Low Temperature Physics/Fizika Nizkikh Temperatur, 2016, v. 42, No. 12, pp. 1397–1405

Effect of nonlinearity, magnetic and nonmagnetic impurities, and spin-orbit scattering on the nonlocal microwave response of a *d*-wave superconductor

H. Yavari, M. Biderang, and M. Kouhfar

Department of Physics, University of Isfahan, Isfahan 81744, Iran
E-mail: h.yavary@sci.ui.ac.ir

Received October 23, 2015, revised June 25, 2016, published online October 24, 2016

By using linear response theory the low-temperature microwave response of a nonlocal and nonlinear *d*-wave superconductor with magnetic and nonmagnetic impurities is calculated. We will show that for the local, linear, and pure sample, penetration depth, $\Delta\lambda(T)$, and conductivity, $\Delta\sigma_1(T)$, vary linearly with temperature, consequently the resistance, $\Delta R(T)$, would change linearly with temperature in agreement with experimental results and for the nonlocal, nonlinear sample the linear temperature dependences $\Delta R(T)$ change to quadratic function. For impure samples the nonlocality and nonlinearity effects are completely hidden by impurities and the temperature dependences $\Delta\lambda(T)$ and $\Delta\sigma_1(T)$ are determined by temperature interval namely the ranges of $T < T^*$ and $T^* << T << T_c$ which T^* is determined by nonmagnetic impurity concentration and the strength of impurity scattering. For $T < T^*$, $\Delta R(T)$ varies as T^2, on the other hand for, $T^* << T << T_c$, $\Delta R(T)$ varies linearly with temperature. We will also show that the temperature dependence of surface resistance is unaffected by spin-orbit interaction and magnetic impurities.

PACS: 85.25.–j Superconducting devices;
74.25.nn Surface impedance;
74.25.Ha Magnetic properties including vortex structures and related phenomena.

Keywords: surface impedance, *d*-wave superconductor, linear response, impurities.

1. Introduction

The origins of the growing interest in the study of high-temperature superconducting (HTSC) microwave resonators to many scientific communities mainly are their applicability to a wide range of devices and providing a better understanding of the physics of these materials. Superconducting resonators have many important applications such as photon detection, SQUID multiplexed read out for astronomy [1,2], quantum computation [3–6] and nanomechanical resonator read out for motion sensing [7]. Today the absorption and noise due to tunneling two-level systems (TLS) [8–11] which are often found at surfaces or in dielectric materials [1,12–14] and tunneling barriers [15] are important problems in improving the performance of these devices. Despite the recent experimental surge investigating the nature of excess noise in these resonators, a full theoretical characterization of noise due to these TLS defects still remains largely a mystery and a deeper understanding is extremely needed.

In developing passive superconducting devices theoretical and experimental analysis are very important for generating sufficient design data or for devising good design models. The high-temperature superconductors are attractive for use in microwave circuits because of their low surface resistance as compared to those of normal metals [16]. The key development of quality microwave resonators filters and delay lines is the low loss HTSC epitaxial thin films on appropriate dielectric substrates [17–19]. To access the ideal optimum device performance, a knowledge of absolute values of the specimen's microwave surface resistance R and penetration depth λ is necessary. Simultaneously valuable information about intrinsic and extrinsic properties of the superconductor can be deduced from these quantities [20]. Another important issue is the establishment of a standard characterization technique for HTSC thin films for microwave applications [21].

Measurements of the temperature behavior of the surface impedance $Z(T) = R(T) + iX(T)$ of HTSC, give information about the nature of quasiparticles in the superconducting state, their scattering, density of states, and about the superconducting pairing mechanism in these materials. The real part of the surface impedance, i.e., the surface resistance R, arise due to normal carriers is proportional to the loss of the microwave power. The imaginary part of the surface impedance, i.e., the surface reactance X, is largely determined by the response of superconducting carriers and characterizes the nondissipating energy stored in the superconductor surface layer.

Since low-temperatures surface resistance of s-wave superconductors at low-frequency involves mechanisms of very weak dissipation instrumental for many applications such as decoherence in superconducting qubits [22], it has recently attracted much attention. For example, niobium superconducting radio-frequency (SRF) cavities for particle accelerators exhibit fantastic high-quality factors due to a very low averaged surface resistance.

By developing a fully microscopic theory, the surface impedance of s-wave superconductors with magnetic impurities was investigated by M. Kharitonov *et al.* [23]. They explicitly demonstrated that, in the regime of gapless superconductivity, the system exhibits saturation of the surface resistance at zero temperature.

Nonlocal effect on the low-temperature surface resistance of d-wave superconductors was calculated as a function of frequency assuming normal state quasiparticle mean free paths in excess of the penetration depth [24]. It was shown that the nonlocal effects can be observed in the surface resistance of high-temperature superconductors if the order parameter has nodes.

The effect of magnetic impurities on the surface impedance of superconducting multilayer and unconventional superconductors has been investigated experimentally [25–27]. Using a simple two-fluid model the results on impedance [25–28] were analyzed theoretically. This model consider a simplified frequency-independent Drude-type dissipative conductivity σ, while the influence of magnetic impurities on the electron spectrum should lead to a nontrivial σ(ω) dependence on the frequency ω.

The effects of both nonmagnetic and magnetic impurities on the complex conductivity of a superconductor was studied by Skalski *et al.* [29]. Their general results can be used for weak magnetic scattering; however, equivalently to the Abrikosov and Gor'kov theory, scattering off magnetic impurities was considered in the Born limit, which did not allow to treat the effect of localized states on the impedance.

By employing the quasiclassical approach, the effects of frequency, the density and strength of magnetic impurities, and the density and temperature of quasiparticles on the dissipative part of the surface impedance for conventional superconductors have been studied [30]. In the limit of small temperatures and small frequencies they have shown that at equilibrium, the dissipation is always proportional to the density of thermally excited quasiparticles and thus exponentially suppressed due to gapped character of the spectrum [30].

To the best of our knowledge, all of the pervious theoretical researches did not take in to account the effects of nonlinearity, spin orbit and magnetic impurities on the surface impedance of a d-wave superconductor. In this paper by using linear response theory we will determine the low-temperature behavior of the real part of the conductivity and consequently surface impedance of a nonlocal and nonlinear d-wave superconductor in the presence of magnetic and nonmagnetic impurities.

2. Formalism

In the Bardeen–Cooper–Schrieffer (BCS) formalism the Hamiltonian is (throughout the paper we use the units $\hbar = k_B = 1$)

$$H = \sum_{k,\alpha} \varepsilon(k) c^{\dagger}_{k\alpha} c_{k\alpha} - \sum_{k} \Delta_k \left(c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} + c_{-k\downarrow} c_{k\uparrow} \right) + + \sum_{kk'i\alpha\beta} (U_{\text{imp}})_{\alpha\beta} \, e^{-i(\mathbf{k}-\mathbf{k}')\mathbf{R}_i} c^{\dagger}_{k\alpha} c_{k'\beta} + + i \frac{v_{so}}{k_F^2} \sum_{kk'i\alpha\beta} e^{-i(\mathbf{k}-\mathbf{k}')\mathbf{R}_i} \left[(\mathbf{k}\times\mathbf{k}') \boldsymbol{\sigma}_{\alpha\beta} \right] c^{\dagger}_{k\alpha} c_{k'\beta}, \quad (1)$$

where $\varepsilon(k)$ is the energy spectrum of an electron in the normal state, $c^{\dagger}_{k\alpha}$ ($c_{k\alpha}$) is the creation (distraction) operator, $\Delta_k = \Delta \cos(2\varphi)$ is the gap for a d-wave superconductor ($d_{x^2-y^2}$ symmetry), $(U_{\text{imp}})_{\alpha\beta}$ is the potential due to nonmagnetic and magnetic impurities (with randomly oriented magnetic impurities which does not preserves the electron spin), and v_{so} refer to spin-orbit coupling to the impurities (k_F is the Fermi wave vector).

The general expression for the surface impedance of a uniformly disordered system reads [31,32]

$$Z = -\frac{i8\omega}{c^2} \int \frac{dq}{q^2 + \dfrac{4\pi K(\omega, \mathbf{q}, \mathbf{v}_s, T^*, T)}{c^2}}. \quad (2)$$

The response function $K(\omega, \mathbf{q}, \mathbf{v}_s, T^*, T)$ defines the relation

$$\mathbf{J}(q,\omega) = -\frac{1}{c} K(\omega, \mathbf{q}, \mathbf{v}_s, T^*, T) \mathbf{A}(q,\omega) \quad (3)$$

in the Fourier representation between the electric current $\mathbf{J}(q,\omega)$ and the electro-magnetic field, described by the vector potential $\mathbf{A}(q,\omega)$. The ohmic dissipation is determined by the imaginary part of the response function.

Given the current, we proceed to calculate the corresponding current-current correlation function. This correlation function can be expressed diagrammatically as a fermionic bubble with fully dressed propagators and a fully dressed vertex. Assuming that the impurity scattering potential is isotropic in k space, the correction to the vertex vanishes. Thus, in this approximation, the correlation function can be obtained from the calculation of bubble with dressed propagators (i.e., Green's functions with self-energy included, but have vertices or bare bubble diagram). With this replacement the response function becomes

$$K\left(\mathbf{q},\mathbf{v}_s,T^*,T,\omega_m\right)=$$

$$=-\frac{ne^2}{mc}\left(1+\frac{2\pi}{m}\left\langle T\sum_{n,k}\hat{k}_{11}^2[\Im(k_+,\omega_n)\Im(k_-,\omega_n+\omega_m)]\right\rangle\right), \quad (4)$$

where $k_\pm = k \pm q/2$, $\omega_n = (2n+1)\pi T$ is the Matsubara fermionic frequencies, $\hat{k}_{11}$ is the direction of the supercurrent and $\langle\ldots\rangle$ represents a Fermi surface average.

In the presence of impurities, the bare Green's function is dressed via scattering from the impurities and obtains a Matsubara self-energy $\Sigma_{\text{imp}}(i\omega)$ due to impurities. Thus the dressed Matsubara Green's function is given by (clearly the $\Im(k,\omega_n+\omega_m)$ is obtained from $\Im(k,\omega_n)$ with replacing $i\omega_n$ by $i\omega_n+i\omega_m$)

$$\Im\left(\mathbf{k},i\omega_n\right)=\frac{1}{\left[\left(i\omega_n-\Sigma_{\text{imp}}+\mathbf{v}_s\mathbf{k}_F\right)^2-E_k^2\right]}\times$$

$$\times\begin{pmatrix} i\omega_n-\Sigma_{\text{imp}}+\mathbf{v}_s\mathbf{k}_F+\xi_k & \Delta_k \\ \Delta_k & i\omega_n-\Sigma_{\text{imp}}+\mathbf{v}_s\mathbf{k}_F+\xi_k \end{pmatrix}, \quad (5)$$

where $\xi_k = \varepsilon_k - \mu$ is the normal state quasiparticle energy and $E_k^2 = \xi_k^2 + \Delta_k^2$. All quasiparticles Matsubara energies modified by the semiclassical Doppler shift $\mathbf{v}_s \cdot \mathbf{k}_F$. The basic idea is that, like impurity effects, the magnetic field itself may serve as a Cooper pair breaking that creates nodal quasiparticles, leading to a quadratic temperature-dependent penetration depth at temperature $(\Delta\lambda(T) \propto T^2)$ below the scale for nonlinear electrodynamics $E_{\text{nonlin}} \approx v_s k_F$ with a typical supercurrent velocity v_s and the Fermi wave vector k_F.

We consider an impurity potential combining the nonmagnetic and the magnetic scattering,

$$U_{\text{imp}}\left(\mathbf{k}-\mathbf{k}'\right)=U_{\text{nonmag}}\left(\mathbf{k}-\mathbf{k}'\right)+U_{\text{mag}}\left(\mathbf{k}-\mathbf{k}'\right), \quad (6)$$

where the nonmagnetic potential is often assumed to be completely local $U_{\text{nonmag}}(\mathbf{r}-\mathbf{r}') = U_0\delta(\mathbf{r}-\mathbf{r}_0)$ with the impurity at $\mathbf{r}_0$ and $U_{\text{mag}} = J(\mathbf{r})\mathbf{S}\boldsymbol{\alpha}$, with $\mathbf{S}$ is the spin operator of the conduction electron; $J(\mathbf{r}) = J_0\delta(\mathbf{r}-\mathbf{r}_0)$ is the exchange interaction between the local spin on the impurity site and the conduction electrons, $\boldsymbol{\alpha} = [(1+\tau_3)\boldsymbol{\sigma}+(1-\tau_3)\sigma_3\boldsymbol{\sigma}\sigma_3]/2$, σ_i are the Pauli matrices acting in spin space, τ_3 are the Pauli matrices in the particle-hole space, and $\tau_i\sigma_i$ denotes a direct product of the matrices operating in the 4-dimensional Nambu space. We assume that the impurity spins are oriented arbitrarily. Difference of the random-spin case from the spin-polarized case is the absence of the average Zeeman energy. Thus the presence of magnetic impurities with randomly oriented can result in the spin-flip scattering.

The self-energy in the second order (Born approximation) can be written as

$$\Sigma_{\text{imp}}\left(\omega,\mathbf{k}\right)=n_{\text{imp}}\int\frac{d^3\mathbf{k}'}{(2\pi)^3}U\left(\mathbf{k}-\mathbf{k}'\right)\Im\left(\mathbf{k}',\omega\right)U\left(\mathbf{k}'-\mathbf{k}\right), \quad (7)$$

where

$$U\left(\mathbf{k},\mathbf{k}'\right)=U_{\text{imp}}+i\frac{v_{so}}{k_F^2}\left[\mathbf{k}\times\mathbf{k}'\right]\hat{z}, \quad (8)$$

here the spin-orbit coupling to the impurities is described as $v_{so} = U_{\text{imp}}\Delta g$ with Δg is the shift of the g factor which, for cuprates, is of order 0.1.

Using Eq. (8) into Eq. (7) we obtain

$$\Sigma_{\text{imp}}(\omega,\mathbf{k})=n_{\text{imp}}\int\frac{d^3\mathbf{k}'}{(2\pi)^3}U_{\text{imp}}(\mathbf{k}-\mathbf{k}')\Im(\mathbf{k}',\omega)U_{\text{imp}}(\mathbf{k}'-\mathbf{k})+$$

$$+n_{\text{imp}}\frac{v_{so}^2}{k_F^4}\sum_{k'}\left|\mathbf{k}\times\mathbf{k}'\right|^2\Im(\mathbf{k}',\omega). \quad (9)$$

The impedance can be written as $Z(T) = R(T) + iX(T)$, where R and X are the surface resistance and reactance, respectively [33]. The complex conductivity can be written as

$$\sigma\left(q,\omega,T\right)=\sigma_1-i\sigma_2=\frac{i}{\omega}K\left(q,\omega,T\right). \quad (10)$$

The dissipation is determined by the real part of conductivity

$$\sigma_1(\mathbf{q},\mathbf{v}_s,T^*,T)=\text{Re}\,\sigma(\mathbf{q},\mathbf{v}_s,T^*,T)=$$

$$=-\frac{1}{\omega}\text{Im}\,K(\mathbf{q},\mathbf{v}_s,T^*,T). \quad (11)$$

The imaginary part of the conductivity which is determined by the real part of the response function $(\text{Re}\,K(\mathbf{q},\mathbf{v}_s,T^*,T))$, in terms of a frequency and temperature-dependent penetration depth can be written as

$$\sigma_2(\mathbf{q},\mathbf{v}_s,T^*,T)=\frac{c^2}{4\pi\omega\lambda^2(\mathbf{q},\mathbf{v}_s,T^*,T)}. \quad (12)$$

For superconductors $(\sigma_2 >> \sigma_1)$, we have

$$R \approx \frac{8}{c^4}\pi^2\omega^2\lambda^3\sigma_1, \tag{13}$$

$$\chi \approx \frac{4\pi}{c^2}\omega\lambda. \tag{14}$$

According to Eq. (13), to determine the effects of nonlocality, nonlinearity, impurity and spin-orbit scattering on the surface resistance we need to calculate the penetration depth and the real part of the conductivity. Following the method of Ref. 34, first we evaluate the temperature behavior of magnetic penetration depth.

By using Eq. (5) into Eq. (4) we have

$$K(\mathbf{q},\mathbf{v}_s,T^*,T) = \frac{ne^2}{mc}\left(1+\frac{2T\pi}{m}\sum_n\int\frac{d^2k}{(2\pi)^2}\hat{k}_{\parallel}^2 \times \right. \\ \left. \times\frac{\left(i\omega_n-\Sigma_{\text{imp}}+\mathbf{k}_F\mathbf{v}_s\right)^2+\xi_+\xi_-+\Delta_+\Delta_-}{\left[\left(i\omega_n-\Sigma_{\text{imp}}+\left(\mathbf{k}+\frac{\mathbf{q}}{2}\right)\mathbf{v}_s\right)^2-E_{k+q/2}^2\right]\left[\left(i\omega_n-\Sigma_{\text{imp}}+\left(\mathbf{k}-\frac{\mathbf{q}}{2}\right)\mathbf{v}_s\right)^2-E_{k-q/2}^2\right]}\right), \tag{15}$$

where $\xi_\pm = \xi_{k\times} = \varepsilon_{k\pm q/2}-\mu$.

In the absence of magnetic impurities and spin-orbit interaction which are included in the self-energy, Eq. (15) reduces to Eq. (6) of Ref. 34.

We separate out the $T=0$ local, linear, and pure response as

$$K(\mathbf{q},\mathbf{v}_s,T^*,T) = K(0,0,0,0)+\delta K(\mathbf{q},\mathbf{v}_s,T^*,T), \tag{16}$$

where $K(0,0,0,0) = c^2/(4\pi\lambda_0^2)$.

By summing over the Matsubara frequencies in Eq. (15) we get [34]

$$\delta K(\mathbf{q},\mathbf{v}_s,T^*,T) = -\frac{2ne^2}{mc}\left\langle \hat{k}_{\parallel}^2\int_0^\infty d\varepsilon\,\text{Re}\frac{\left[f\left(\varepsilon_+-\mathbf{v}_s\mathbf{k}_F\right)-f\left(-\varepsilon_--\mathbf{v}_s\mathbf{k}_F\right)\right]\Delta_k^2}{\sqrt{\left(\omega-i\Sigma_{\text{imp}}\right)^2-\Delta_k^2}\left[\Delta_k^2+\left(\frac{\mathbf{q}\mathbf{k}_F}{2m}\right)^2-\left(\omega-i\Sigma_{\text{imp}}\right)^2\right]}\right\rangle =$$

$$= -\frac{ne^2}{mc}\left\{1-2\left\langle \hat{k}_{\parallel}^2\frac{\sinh^{-1}\left(\frac{\mathbf{q}\mathbf{k}_F}{2m\Delta_k}\right)}{\left(\frac{\mathbf{q}\mathbf{k}_F}{2m\Delta_k}\right)\sqrt{1+\left(\frac{\mathbf{q}\mathbf{k}_F}{2m\Delta_k}\right)^2}}\right\rangle + \right.$$

$$+\left\langle \hat{k}_{\parallel}^2\text{Ln}\left[\frac{\sqrt{\Delta_k^2+\left(\frac{\mathbf{q}\mathbf{k}_F}{2m}\right)^2}-\left(1+\left(\frac{\mathbf{q}\mathbf{k}_F}{2m\Delta_k}\right)^2\right)\Sigma_{\text{imp}}-\left(\frac{\mathbf{q}\mathbf{k}_F}{2m\Delta_k}\right)\sqrt{1+\left(\frac{\mathbf{q}\mathbf{k}_F}{2m\Delta_k}\right)^2}\sqrt{\Sigma_{\text{imp}}^2-\Delta_k^2}}{\sqrt{\Delta_k^2+\left(\frac{\mathbf{q}\mathbf{k}_F}{2m}\right)^2}+\left(1+\left(\frac{\mathbf{q}\mathbf{k}_F}{2m\Delta_k}\right)^2\right)\Sigma_{\text{imp}}-\left(\frac{\mathbf{q}\mathbf{k}_F}{2m\Delta_k}\right)\sqrt{1+\left(\frac{\mathbf{q}\mathbf{k}_F}{2m\Delta_k}\right)^2}\sqrt{\Sigma_{\text{imp}}^2-\Delta_k^2}}\right]\right\rangle +$$

$$\left. +2\left\langle \hat{k}_{\parallel}^2\int_0^\infty d\varepsilon\,\text{Re}\frac{\left[f\left(\varepsilon_+-\mathbf{v}_s\mathbf{k}_F\right)+f\left(\varepsilon_-+\mathbf{v}_s\mathbf{k}_F\right)\right]}{\sqrt{\left(\omega-i\Sigma_{\text{imp}}\right)^2-\Delta_k^2}\left[\Delta_k^2+\left(\frac{\mathbf{q}\mathbf{k}_F}{2m}\right)^2-\left(\omega-i\Sigma_{\text{imp}}\right)^2\right]}\right\rangle\right\}, \tag{17}$$

where $\varepsilon_\pm = \varepsilon\pm\omega/2$.

The first two terms in Eq. (17) represent the nonlocal correction to the London penetration depth and the third represents the nonlocal and impure renormalization of the response while the forth combined nonlocal, nonlinear, and impure corrections to the temperature dependence. Note that the third and fourth terms also include the effect of spin-orbit correction.

As mentioned in Ref. 34, the resulting response function includes nonlinearity, nonlocality, and impurity, from which a three parameter scaling function of the penetration depth is obtained. The well-known linear T-dependence of the penetration depth in the pure system should be expected at ($T >> E_{\rm nonloc}$, $E_{\rm nonlin}$). But in the opposite limit, i.e., at extremely low temperature, either nonlinear or nonlocal effects play a crucial role in modifying the linear temperature behavior to quadratic behavior.

For the impure sample, as shown in Ref. 35 in the Born limit, the contribution of spin-orbit scattering is Δg^2 times smaller than that of nonmagnetic impurity scattering, while in the unitarity limit (strong scattering), the spin-orbit scattering leads to a renormalization of the same order of the impurity scattering. Thus, it is necessary to treat the impurity and spin-orbit interactions on the same level by generalizing the usual t-matrix approach for the impurity scattering also to the spin-orbit contribution. To characterize the strength of the impurity scattering for the nonmagnetic and the magnetic components we introduce $C_n = 1/(\pi N_0 U_n)$ and $C_m = 1/(\pi N_0 J\sqrt{s(s+1)})$.

Following the notation of Ref. 35 for arbitrary strength of the scattering potential the renormalized frequency and gap equations, respectively, become

$$i\tilde{\omega}_n = i\omega_n + \Gamma\frac{g_{0n}}{C_n^2 + C_m^2 - g_{0n}^2} + \Gamma\frac{\left(\dfrac{C_n^2+C_m^2}{\Delta g^2}\right)+(f_n^2-g_n^2)}{\left[\left(\dfrac{C_n^2+C_m^2}{\Delta g^2}\right)+(f_n^2-g_n^2)\right]^2-4f_ng_n}g_n, \quad (18)$$

$$\tilde{\Delta} = \Delta + \Gamma\frac{\left(\dfrac{C_n^2+C_m^2}{\Delta g^2}\right)-(f_n^2-g_n^2)}{\left[\left(\dfrac{C_n^2+C_m^2}{\Delta g^2}\right)-(f_n^2-g_n^2)\right]^2-4f_ng_n}f_n, \quad (19)$$

$$\tilde{\Omega} = -2\Gamma\frac{f_ng_n\left(\dfrac{\sqrt{C_n^2+C_m^2}}{\Delta g}\right)}{\left[\left(\dfrac{C_n^2+C_m^2}{\Delta g^2}\right)-(f_n^2-g_n^2)\right]^2-4f_ng_n}f_n, \quad (20)$$

where

$$\Gamma = \Gamma_n + \Gamma_m = \frac{\Gamma}{1+C_n^2} + \frac{\Gamma}{1+C_m^2} \qquad \left(\Gamma = \frac{n_i}{\pi N_0}\right),$$

$$f_n = \left\langle \frac{\tilde{\Delta}\sin^2\varphi}{\sqrt{\tilde{\Delta}^2+\tilde{\Omega}^2+\tilde{\omega}_n^2}} \right\rangle, \qquad g_n = \left\langle \frac{i\tilde{\omega}_n\sin^2\varphi}{\sqrt{\tilde{\Delta}^2+\tilde{\Omega}^2+\tilde{\omega}_n^2}} \right\rangle,$$

$$g_{0n} = \left\langle \frac{i\tilde{\omega}_n}{\sqrt{\tilde{\Delta}^2+\tilde{\Omega}^2+\tilde{\omega}_n^2}} \right\rangle. \quad (21)$$

By considering the self-consistent equation for the order parameter (Eq. (19)) the following expression is obtained for the critical temperature

$$\mathrm{Ln}\left(\frac{T_c}{T_{c0}}\right) = \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \frac{\Gamma_n+\Gamma_n+\Gamma_{so}}{2\pi T_c}\right), \quad (22)$$

where

$$\Gamma_{so} = \Gamma\frac{3(2C/\Delta g)^2+1}{[(2C/\Delta g)^2+1]^2} \qquad (C^2 = C_n^2 + C_m^2).$$

In the absence of magnetic impurities Eq. (22) reduces to Eq. (20) of Ref. 35.

In the Born limit $(C_n, C_m >> 1)$ Eq. (22) becomes

$$\mathrm{Ln}\left(\frac{T_c}{T_{c0}}\right) = \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \frac{1}{2\pi T_c}\left(\frac{1}{2\tau_n} + \frac{1}{2\tau_m} + \frac{3}{4\tau_{so}}\right)\right), \quad (23)$$

here

$$\frac{1}{2\tau_n} = \pi n_n N_0 U_n^2, \ \frac{1}{2\tau_m} = \pi n_m N_0 s(s+1)|J|^2, \text{ and } \frac{1}{\tau_{so}} = \Gamma\left(\frac{\Delta g}{C}\right)^2$$

(not that, τ_n and τ_m are spin-independent and spin-dependent relaxation times due to nonmagnetic and magnetic impurities, respectively).

By solving the gap equation in the Ginzburg–Landau region, the transition temperature in the presence of magnetic and nonmagnetic impurities was also computed in Ref. 36 and the obtained result in the Born limit (Eq. (31)) is the same as Eq. (23) of our paper in the absence of spin-orbit coupling $(\tau_{so} \to \infty)$. On the other hand, Eq. (23) without magnetic impurities $(\tau_m \to \infty)$ reduces to Eq. (31) of Ref. 37.

In the unitarity limit $(C_n, C_m << 1)$ Eq. (23) becomes

$$\mathrm{Ln}\left(\frac{T_c}{T_{c0}}\right) = \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \frac{1}{2\pi T_c}\left(\frac{1}{2\tau_n} + \frac{1}{2\tau_m} + \frac{1}{2\tau_{so}}\right)\right), \quad (24)$$

where $\frac{1}{2\tau_n}=\frac{1}{2\tau_m}=\frac{1}{2\tau_{so}}=\frac{n_i}{\pi N_0}$.

In the unitarity limit, the superconductivity is destroyed for the critical value $\Gamma^*=\Gamma_n^*+\Gamma_m^*+\Gamma_{so}^*=0.88T_{c0}$, i.e., to destroy the superconductivity one needs the impurity concentration about the third value of the impurity concentration in the unitarity limit without magnetic impurities and spin-orbit scattering.

As can be seen from Eqs. (23) and (24), the spin-orbit scattering rate negligible with respect to magnetic and nonmagnetic impurity scattering rates in the Born limit, while close to the unitarity limit it has the same order of magnitude than these relaxation rates and cannot be neglected.

Now the kernel $\delta K(\mathbf{q},\mathbf{v}_s,T^*,T)$ can be written as

$$\delta K(\mathbf{q},\mathbf{v}_s,T^*,T)=2\pi T\sum_n\int d\varphi\hat{k}_{11}^2\frac{\tilde{\Delta}_k^2}{\sqrt{(\tilde{\omega}_n+i\mathbf{k}_F\mathbf{v}_s)^2+\tilde{\Omega}^2+\Delta_k^2}\left[(\tilde{\omega}_n+i\mathbf{k}_F\mathbf{v}_s)^2+\Delta_k^2+\tilde{\Omega}^2+(\mathbf{q}\mathbf{v}_F)^2\right]^2}. \quad (25)$$

In the impurity dominated gapless regime, the renormalized frequency $\tilde{\omega}$ takes the limiting form $\tilde{\omega}\to\omega+i\gamma$, where $\gamma=\Sigma_{\mathrm{imp}}(0)$. For dilute impurity concentration $(\gamma\to 0)$ the gapless behavior restricted to a vanishingly small range below $T<T^*\approx\gamma$ and in this range of temperature we can replace the normalized frequency everywhere by its low-frequency limiting form and after integration over frequency we find $\delta K(\mathbf{q},T^*,T)\propto\Delta\lambda\propto T^2$.

In the impure sample below the crossover temperatures, $T<T^*\approx\gamma$, the impurity effect determines the temperature dependence of the penetration depth, and nonlocality and nonlinearity behavior should be hidden by the impurity effects.

To calculate the real part of the conductivity we need to determine the imaginary part of the response function $(i\omega_n=\varepsilon+i\delta)$. From Eq. (15) we obtain

$$\mathrm{Im}\,K(\mathbf{q},\mathbf{v}_s,T^*,T)=\frac{ne^2}{mc}\int\frac{d\varphi}{2\pi}\hat{k}_{11}^2\int d\varepsilon\left[f(\varepsilon+\omega-\mathbf{k}_F\mathbf{v}_s)-f(\varepsilon-\mathbf{k}_F\mathbf{v}_s)\right]\times$$
$$\times\mathrm{Im}\left\{\frac{(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)(\tilde{\varepsilon}'-\mathbf{k}_F\mathbf{v}_s+i\delta)+(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)^2-\mathbf{q}\mathbf{v}_F\sqrt{(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)^2-\Delta_k^2}}{\left[\left(\sqrt{(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)^2-\Delta_k^2}-\mathbf{q}\mathbf{v}_F\right)^2-(\tilde{\varepsilon}'-\mathbf{k}_F\mathbf{v}_s+i\delta)^2+\Delta_k^2\right]\sqrt{(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)^2-\Delta_k^2}}+\right.$$
$$+\frac{(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)(\tilde{\varepsilon}'-\mathbf{k}_F\mathbf{v}_s+i\delta)+(\tilde{\varepsilon}'-\mathbf{k}_F\mathbf{v}_s+i\delta)^2+\mathbf{q}\mathbf{v}_F\sqrt{(\tilde{\varepsilon}'-\mathbf{k}_F\mathbf{v}_s+i\delta)^2-\Delta_k^2}}{\left[\left(\sqrt{(\tilde{\varepsilon}'-\mathbf{k}_F\mathbf{v}_s+i\delta)^2-\Delta_k^2}+\mathbf{q}\mathbf{v}_F\right)^2-(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)^2+\Delta_k^2\right]\sqrt{(\tilde{\varepsilon}'-\mathbf{k}_F\mathbf{v}_s+i\delta)^2-\Delta_k^2}}-$$
$$-\frac{(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)(\tilde{\varepsilon}'-\mathbf{k}_F\mathbf{v}_s-i\delta)+(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)^2-\vec{q}\mathbf{v}_F\sqrt{(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)^2-\Delta_k^2}}{\left[\left(\sqrt{(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)^2-\Delta_k^2}-\mathbf{q}\mathbf{v}_F\right)^2-(\tilde{\varepsilon}'-\mathbf{k}_F\mathbf{v}_s-i\delta)^2+\Delta_k^2\right]\sqrt{(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)^2-\Delta_k^2}}-$$
$$\left.-\frac{(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)(\tilde{\varepsilon}'-\mathbf{k}_F\mathbf{v}_s-i\delta)+(\tilde{\varepsilon}'-\mathbf{k}_F\mathbf{v}_s-i\delta)^2-\mathbf{q}\mathbf{v}_F\sqrt{(\tilde{\varepsilon}'-\mathbf{k}_F.\mathbf{v}_s-i\delta)^2-\Delta_k^2}}{\left[\left(\sqrt{(\tilde{\varepsilon}'-\mathbf{k}_F\mathbf{v}_s-i\delta)^2-\Delta_k^2}+\mathbf{q}\mathbf{v}_F\right)^2-(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta)^2+\Delta_k^2\right]\sqrt{(\tilde{\varepsilon}'-\mathbf{k}_F\mathbf{v}_s-i\delta)^2-\Delta_k^2}}\right\}, \quad (26)$$

where $\varepsilon'=\varepsilon-\omega$ and $\tilde{\varepsilon}=\varepsilon-\Sigma_{\mathrm{imp}}(\varepsilon)$.

In the absence of magnetic impurities $(\tau_m\to\infty)$, spin-orbit coupling $(v_{so}\to 0)$ and nonlinear effect $(v_s\to 0)$, Eq. (26) reduces to Eq. (6) of Ref. 38. In the local limit they obtained the Drude-like form of the conductivity of a *d*-wave superconductor in the range of temperature $T^*<<T<<T_c$. In this limit the quasiparticle density varies linearly with temperature, and if the average lifetime were constant, σ_1 would vary linearly with temperature. For $T<T^*$, they found that the real part of the conductivity varies as T^2 in the unitarity limit.

The real part of the conductivity in the microwave regime can be written as

$$\sigma_1\left(\mathbf{q},\mathbf{v}_s,T^*,T,\omega\to 0\right)=-\lim_{\omega\to 0}\frac{\operatorname{Im}K(\mathbf{q},T^*,T,\ \omega)}{\omega}=-\frac{ne^2}{mc}\int\frac{d\varphi}{2\pi}\hat{k}_{\parallel}^2\int d\varepsilon\left[\frac{\partial f\left(\varepsilon-\mathbf{k}_F\mathbf{v}_s\right)}{\partial\varepsilon}\right]\times$$

$$\times\operatorname{Im}\left\{\frac{2\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2-\mathbf{q}\mathbf{v}_F\sqrt{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2-\Delta_k^2}}{\left[\left(\sqrt{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2-\Delta_k^2}-\mathbf{q}\mathbf{v}_F\right)^2-\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2+\Delta_k^2\right]\sqrt{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2-\Delta_k^2}}+\right.$$

$$+\frac{2\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2+\mathbf{q}\mathbf{v}_F\sqrt{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2-\Delta_k^2}}{\left[\left(\sqrt{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2-\Delta_k^2}+\mathbf{q}\mathbf{v}_F\right)^2-\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2+\Delta_k^2\right]\sqrt{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2-\Delta_k^2}}-$$

$$-\frac{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s-i\delta\right)+\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2-\mathbf{q}\mathbf{v}_F\sqrt{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2-\Delta_k^2}}{\left[\left(\sqrt{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2-\Delta_k^2}-\mathbf{q}\mathbf{v}_F\right)^2-\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s-i\delta\right)^2+\Delta_k^2\right]\sqrt{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2-\Delta_k^2}}-$$

$$\left.-\frac{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s-i\delta\right)+\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s-i\delta\right)^2-\mathbf{q}\mathbf{v}_F\sqrt{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s-i\delta\right)^2-\Delta_k^2}}{\left[\left(\sqrt{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s-i\delta\right)^2-\Delta_k^2}+\mathbf{q}\mathbf{v}_F\right)^2-\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s+i\delta\right)^2+\Delta_k^2\right]\sqrt{\left(\tilde{\varepsilon}-\mathbf{k}_F\mathbf{v}_s-i\delta\right)^2-\Delta_k^2}}\right\}. \quad (27)$$

Equation (27) is a general expression which contains all effect (temperature, nonlocality, nonlinearity, impurity, and spin-orbit coupling) on the real part of the conductivity and may be evaluated numerically. Here we focus on the low-temperature behavior of surface resistance which can be derived an approximated result.

In the normal state $(\Delta_k = 0)$, from Eq. (27) the normal state conductivity

$$\sigma_1 = \frac{3\pi ne^2}{4mc}\frac{1}{qv_F}$$

can be obtained.

According to Eq. (13) we can write

$$\Delta R(T) \propto \left(\frac{\Delta\sigma_1(T)}{\sigma_1}+\frac{3\Delta\lambda(T)}{\lambda}\right). \quad (28)$$

For the local, linear, and pure sample, $\Delta\lambda(T)$ and $\Delta\sigma_1(T)$ vary linearly with temperature, consequently $\Delta R(T)$ would change linearly with temperature in agreement with experimental results for high quality sample [39].

For the nonlocal, nonlinear, and pure sample, according to Eqs. (19) and (20), $\Delta\lambda(T)\propto T^2$ and $\Delta\sigma_1(T)\propto T^2$, consequently the linear temperature dependence of $\Delta R(T)$ change to quadratic function.

For impure samples without magnetic impurities, as we mentioned before the nonlocal and nonlinear effects are completely hidden by impurities and the temperature dependences $\Delta\lambda(T)$ and $\Delta\sigma_1(T)$ are determined by temperature interval namely the ranges of $T<T^*$ and $T^* << T << T_c$. The energy range between zero and the gap edge may be partitioned crudely into two regimes, separated by a crossover energy or temperature T^*, dependent on the nonmagnetic impurity concentration and phase shift. It can be shown that T^* to be of order of $\gamma = \Sigma_{\text{imp}}(0)$. Since in the Born limit the effect of spin-orbit interaction is small and in the unitarity limit it has a considerable effect, the crossover temperature in this two limits, respectively, is $T^{\bullet} \approx \gamma \approx \Gamma_n n_i$ and $T^{\bullet} \approx \gamma \approx (\Gamma_n + \Gamma_{so})/n_i$. For $T<T^*$ both $\Delta\lambda(T)$ and $\Delta\sigma_1(T)$ vary as T^2, on the other hand, for $T^* << T << T_c$ both $\Delta\lambda(T)$ and $\Delta\sigma_1(T)$ vary linearly with temperature.

In Fig. 1 it is shown the real part of the conductivity as a function of temperature at several different relaxation times (the imaginary part of the self-energy in Eq. (27) $\tau_n^{-1} \propto \Sigma_{\text{imp}}(i\omega)$ $(\tilde{\varepsilon} = \varepsilon - \Sigma_{\text{imp}}(\varepsilon))$) due to scattering by nonmagnetic impurities. As can be seen from Fig. 1, the T^2 behavior of σ_1 below the crossover temperature $T^* \approx 0.34\,T_c$ is changed to the linear above T^* in agreement with previ-

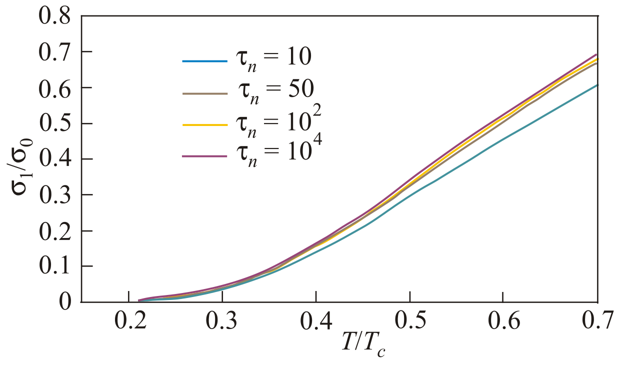

Fig. 1. (Color online) Real part of the complex conductivity $\sigma_1(T)$, normalized to the Drude conductivity vs reduced temperature T/T_c for various impurity concentrations.

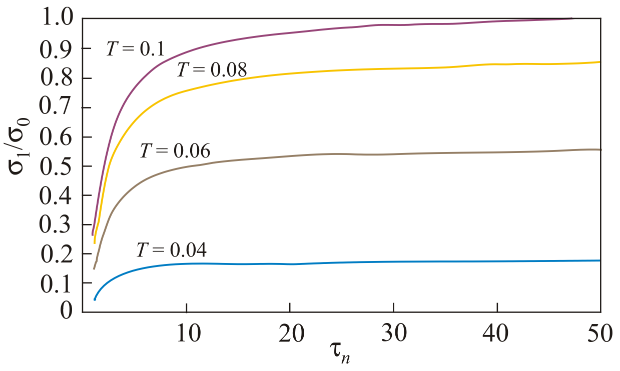

Fig. 2. (Color online) The real part of the conductivity as function of the relaxation time due to nonmagnetic impurity scattering at different temperatures. To perform the numerical evaluations, we assumed that the Fermi energy ε_F which is the largest scale of energy is equal to 1, and because in the superconducting case $k_BT_c << \varepsilon_F$, we find $T_c = 0.1$.

ous results [38,40]. In Fig. 2 we also present the real part of the conductivity as a function of impurity relaxation time for different values of temperature.

It is recognized that the quadratic dependence $\Delta R(T)$ is largely due to the presence of defects in samples (extrinsic origin), and the linear temperature dependences $\Delta R(T)$ is due to intrinsic microscopic properties of unconventional superconductors. This conjecture was confirmed by systematic research of YBCO thin films [41] as their quality improved, the quadratic dependence in the low-temperature range was replaced by a linear function.

Magnetic impurities do not show the same scheme of resonant scattering that seems to give a reasonable description of the nonmagnetic impurity effects. The magnetic impurity concentration which needs to suppresses T_c about the nonmagnetic impurities is nearly two times and provides at least as much scattering as the nonmagnetic impurities, but does not produce any quadratic, gapless behavior in either $\Delta\lambda(T)$ or $\Delta\sigma_1(T)$. It is attractive to suggest that magnetic impurities might be a nonresonant scatterer and that results for the Born limit rather than the unitary limit are more appropriate. In the Born limit $T^{\bullet} \approx \gamma \approx \Delta_0 e^{-\Delta_0/\Gamma_N}$, where Δ_0 is the gap maximum over the Fermi surface and $\Gamma_N = \Gamma_m/(1+C_m^2)$ gives a much lower crossover temperature for a given scattering rate Γ_N than does the resonant scattering result.

In the presence of magnetic impurities the gapless superconductor exhibits saturation of the surface resistance at zero temperature and at low temperatures the linear temperature dependence in the magnetic impurity-doped sample does not change which is in agreement with experiment results [35].

3. Conclusions

The effect of magnetic and nonmagnetic impurities and spin-orbit scattering on the low-temperature surface impedance of a nonlocal and nonlinear *d*-wave superconductor in the framework of linear response theory was investigated. At first, the general expressions for penetration depth and the real part of the conductivity were obtained. For arbitrary strength of the scattering potentials the renormalized frequency and gap equations were derived. For the impure sample, in the Born limit, the contribution of spin-orbit scattering is smaller than that of nonmagnetic impurity scattering, while in the unitarity limit (strong scattering), the spin-orbit scattering leads to a renormalization of the same order of the impurity scattering. The magnetic impurity concentration which needs to suppresses T_c about the nonmagnetic impurities is nearly two times and provides at least as much scattering as the nonmagnetic impurities, but does not produce any quadratic, gapless behavior in either $\Delta\lambda(T)$ or $\Delta\sigma_1(T)$.

For local, linear, and pure sample the surface resistance change linearly with temperature which is consist with the experimental result for high quality sample. For impure sample the nonlocal and nonlinear effects are completely hidden by impurities and surface resistance varies as T^2 below the crossover temperature ($T^{\bullet}$) which is determined by the impurity concentration and phase shift.

Our results show that the quadratic dependence of $\Delta R(T)$ is due to nonlocality, nonlinearity and impurity effects (extrinsic origin), and the linear temperature dependence $\Delta R(T)$ is due to intrinsic microscopic properties of unconventional superconductors.

1. P.K. Day, H.G. LeDuc, B.A. Mazin, A. Vayonakis, and J. Zmuidzinas, *Nature* (*London*) **425**, 817 (2003).
2. K.D. Irwin and K.W. Lehnert, *Appl. Phys. Lett*. **85**, 2107 (2004).
3. D.I. Wallraff, D.I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S.M. Girvin, and R.J. Schoelkopf, *Nature* (*London*) **162**, 431 (2004).
4. A.M. Zagoskin, E. Il'ichev, M.W. McCutcheon, J.F. Young, and F. Nori, *Phys. Rev. Lett*. **101**, 253602 (2008).
5. K. Moon and S.M. Girvin, *Phys. Rev. Lett*. **95**, 140504 (2005).
6. M.A. Sillanpää, J.I. Park, and R.W. Simmonds, *Nature* (*London*) **449**, 438 (2007).
7. C.A. Regal, J.D. Teufel, and K.W. Lehnert, *Nat. Phys*. **4**, 555 (2008).
8. W.A. Philipps, *Rep. Prog. Phys*. **50**, 1657 (1987).
9. P.W. Anderson, B.I. Halperin, and C.W. Varma, *Philos. Mag*. **25**, 1 (1972).
10. W.A. Phillips, *J. Low Temp. Phys*. **7**, 351 (1972).
11. M.V. Schickfus and S. Hunklinger, *J. Phys. C* **9**, L439 (1976).
12. J. Gao, J. Zmuidzinas, B.A. Mazin, H.G. LeDuc, and P.K. Day, *Appl. Phys. Lett*. **90**, 102507 (2007).
13. H. Wang, M. Hofheinz, J. Wenner, M. Ansmann, R.C. Bialczak,M. Lenander, E. Lucero, M. Neeley, A.D.O. Connell, D. Sank, M. Weides, A.N. Cleland, and J.M. Martinis, *Appl. Phys. Lett*. **95**, 233508 (2009).

14. H. Paik and K.D. Osborn, *Appl. Phys. Lett*. **96**, 072505 (2010).
15. R.W. Simmonds, K.M. Lang, D.A. Hite, S. Nam, D.P. Pappas, and J.M. Martinis, *Phys. Rev. Lett*. **93**, 077003 (2004).
16. M.J. Lancaster, *Passive Microwave Device Applications of High-Temperature Superconductors,* Cambridge University Press (1997).
17. N. Newman and W.G. Lyons, *J. Supercond*. **6**, 119 (1993).
18. *Microwave Studies of High-Temperature Superconductors*, A.V. Narlikar (ed.), Nova Science Publishers (1996).
19. A. Jha, *Superconductor Technology-Applications to Microwave, Electro-Optics, Electrical Machines, and Propulsion Systems*, John Wiley & Sons (1998).
20. D.A. Bonn and W.N. Hardy, *Microwave Surface Impedance of High-Temperature Superconductors*, in: Physical Properties of High-Temperature Superconductors, Vol. 5, D.M. Ginsberg (ed.), World Scientific, Singapore (1996).
21. J. Mazierska, *J. Supercond*. **10**, 73 (1997).
22. M. Reagor, H. Paik, G. Catelani, L. Sun, C. Axline, E. Holland, I.M. Pop, N.A. Masluk, T. Brecht, L. Frunzio, M.H. Devoret, L. Glazman, and R.J. Schoelkopfd, *Appl. Phys. Lett*. **102**, 192604 (2013).
23. M. Kharitonov, T. Proslier, An. Glatz, and M.J. Pellin, *Phys. Rev. B* **86** (2), 024514 (2012).
24. C.T. Rieck, D. Straub, and K. Scharnberg, *J. Supercond*. **12**(2), 385 (1999).
25. F. Palomba, A. Andreone, G. Pica, M. Saluzzo, C. Attanasio,T. Di Luccio, L. Maritato, and R. Russo, *Physica B* **284–288**, 955 (2000).
26. H. Srikanth, S. Sridhar, D.A. Gajewski, and M.B. Maple, *Physica C* **291**, 235 (1997).
27. V. Prokhorov, Y.P. Lee, and G. Kaminsky, *IEEE Trans. Magn*. **35**, 3166 (1999).
28. M. Hein, T. Kaiser, and G. Müller, *Phys. Rev. B* **61**, 640 (2000).
29. S. Skalski, O. Betbeder-Matibet, and P.R. Weiss, *Phys. Rev*. **136**, A1500 (1964).
30. Ya.V. Fominov, M. Houzet, and L.I. Glazman, *Phys. Rev. B* **84**, 224517 (2011).
31. G.E. Reuter and E. H. Sondheimer, *Proc. R. Soc. A* **195**, 336 (1948),
32. A. B. Pippard, *Proc. R. Soc. A* **216**, 547 (1953).
33. L.L. Landau and E.M. Lifshitz, *Physical Kinetics Landau and Lifshitz Course of Theoretical Physics*, vol. 10, Pergamon (1981).
34. H. Yavary, *Physica C* **450**, 129 (2006).
35. C. Grimaldi, *Europhysics Lett*. **48**(3), 306 (1999).
36. C.H. Choi, *J. Korean Phys. Soc*. **37**, No. 5, 552 (2000).
37. C. Grimaldi, *J. Phys.: Condens. Matter* **12**(7), 1329 (2000).
38. P.J. Hirschfeld, W.O. Putikka, and D.J. Scalapino, *Phys. Rev. B* **50**, 10250 (1994).
39. M.R. Trunin, *J. Supercond*. **11**, 381 (1998).
40. P.J. Hirschfeld, and N. Goldenfeld, *Phys. Rev. B* **48**, 4219 (1993).
41. L.A. de Vaulchier, J.P. Vieren, Y. Guldner, N. Bontemps, R. Combescot, Y. Lemaitre, and J.C. Mage, *Europhys. Lett*. **33**, 153 (1996).