-band model and assuming the moderate values of coupling constants, one can obtain high values of T_c, the two energy gaps $2\Delta_1/T_c > 3.5$ and $2\Delta_2/T_c < 3.5$, large values of negative $dlnT_c/dlnV$ (V is the volume), positive curvature of the upper critical field near the transition temperature, *etc.* [34–38]. Furthermore, in the two-band model, it is possible to describe a decrease of T_c with increase of the system disorder [39–41].

(5) The location of the Fermi level that can be changed by doping plays an important role in the determination of thermodynamic and magnetic properties of a two-band superconductor. Having assumed a non-phonon pairing mechanism of superconductivity, as well as the phonon mechanism in the many-band systems with lowered densities of charge carriers, the account of the singularities mentioned above is very crucial. A particular interest is attached to the possibility to obtain a bell-shaped dependence of T_c and a jump of the heat capacity $(C_S - C_N)/C_N$ at the point $T = T_c$ on the carrier density [42–44]. In the three-band model, it is possible to obtain a "step" in the dependence of T_c on the carrier density [45, 46]. The investigation of the properties of superconductors with energy bands on the Fermi surface and the electronic topological transitions, was reviewed in [47]. This review contains the classical results of the problem.

An increase in the number of energy bands on the Fermi surface increases the overall density of electron states and leads to the onset of an additional interband electron-electron interaction that contributes to the onset of superconductivity. This interaction violates the universal BCS relations and leads to the substantial dependence of a number of the physical characteristics on properties of the anisotropic system [12, 13, 15, 47–49].

Note that the experimental confirmation of two-band superconductivity has been made long before the discovery of the high-T_c superconductivity. For example, there have been studied the tunnel characteristics of $SrTiO_3$: NbIn in [50], and two order parameters have been found. The appearance of two energy bands that overlap on the Fermi surface in this compound is likely to be caused by the great value of the dielectric constant of strontium titanite. This results in the weak interband scattering of electrons on the ionized donors, and even their high concentration does not initiate the transition of the system to the one-band case. When the carrier concentration is gradually increased by adding niobium, the consequent filling of the first and second energy bands takes place.

The discovery of superconductivity in the intermetallic borocarbide nitrides was the important whirling point in superconductivity. These compounds possess interesting superconducting and magnetic properties. The more detailed information about the discovery of diverse new intermetallic compounds and their properties can be found in [51].

2. Superconductivity in compound MgB_2 and in the Systems with Reduced Density of Charge Carriers

The discovery of superconductivity in the intermetallic compound MgB_2 was even more intriguing [52]. The main physical properties and singularities are as follows (see, e.g., [53] and [54]):

(1) The high temperature of the superconducting transition, $T_c \sim 39$ K [52].

(2) The interaction responsible for the formation of superconducting pairs is caused by the exchange of phonons [55, 56], and the symmetry of Cooper pairs is of the s-wave type [57].

(3) The average phonon frequencies are 2–3 times as high as those for classical superconductors Nb_2Sn [58, 59], the mass renormalization factor $(1 + \lambda)$ is small, the Sommerfeld's coefficient γ_n and the condensation energy also appear to be inconsistent with superconductivity near ~ 40 K.

Researchers have concluded that the superconductivity in MgB_2 cannot be understood from the isotropic one-band BCS model. The anisotropy of the system is

the very key factor here that is revealed in the anisotropy of energy bands (the overlapping of energy bands on the Fermi surface) and in the anisotropy of the order parameter, i.e. its dependence on the momentum direction.

As was mentioned above, the properties of the isotropic two-band (multiband) superconductors have been studied long before the discovery of MgB_2 compounds by V. Moskalenko and his co-workers. These studies showed that the thermodynamic and electromagnetic properties of a two-band superconductor are qualitatively different from those of a one-band superconductor.

Another field of studies in the BCS theory is an account for an anisotropy of the electron-phonon interaction [60, 66]. As shown in the works mentioned above, the band anisotropy, as well as the anisotropy of the matrix element of the electron-phonon interaction, decreases the relative jump of the electron heat capacity $(C_S - C_N)/C_N$ at $T = T_c$ in comparison with the value 1.43, which is specific of the isotropic case.

Works [62–64] have developed the method taking the existence of a greater number of bands and the anisotropy of energy gaps in each band into consideration in the calculation of the electron heat capacity.

We did not intend to make a thorough analysis of the works mentioned above. Note only that the clear picture with regard for the band structure, the topology of the Fermi surface, the values of densities of electron states, the averaged velocities of electrons on the Fermi surface, and other characteristics necessary for consistency of theory with experiment has been obtained. In MgB_2, two energy gaps $\Delta_1(0) = 6.8$ meV $(2\Delta(0)/k_BT_c = 4.0)$; $\Delta_2(0) = 1.8$ meV $(2\Delta_2(0)/k_BT_c = 1.06)$ have been experimentally determined [65]. As a result, some singularities of thermodynamic properties have been observed in these compounds. The shoulder-type anomaly, for example, appears in the temperature dependence of the heat capacity around $0.25\,T_c$ as well as $(C_S - C_N)/C_N \approx 0.8$ at $T = T_c$ [59], which is consistent with the theory [5, 9, 64]. Another anomaly observed experimentally is the positive curvature of the upper critical magnetic field $H_{c_2}(T)$ near the temperature of superconducting transition (see theoretical studies [47, 65]). Note also the breakdown of the Anderson theorem in the two-band system with a non-magnetic impurity due to the interband scattering of electrons on the impurity, which leads to a disorder in the system [7, 8, 13]. These and other anomalies in MgB_2 can be understood only by considering the overlapping of energy bands on the Fermi surface. The two-band model describes qualitatively the main singularities in the behavior of the physical characteristics of MgB_2.

A possibility to describe superconductivity on the basis of the two-band model in other compounds cannot be excluded. This fact validates the following generalizations and the development of superconductivity theory with regard for the overlapping of energy bands.

In all the above-mentioned works, the two-band model can be used to describe the properties of the superconductors, for which the relation $\mu \gg T_c$ is satisfied (μ is the chemical potential). This description is made in the diagonal approximation over the band indices [7, 10].

In systems with low carrier densities, however, the relation $\mu \gg T_c$ does not hold. Therefore, it is necessary to develop the theory of superconductivity for the two-band systems without constraints on the Fermi energy. We consider simultaneously two possible superconductivity mechanisms – the phonon and electron ones. The substantial dependence of the chemical potential μ on the order parameter in the superconducting phase is an inherent feature of the systems with low carrier densities. These circumstances have been noted in many papers, and the feasibility of experimental observation of these anomalies in the temperature dependence of the chemical potential was first suggested in [66]. It was shown there with the BCS model as an example that the $\mu(T)$ curve has an experimentally observable bend at the point $T = T_c$. Below, we show that, in the two-band case, this effect is enhanced by the presence of two or four order parameters (Δ_{nm}; $m = 1, 2$), and is manifested at μ values more easily observed in experiment [67].

In the works cited above, the investigations were carried out by the Cooper pairing scenario. In the systems with small carrier concentrations, the bound states may arise following a decrease in the carrier concentration, and the transition to the Bose condensate of localized pairs with a finite bound energy may occur (the Schafroth scenario [68]). The possibility of such a transition in the one-band systems was discussed in a number of works (see, e.g., [67–74]).

As was shown in [69, 72], a change of the sign of the chemical potential with decrease in the carrier concentration corresponds to the transition from the BCS to Schafroth scenario. The condensation of localized pairs occurs at the concentrations of carriers, for which $\mu \leq 0$.

We present the theory of superconductivity of the two-band systems that is valid at any carrier density and considers all possible pairing of electrons due to the intra-

band and interband interactions by the Cooper pairing scenario. The critical temperature T_c, chemical potential μ, and heat capacity $(C_S - C_N)$ at the point $T = T_c$ as functions of the carrier density are shown in [47, 67, 75]. This theory is used to describe the properties of MgB_2-based compounds, where magnesium and boron are replaced by different elements of the Periodic table. The works [76,77] are designed for the self-consistent discussion, in the mean-field approximation, of the system of equations for the order parameters Δ_n and μ at $T = 0$. There have been revealed the influence of the overlapping of energy bands on these quantities and the carrier concentration, at which the system experiences the transition from the Cooper pairing ($\mu > 0$) to the Schafroth ($\mu < 0$) scenario. The equation for the binding energy ε_b of the two-particle state is also obtained, and the relation between ε_b and μ is established.

The path integral method as applied to the two-band model is also developed, and, on this basis, the procedure for the transition from the Fermi to Bose elementary excitations at $T = 0$ is given in [75, 78, 79]. The Bose system condensation temperature T_k is also determined. The theory of superconductivity in two-band non-adiabatic systems with strong electron correlations in the linear approximation over non-adiabaticity is built [80].

The superconducting ordering in the systems with two characteristic features – the small concentration of charge carriers and the overlapping of energy bands on the Fermi surface – is investigated.

Along with the above-described phenomena related to the overlapping of energy bands on the Fermi surface, a very interesting one has to be examined – the appearance of collective oscillations due to the phase fluctuations of the order parameters of different bands. Fistly, this phenomenon was researched in the theoretical work by Leggett [74]. According to its nature, it can appear only in the systems with two or more energy bands on the Fermi surface. Our following results on collective oscillations in three-dimensional systems and in the systems with reduced dimensionality develop the Leggett's researches, by considering two or more energy bands, within the phonon and non-phonon superconducting mechanisms, supposing a reduced and a weak carrier concentration until the transition from BCS state ($\mu > 0$) to the Schafroth state ($\mu < 0$). The collective exciton-type modes differ quantitatively in different systems and are determined by physical features of the examined systems [81–85].

3. Thermodynamic and Magnetic Properties of Doped Compound MgB_2

It should be noted that the above-discussed two-band model [5] was found to be very fruitful, since it had explained a lot of abnormal physical properties of superconducting anisotropic systems and had given a quite good accordance with the experimental data.

Let's give an example of the determination of thermodynamic and magnetic properties in MgB_2, when Mg and B are replaced by other chemical elements.

a. Thermodynamic properties of doped compound MgB_2

The theory of two-band superconductors with variable or small density of charge carriers [42, 33, 28] can describe the behavior of thermodynamic quantities such as T_c, Δ_1, Δ_2, $(C_S - C_N)/C_N$ at $T = T_c$ as functions of the chemical potential μ or the charge carrier density in MgB_2.

To this end, it is necessary to do the following:
(1) Start from the system of equations for the BCS-type order parameters Δ_n $(n = 1, 2)$ for the two-band model with the electron-phonon interaction constants λ_{nm} corresponding to the strong electron–phonon coupling renormalized in the two-band model and to the presence of the Coulomb interaction μ^*_{nm} [86, 87].
(2) Add an equation that determines the chemical potential to the system of equations for the quantities Δ_n. This addition is necessary for systems with the low charge carrier density $\mu \sim \Delta_n$. The system MgB_2 is not such a system, compound because $\mu = \mu_0 \approx 0.74$ eV for a pure substance; that is, $\mu \gg \Delta_n$. However, the additional equation would be introduced due to a specific band structure of the considered system: the upper boundary of the σ-band, which is responsible for superconductivity in MgB_2, is situated in the vicinity of μ_0. This circumstance plays the decisive role in the dependence of thermodynamic quantities on the parameter μ varying at the substitution of magnesium and boron atoms by chemical elements of another valence.
(3) Consider the overlapping of the two-dimensional σ- and three-dimensional π-bands on the Fermi surface. The dependence of thermodynamic quantities on the chemical potential μ should be built with regard for its proximity to $\mu_0 \approx 0.74$ eV for pure MgB_2. From the experimental data for the renormalized constant of the electron-phonon interaction, we obtain $\lambda_{11} = 0.302$, $\lambda_{22} = 0.135$, $\lambda_{12} = 0.04$, and $\lambda_{21} = 0.038$.

(4) Introduce the relative charge carrier density $\delta = (\mu - \mu_0)/\mu_0$ that coincides with the corresponding value which is calculated with regard for the valence of the elements constituting compounds $Mg_{1-x}Li_xB_2$, $Mg_{1-x}Cu_xB_2$, $Mg_{0.8}Li_{0.2}B_{2-x}C_x$, $Mg_{0.95}Cu_{0.05}B_{2-x}C_x$, and $MgB_{2-x}C_x$ at different values of x. The dependences of the T_c values on δ built in this way allow us to compare our theoretical results with the experimental data (see, e.g., [88]). As follows from our calculations, the doping of MgB_2 with electrons ($\delta > 0$) leads to a decrease of the critical temperature T_c ($MgB_{2-x}C_x$ and $Mg_{0.95}Cu_{0.05}B_{2-x}C_x$). But, at the doping with holes ($\delta < 0$), the value of T_c ($Mg_{1-x}Li_xB_2$) does not vary with the parameter δ. This scheme does not cover compound $Mg_{0.8]}Li_{0.2}B_{2-x}C_x$, in which T_c takes the value 39.4 (that corresponds to MgB_2) at $\delta = -0.02$ and decreases with the hole density.

Our theory takes the occupation of energy bands (variation of the chemical potential μ) into account, as well as the scattering of charge carriers on the impurity potential [8, 38], when carbon atoms are introduced into the layered structure instead of boron atoms responsible for superconductivity. In view of these two mechanisms, we obtain the dependence which adequately describes the experimental data ($MgB_{2-x}C_x$ and $Mg_{0.95}Cu_{0.05}B_{2-x}C_x$). The doping with holes ($\delta < 0$) ($Mg_{1-x}Li_xB_2$ and $Mg_{1-x}Cu_xB_2$) does not change the T_c value, because the impurity is not introduced into the layer responsible for superconductivity, and lithium and copper introduced instead of magnesium cause only the variation of the effective valence of boron.

A decrease of T_c in $Mg_{0.8}Li_{0.2}B_{2-x}C_x$ is due to the scattering of electrons on the impurity potential of carbon atoms. This jump of the electron heat capacity $(C_S - C_N)/C_N$ at $T = T_c$ is very small (0.8) at $\delta = 0$ that corresponds to MgB_2 without impurity. This small value is due to the overlapping of energy bands on the Fermi surface. As this overlapping decreases (to the right or to the left from this point), the value of this jump increases and becomes equal to 1.43 (at $\delta \approx 0.06$, e.g.) that corresponds to the one-band system. These estimations have been realized with regard for the effect of occupation of energy bands only, without consideration of the impurity scattering. The obtained results correctly reflect the transition from the two-band system to the one-band one and are in qualitative agreement with the experimental data (as for details, see [86, 87]). The above-given experimental and theoretical results describe well enough the dependences [88, 89] which were observed during the experiment.

b. The upper critical fields $H_{c2}^{(ab)}$ and $H_{c2}^{(c)}$ in intermetallic compound MgB_2

The experimental investigations of the magnetic properties of MgB_2 show the bright appearance of an anisotropy of the upper critical field H_{c2} [90]. The upper critical field $H_{c2}^{(ab)}$, which corresponds to the external magnetic field in the plane (ab), exceeds $H_{c2}^{(c)}$ with the magnetic field along the c-axis by several times.

We pose the problems to build the microscopic theory of the upper critical field H_{c2} of a pure anisotropic two-band superconductor applicable on the whole temperature interval $0 < T < T_c$, to describe the pattern of the H_{c2} behavior as a function of the temperature in MgB_2, to determine the curvature of the upper critical field $H_{c2}^{(ab)}$ and $H_{c2}^{(c)}$ close to the temperature of superconducting transition, and to reveal then the anisotropy of the temperature dependence of the coefficient $\gamma_H = H_{c2}^{(ab)}/H_{c2}^{(c)}$. We determine also the influence of the mechanism of occupation of energy bands on T_c and H_{c2}^{ab}, when the system is doped with electrons or holes. We note that the researches of two-band systems are based on the microscopic approximation of the theory of superconductivity [91–93].

Herewith, the following peculiarities of the MgB_2 band structure are taken into account: the mutual arrangement of energy bands, presence of the overlapping of the two-dimensional σ-band and the three-dimensional π-band, and differences of the topologies of Fermi surface cavities of the bands under consideration.

The values of the above-given magnetic fields are determined on the base of the Ginzburg–Landau equations for a two-band system. In this case, we apply the method of Maki and Tsuzuki extended to the two-band case [92] with the account for MgB_2 compound band structure peculiarities (as for details, see [94–96]). This method allows one to obtain the analytic solutions for the critical fields $H_{c2}^{(ab)}$ and $H_{c2}^{(c)}$ in the low-temperature range ($T \ll T_c$) and near the critical temperature ($T_c - T \ll T_c$). The account for the anisotropy results in an anomaly of physical characteristics of compound MgB_2. At the same time, the proposed method allows one to consider both a pure anisotropic two-band superconductor and intermetallic compound MgB_2 with the Mg and B atoms replaced by the other elements of the Periodic table. We now give the results of calculations of the upper critical fields $H_{c2}^{(ab)}$ and $H_{c2}^{(c)}$ which were obtained on the base of the constructed two-band theory. We use the following constants of the electron-phonon interaction which correspond to MgB_2: $\lambda_{11} = 0.302$; $\lambda_{22} = 0.135$;

$\lambda_{12} = 0.04$; $\lambda_{22} = 0.038$, and $\lambda = \frac{v_1}{v_2} = 0.8$ (the ratio of electron velocities on the different cavities of the Fermi surface) [86]. We assumed that the chemical potential in MgB_2 without doping amounts to $\mu_0 = 0.74$ eV. The parameter is selected as $\varepsilon = 0.31$. This value gives the closest approach to the experimental results. The parameter ε defines by the declination of the σ-band from two-dimensionality.

We obtain the values of $H_{c2}^{(ab)} \gg H_{c2}^{(c)}$. This result corresponds well both to the results of many theoretical works and to the experimental data. The strong anisotropy of the upper critical field is explained by a weak dispersion of the electron energy in the axis z direction and by the low value of the average electron velocity on the Fermi surface in this direction. In the case of Mg and B replacement by the other chemical elements, which furthers the doping by electrons (increasing the chemical potential), the behavior of the upper critical field as a function of the temperature is similar to that in the case of pure MgB_2. However, the values of these quantities decrease in comparison with those in the case of pure MgB_2. The correlation of the superconducting phase transition temperature T_c and the critical fields takes place with increasing the chemical potential. An increase in the hole conduction has no effect on the critical temperature and the upper critical field. The interrelation of the upper critical temperatures of doped and pure MgB_2, as well as the dependence of upper critical fields on the electron density (the chemical potential μ), is considered in [96]. We obtain that all the quantities decrease with increase in the electron density of charge carriers at $\mu > 0.74$ eV and remain constant at $\mu < 0.74$ eV. Consequently, the hole doping leaves constant the values of the temperature of the superconducting phase transition and the upper critical field. The essential dependence of the anisotropy coefficient γ_H on the temperature in pure MgB_2 ($\mu_0 = 0.74$ eV) and doped MgB_2 ($\mu = 0.76$ eV) was obtained. The above-presented results correspond well to the experimental data on the magnetic properties of both pure intermetallic compound MgB_2 and doped by electrons and holes (see [97], e.g.). This tells about the ability of the two-band model to describe the properties of real materials and the ability to calculate the anomalies of physical properties which were generated by the anisotropy of a system. We note that the filling of energy bands was concerned as the main mechanism of action of a substitutional impurity. It was assumed that the scattering on the impurity potential is weak. The account for the electron scattering on impurities essentially complicates the results for the systems, where the impurity scattering is strong.

4. Some Kinetic Properties of Two-Band Superconductors

The discovery and the experimental investigation of superconducting properties of MgB_2 have attracted a special interest to the model with overlapping energy bands. In some cases, this model is extended to take various anisotropies of the order parameters into account, as well as the strong electron-lattice coupling. The two-band model is used to interpret experimental data on tunneling, specific heat, electron Raman scattering, thermal conductivity, penetration depth of a magnetic field, and other properties of MgB_2.

Prof. V.A. Moskalenko and his coworkers from the Institute of Applied Physics of the Academy of Sciences of Moldova have studied most of these properties on the basis of the above model. The equilibrium problem is described by the two-band Hamiltonian [5]. This Hamiltonian has been extended to consider the scattering of electrons by non-magnetic impurities [8]. The results for pure and doped systems are valid for arbitrary values of the two-band parameters.

We present here only some qualitatively new kinetic properties which have been obtained within the model with overlapping energy bands.

1. The GL system of equations for the two-band model has been formulated to cover the whole range of parameters from the pristine to the dirty limit. On this basis, the magnetic penetration depth of a superconductor, the jump of the specific heat per unit cell at the critical temperature, and other properties have been investigated in [9, 15, 98].

For a high concentration of non-magnetic impurities, the system of GL equations for the two-band model is similar to the GL system of the one-band model. However, the critical temperature T_c and the dimensionless GL-parameter k in the equations are determined by the two-band model. In this case, the expression for the relative jump of the specific heat at T_c coincides with the corresponding expression for the pristine one-band model, the density of states being the sum of densities of two bands, and the dependence of T_c on impurities being specified by the two band model. The effect of magnetic impurities is to decrease the relative specific heat jump and to increase considerably both the parameter k and the magnetic field penetration depth in the superconducting phase.

2. The influence of impurities (non-magnetic and paramagnetic) on the thermodynamic properties of two-band superconductors at zero, close to zero, and at the critical temperatures has been considered in [11, 16, 99]. It has been found that, due to the interband scattering of electrons on impurities, the superconducting state in the two-band model is described by a single-energy gap. Thus, when one of the densities of states becomes zero, the other density vanishes too. In particular, the energy gap in the "dirty" limit for a non-magnetic impurity decouples in a product of averages of the order parameters of individual bands with their densities serving as weight factors.

3. The non-equilibrium process of charge imbalance in a two-band superconductor has been investigated by employing the technique of Keldysh-Green functions. The kinetic equation, penetration depth of a longitudinal electric field, and distribution of this field in the superconductor are given in [100].

A new mechanism of relaxation of the charge imbalance in non-equilibrium two-band superconductors has been revealed. This mechanism is due to the inter-band electron-impurity scattering and leads to a decrease of the penetration depth of a longitudinal electric field into the superconductor.

4. A model of a superconductor with two dielectric gaps and two superconducting order parameters corresponding to two parts of the Fermi surface has been formulated. The phase diagram obtained for this model contains an area of coexistence of structural, antiferromagnetic, and superconducting phase transitions versus the non-magnetic impurity concentration, which agrees qualitatively with the experimental data on high-temperature superconductors [101].

5. In [102], the electron Raman scattering in superconductors, taking the collective oscillations, Coulomb screening, and scattering of electrons by non-magnetic impurities into account, has been studied in the framework of the two-band model.

Two contributions to the scattered light intensity have been singled out: an additive contribution from each of the two bands, and a term caused by the interband transitions of Cooper pairs which exists for an arbitrary light polarization. Experimentally, this means that the lowest gap should be active for any light polarization.

6. The propagation of a longitudinal ultrasound in the one- and two-band models of superconductors at arbitrary temperatures has been investigated by considering the collective oscillations in the presence of nonmagnetic impurities for an arbitrary mean free path. The effect of superconductivity and impurities on the relative shift of the sound velocity turned out to depend strongly on the choice of a model [103,104].

In particular, we have predicted a more efficient suppression of fluctuations of the superconducting gaps by impurities within the two-band model, than that within the one-band model. The two-band model has also allowed us to explain such a spectacular feature of high-T_c superconductors as an increase of the sound velocity for all the temperature interval below T_c.

7. For a two-band superconductor, the amplitude of the multiple electron scattering by nonmagnetic impurities has no electron-hole symmetry with respect to the Fermi surface, and this may be the cause of an increase in the thermoelectric effect in superconductors. As a result, the temperature dependence of the additional contribution to the thermoelectric coefficient reaches a maximum in the region of temperatures $T < T_c$ [105].

8. The collective modes related to phase fluctuations near T_c have been investigated by assuming the existence of a two-component neutral superfluid. The equation for collective modes describes the interference of two processes: small fluctuations of the relative density of the condensate of electrons (Leggett-type) and small fluctuations of the charge imbalance of the electron-hole branches. This equation is analogous to the well-known equation in solid state physics describing, e.g., the collective modes of polaritons [106].

The amplitudes of the collective modes of the two-band model have also been studied. We mention that these modes in the case of non-identical traditional two-band superconductors do not occur [107].

5. Conclusion

This work is put forward for the publication with the purpose to turn scientists' attention to the information which is related to the two-band superconductors' properties and which was obtained by the Moldavian physicists guided by Prof. V.A. Moskalenko who is the creator of the multiband superconductivity model. Thus, we underline the essential contribution of the N.N. Bogoliubov's school to the development of superconductivity in this direction. We note that our theory contains the classical results which are related to the essential distinction of the two-band superconductor properties from the one-band ones not only in the quantitative sense, but also in the qualitative sense. These results had been obtained long before the discovery of HTSC and superconductivity in MgB_2. The theory describes well enough all kinds of anomalies of physical characteristics of multiband superconductors (e.g., MgB_2). The

large number of noncuprated compounds is obtained nowadays, such as $LaFeAsO_{1-x}F_x$, $Pr[O_{1-x}F_x]Fe_4As$, $CeO_{1-x}F_xFeAs$, and others, for which, in particular, the presence of energy bands which overlap on the Fermi surface is essential. Consequently, the works on the two-band theory of superconductivity are the base, to some extent, for the investigation of properties of these new compounds. We adduced by far not all of the investigations on the theory of multiband superconductors in this work. We'd like to express our thanks to all the Moldavian physicists who manifested the interest in this problem and who made the contribution to its solution.

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ТЕОРІЯ ВИСОКОТЕМПЕРАТУРНОЇ НАДПРОВІДНОСТІ В БАГАТОЗОННИХ СИСТЕМАХ. MgB_2

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Р е з ю м е

Сформульовано основні етапи розвитку теорії надпровідних систем із зонами, що перекриваються. Наведено основні посилання на класичні роботи автора цієї теорії, професора В.Л. Москаленко, і його співробітників разом із роботами з високотемпературної надпровідності. Відзначено особливості двозонної моделі, яка дає якісно нові результати порівняно зі звичайною однозонною. Обговорено застосування двозонної моделі для опису термодинамічних властивостей сполуки MgB_2. Дано огляд наших досліджень кінетики надпровідників із зонами, що перекриваються. Зокрема, наведено рівняння Гінзбурга–Ландау для двозонних надпровідників з домішками та результати впливу домішок на ширину забороненої зони. Розглянуто динамічні властивості двозонних надпровідників.