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Phase fluctuations and pseudogap properties: influence of nonmagnetic impurities

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The presence of nonmagnetic impurities in a $2D$ «bad» metal depresses the superconducting Berezinskii-Kosterlitz-Thouless transition temperature while leaving the pairing energy scale unchanged. Thus the region of the pseudogap nonsuperconducting phase, in which the modulus of the order parameter is nonzero but its phase is random and which arises at the pairing temperature, is substantially bigger than for the clean system. This supports the premise that fluctuations in the phase of the order parameter can in principle describe the pseudogap phenomena in high-T_c materials over a rather wide range of temperatures and carrier densities. The temperature dependence of the bare superfluid density is also discussed.

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Introduction

The differences between the BCS scenario of superconductivity and superconductivity in high-T_c materials are well accepted as experimental facts, although there is no theoretical consensus about their origin. One of the most convincing manifestations is the pseudogap, or a depletion of the single-particle spectral weight around the Fermi level (see, for example, [1]). Another transparent manifestation is the temperature and carrier-density dependences of the superfluid density in high-T_c superconductors (HTSC) [2–4], which do not fit the canonical BCS behavior. In particular, the value of the zero-temperature superfluid density is substantially less than the total density of doped carriers [5]. Currently there are many possible explanations for the unusual properties of HTSC. One of these is based on the nearly antiferromagnetic Fermi liquid model [6]. Another explanation, proposed by Anderson, relies on the separation of spin and charge degrees of freedom. One more approach, which we will follow in this paper, relates the observed anomalies to precursor superconducting fluctuations. Some authors argue that alternative types of superconducting fluctuations are responsible for the pseudogap (e.g. [7]) while Emery and Kivelson [8] suggest a scenario based on fluctuations of the phase of the order parameter. The latter scenario we believe to be more relevant due to the low superfluid density and practically $2D$ character of the conductivity in HTSC mentioned above. A microscopic $2D$ model which elaborates the above-mentioned scenario [8] has been studied in the papers [9,10]. The results obtained show that the condensate phase fluctuations indeed lead to features which are experimentally observed in HTSC both in the normal and superconducting states*. It is obvious, however, that the present treatment of the phase fluctuations is incomplete due to both the oversimplified character of the model and the absence of an explanation for the more recent advanced experiments [2–4] on the temperature and doping dependences of the superfluid density. It is well known, however, that the theoretical study of HTSC faces a lot of computational difficulties due to, for example, an unconventional order parameter symmetry, complex frequency-momentum depend-

* Of course, the contribution from the phase fluctuations need not be the only or even the major contribution.

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ence of the effective quasiparticle attraction, general form of the quasiparticle dispersion law, etc. Therefore, in order to obtain analytical results, we have to date only considered nonretarded s-wave pairing in the absence of impurities. (Attempts to consider retardation effects were made in [11].)

Nevertheless, a discussion of the effect of impurities seems to be crucial for a realistic model of the HTSC. Indeed, it is known that the itinerant holes in HTSC are created by doping, which in turn introduces a considerable disorder into the system, for instance, from the random Coulomb fields of chaotically distributed charged impurities (doped ions) [12]. Thus one of the purposes of the present paper is to study the model [9,10] but in the presence of nonmagnetic impurities.

In the theory of «common» metals the Fermi energy ϵ_F and the mean transport quasiparticle time $\tau_{\rm tr}$ are independent quantities which are always assumed to satisfy the criterion $\epsilon_F\tau_{\rm tr} >> 1$. In HTSC, which are «bad» metals [8], both ϵ_F and $\tau_{\rm tr}$ are dependent on the doping and the above-mentioned criterion may fail [12]. As an illustration, we refer to the remarkable linear dependence of the normal-state resistivity [1], which implies that $\epsilon_F\tau_{\rm tr}$ may indeed be ~ 1. It has been shown [12] for strongly disordered metallic systems that superconductivity is absent if the scattering-to-pairing ratio exceeds a critical value and that superconductivity exists in a finite range of doping if this ratio is not exceeded. We shall not study this case but rather consider here the more usual (and in some sense simpler) situation originally studied in the papers of Anderson [13] and Abrikosov-Gor'kov (AG) [14] (see also [15]), when the superconducting order is preexisting and the criterion $\epsilon_F\tau_{\rm tr} >> 1$ is satisfied.

The Anderson theorem [13] states that in 3D the BCS critical temperature is unchanged in the presence of nonmagnetic impurities. However, as discussed in [9], the BCS critical temperature in 2D is the temperature T^* at which the pseudogap opens, while the superconducting transition temperature transition is the temperature T_{BKT} of the Berezinskii-Kosterlitz-Thouless (BKT) transition. In contrast to the former, the latter is defined by a bare superfluid density (given by the delocalized carriers) which is dependent on (see below) the concentration of impurities. Thus in 2D case the superconducting transition temperature T_{BKT} decreases with increasing impurity concentration.

Thus in the model under consideration, the relative size of the pseudogap phase, $(T^* - T_{BKT})/T^*$, is larger in the presence of impurities than in the clean limit [9]. Therefore it can be observed over a wider range of densities. The second result obtained is that the value of the zero-temperature superfluid density is less than the total density of carriers (dopants), so that the presence of impurities may contribute into this diminishing and, in turn, explain the experimental results [5]. Finally, we attempt to interpret qualitatively the recent experiments on the temperature dependence of the superfluid density [2,3] within our scenario.

A brief overview of the paper follows: In Sec. 2 we present the model and derive the main equations. In Sec. 3 we compare the results obtained for the clean and dirty limits. In particular, we compare the values of T_{BKT} , the relative sizes of the pseudogap region, and the values of the bare superfluid density at $T = 0$ and for T close to T_ρ . In Sec. 4 an attempt is made to give an explanation for the experimental results [2,3].

2. Model and main equations

Our starting point is a continuum version of the two-dimensional attractive Hubbard model defined by the Hamiltonian [9,10]:

$$H = \int d^2r \left[\psi_\sigma^+(x) \left(-\frac{\nabla^2}{2m} - \mu \right) \psi_\sigma(x) - \right.$$
$$\left. - V\psi_\uparrow^+(x)\psi_\downarrow^+(x)\psi_\downarrow(x)\psi_\uparrow(x) + U_{\rm imp}(\mathbf{r})\psi_\sigma^+(x)\psi_\sigma(x) \right], \qquad (1)$$

where $x = \mathbf{r}; \tau$ denotes the space and imaginary time variables; $\psi_\sigma(x)$ is a fermion field with spin $\sigma = \uparrow, \downarrow$; m is the effective fermion mass; μ is the chemical potential; V is an effective local attraction constant, and $U_{\rm imp}(\mathbf{r})$ is the static potential of randomly distributed impurities; we take $\hbar = k_B = 1$. The model with the Hamiltonian (1) is equivalent to a model with an auxiliary BCS-like pairing field which is given in terms of the Nambu variables as

$$H = \int d^2r \left\{ \Psi^+(x) \left[\tau_3 \left(-\frac{\nabla^2}{2m} - \mu \right) - \right. \right.$$
$$\left. \left. - \ \tau_+\Phi(x) - \tau_-\Phi^*(x) + \tau_3 U_{\rm imp}(\mathbf{r}) \right] \Psi(x) + \frac{|\Phi(x)|^2}{V} \right\}, \qquad (2)$$

where $\tau_\pm = (\tau_1 \pm i\tau_2)/2$, τ_3 are the Pauli matrices, and $\Phi(x) = V\Psi^+(x)\tau_-\Psi(x) = V\psi_\downarrow(x)\psi_\uparrow(x)$ is the complex ordering field. Then the partition function can be presented as a functional integral over Fermi fields (Nambu spinors) and the auxiliary fields Φ, Φ^*.

However, in contrast to the usual method, the modulus-phase parametrization $\Phi(x) = \rho(x) \exp[i\theta(x)]$ is necessary for the $2D$ model at finite temperatures (see [9,10,16] and references therein). To be consistent with this replacement one should also introduce the spin-charge variables for the Nambu spinors

$$\Psi(x) = \exp[i\tau_3\theta(x)/2]\, \Upsilon(x) , \tag{3}$$

where $\Upsilon(x)$ is the field operator for neutral fermions.

From the Hamiltonian (1), following [9], can derive an effective Hamiltonian which is the Hamiltonian of the classical XY model

$$H_{XY} = \frac{1}{2} J(\mu, T, \rho) \int d^2r[\nabla\theta(x)]^2 \tag{4}$$

where

$$J(\mu, T, \rho) = \frac{T}{16m\pi^2} \sum_{n=-\infty}^{\infty} \int d^2k \,\mathrm{tr}\,[\tau_3\langle \mathcal{G}(i\omega_n, \mathbf{k})\rangle] + \frac{T}{32m^2\pi^2} \sum_{n=-\infty}^{\infty} \int d^2k\mathbf{k}^2 \,\mathrm{tr}\,[\langle \mathcal{G}(i\omega_n, \mathbf{k})\rangle\langle \mathcal{G}(i\omega_n, \mathbf{k})\rangle] \tag{5}$$

is the bare (i.e., unrenormalized by the phase fluctuations, but including pair breaking thermal fluctuations) superfluid stiffness. Here

$$\langle \mathcal{G}(i\omega_n, \mathbf{k})\rangle = -\frac{(i\omega_n\hat{I} - \tau_1\rho)\eta_n + \tau_3\xi(\mathbf{k})}{(\omega_n^2 + \rho^2)\eta_n^2 + \xi^2(\mathbf{k})} \tag{6}$$

with

$$\eta_n = 1 + \frac{1}{2\tau_{\mathrm{tr}}\,(\omega_n^2 + \rho^2)^{1/2}} , \qquad \xi(\mathbf{k}) = \frac{\mathbf{k}^2}{2m} - \mu , \quad \omega_n = \pi(2n + 1)T \tag{7}$$

is the AG [15] Green's function of neutral fermions averaged over a random distribution of impurities and written in the Nambu representation [17,18]. In writing (5) we assumed that $\langle \mathcal{G}(i\omega_n, \mathbf{k})\, \mathcal{G}(i\omega_n, \mathbf{k})\rangle \simeq \langle \mathcal{G}(i\omega_n, \mathbf{k})\rangle\langle \mathcal{G}(i\omega_n, \mathbf{k})\rangle$. This approximation, as shown by AG [15], does not change the final result for J. Note also that the Green's function (6) is valid only when $\epsilon_F\tau_{\mathrm{tr}} \gg 1$, which demands the presence of a well-developed Fermi surface, which in turn implies that $\mu \simeq \epsilon_F$. Thus one cannot use the expression (6) in the so called Bose limit with $\mu < 0$ [9]. On the other hand, a Fermi surface can be formed even in the bad metals when the Ioffe-Regel-Mott criterion proves to be fulfilled [12].

Substituting (6) into (5), and using the inequalities $\mu \gg T$, ρ to extend the limits of integration to infinity, one arrives at

$$J = \frac{\mu}{4\pi} + \frac{T\mu}{4\pi} \times \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dx \left(\frac{1}{x^2 + (\omega_n^2 + \rho^2)\eta_n^2} - \frac{2\omega_n^2\eta_n^2}{[x^2 + (\omega_n^2 + \rho^2)\eta_n^2]^2} \right) . \tag{8}$$

Eq. (8) is formally divergent and demands special care due to the fact that one has to perform the integration over x before the summation [15]. Finally, one can formally cancel the divergence [15] to obtain

$$J = \frac{\mu\rho^2 T}{4} \sum_{n=-\infty}^{\infty} \frac{1}{(\omega_n^2 + \rho^2)(\sqrt{\omega_n^2 + \rho^2} + 1/2\tau_{\mathrm{tr}})} . \tag{9}$$

The temperature of the BKT transition for the XY-model Hamiltonian (4) is determined by the equation

$$T_{BKT} = \frac{\pi}{2} J(\mu, T_{BKT}, \rho(\mu, T_{BKT})) . \tag{10}$$

The self-consistent calculation of T_{BKT} as a function of the carrier density $n_f = m\epsilon_F/\pi$ requires additional equations for ρ and μ, which together with (10) form a complete set [9].

When the modulus of the order parameter $\rho(x)$ is treated in the mean field approximation, the equation for ρ takes the form [9]

$$\frac{2\rho}{V} = \sum_{n=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \,\mathrm{tr}\,[\tau_1\langle \mathcal{G}(i\omega_n, \mathbf{k})\rangle] , \tag{11}$$

which formally coincides with the gap equation of the BCS theory. This coincidence allows one to use the Anderson theorem [13], which states that the dependence of $\rho(T)$ is the same as that for the clean superconductor and is not affected by the presence of nonmagnetic impurities. It is important to recall that this theorem is, of course, valid only for the s-wave pairing and small disorder.

There are, however, both physical and mathematical differences between the gap in the BCS theory and ρ [9,10]. In particular the temperature T_ρ which is estimated from the condition $\rho = 0$ is not related to the temperature of the superconducting transition, but is interpreted as the pseudogap opening temperature T^* (see details in [9]). The main point, which we would like only to stress here, is that by virtue of the Anderson theorem [13] the value of T_ρ does not depend on the presence of impurities, while the temperature T_{BKT}, as we will show, is lowered.

The chemical potential μ is defined by the number equation

$$\sum_{n=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \operatorname{tr} [\tau_3 \langle \mathcal{G}(i\omega_n, \mathbf{k}) \rangle] = n_f . \quad (12)$$

Since we are interested in the high carrier density region, the solution of (12) is $\mu \simeq \epsilon_F$, so that in Eqs. (9)–(11) one can replace μ by ϵ_F.

Having the temperatures T_ρ and T_{BKT} as functions of the carrier density, one can build the phase diagram of the model [9] which consists of three regions. The first one is the superconducting (here BKT) phase with $\rho \neq 0$ at $T < T_{BKT}$. In this region there is algebraic order, or a power-law decay of the $\langle \Phi^* \Phi \rangle$ correlations. The second region corresponds to the pseudogap phase ($T_{BKT} < T < T_\rho$). In this phase ρ is still nonzero but the correlations mentioned above decay exponentially. The third is the normal (Fermi-liquid) phase at $T > T_\rho$ where $\rho = 0$. Note that $\langle \Phi(x) \rangle = 0$ everywhere. While the given phase diagram was derived for the idealized $2D$ model, there are indications that even for such complicated layered systems as HTSC the value of the critical temperature for them may be well estimated using T_{BKT} [19,20], even though the transition undoubtedly belongs to the $3D$ XY class. It was also pointed out in [19] that a nonzero gap in the one-particle excitation spectrum can persist even without long-range order.

3. Comparison of the clean and dirty limits

3.1. Clean limit

The transport time τ_{tr} is infinite in the clean limit, so that

$$J(\epsilon_F, T, \rho(\epsilon_F, T)) = \frac{\epsilon_F \rho^2 T}{4} \sum_{n=-\infty}^{\infty} \frac{1}{(\omega_n^2 + \rho^2)^{3/2}} . \quad (13)$$

Near T_ρ one can obtain from (13)

$$J(\epsilon_F, T \to T_\rho^-, \rho \to 0) = \frac{7\zeta(3)}{16\pi^3} \frac{\rho^2}{T_\rho^2} \epsilon_F , \quad (14)$$

where $\zeta(x)$ is the zeta function. This expression must coincide with the result from [9], which was derived using the opposite order for the summation and integration. Inserting the well-known dependence of $\rho(T)$ (see, for example, [21])

$$\rho^2(T \to T_\rho^-) = \frac{8\pi^2}{7\zeta(3)} T_\rho^2 \left(1 - \frac{T}{T_\rho}\right) \quad (15)$$

and then substituting (14) into (10), one obtains the following asymptotic expression for the BKT temperature in the clean limit for high carrier densities [9,22,23]:

$$T_{BKT} = T_\rho \left(1 - \frac{4T_\rho}{\epsilon_F}\right), \quad T_{BKT} \lesssim T_\rho . \quad (16)$$

In the high-density limit one can also use the equation

$$T_\rho = \frac{\gamma}{\pi} \sqrt{2|\varepsilon_b|\epsilon_F} , \quad (17)$$

where $\gamma \simeq 1.781$ and ε_b is the energy of the two-particle bound state in vacuum, which is a more convenient parameter than the four-fermion constant V [9,24].

It is obvious from (16) and (17) that the pseudogap region shrinks rapidly for high carrier densities* and one may ask (see, for example, [25] whether this scenario can explain the pseudogap anomalies which are observed over a wide range of temperatures and carrier densities, since in the clean limit the relative size of the pseudogap region $(T_\rho - T_{BKT})/T_\rho$ is, for instance, less than 1/2 when the dimensionless ratio $\epsilon_F/|\varepsilon_b| \lesssim 128\gamma^2/\pi^2 \simeq 41$. A crude estimate for the dimensionless ratio for optimally doped cuprates gives $\epsilon_F/|\varepsilon_b| \sim 3 \cdot 10^2$–$10^3$ [26], which indicates that in the clean superconductor the pseudogap region produced by the phase fluctuations is too small. Of course, all these estimates are qualitative due to the simplicity of the model.

The value of the bare superfluid density $n_s(T)$ is straightforwardly expressed via the bare phase stiffness, $n_s(T) = 4mJ(T)$. In particular, it follows from (13) that $n_s(T = 0) = n_f$. This is not surprising, since $n_s(T = 0)$ must be equal to the total density n_f for any superfluid ground state in a translationally invariant system [27] and the clean system is

* In $2D$ for s-wave pairing the high-density limit is in fact equivalent to the weak-coupling BCS limit.

translationally invariant. We note, however, as stated above, that $n_s(T=0) << n_f$ in HTSC [5]. Substituting (15) into (13), one obtains for T close to T_ρ the bare superfluid density as $n_s(\tilde{T} \to T_\rho^-) = 2n_f(1 - T/T_\rho)$. This behavior of the bare superfluid density is formally the same as the behavior of the total superfluid density in the BCS theory. Nevertheless it is important to remember that the total superfluid density in the present model undergoes the Nelson-Kosterlitz jump at T_{BKT} and is zero for $T > T_{BKT}$. We note that one can probe experimentally both the bare superfluid density in high-frequency measurements [2] and the total superfluid density in low-frequency measurements [3].

3.2. Dirty limit

In the dirty limit the quasiparticle transport time τ_{tr} is small ($\tau_{tr} << \rho^{-1}(T=0)$), so that one can neglect the radical inside the brackets in Eq. (9) [15]. The remaining series is easily summed, and one obtains for the bare superfluid stiffness

$$J(\epsilon_F, T, \rho(\epsilon_F, T), \tau_{tr}) = \frac{\epsilon_F \tau_{tr} \rho}{4} \tanh \frac{\rho}{2T} . \quad (18)$$

As explained above, by virtue of the Anderson theorem, the expressions (15) for ρ and (17) for T_ρ remain unchanged in the presence of impurities. Again substituting (15) into (18), one obtains

$$T_{BKT} = T_\rho \left(1 - \frac{14\zeta(3)}{\pi^3} \frac{1}{\epsilon_F \tau_{tr}} \right), \quad T_{BKT} \lesssim T_\rho . \quad (19)$$

One can see that the size of the pseudogap region is now controlled by the new phenomenological parameter τ_{tr}, which is an unknown function of ϵ_F for HTSC. The experimental data [1] suggest that τ_{tr} is almost independent on doping level in the underdoped region.

It is difficult to obtain more than a qualitative estimate using Eq. (19), since in its derivation we have assumed that $\epsilon_F \tau_{tr} >> 1$. In HTSC however, as discussed above (see also [12]), this assumption is not always justified. Bearing in mind that the dirty limit implies that the condition $\tau_{tr}^{-1} >> \rho\ (T=0) \sim T_\rho$ is satisfied, one can easily see that the value of T_{BKT} for this case is less than that given by (16) for the clean superconductor. Since impurities are inevitably present in HTSC, phase fluctuations can in fact give rise to a pseudogap region that is of comparable size to that observed experimentally. We note that our arguments are in fact quite similar to that given in [22] for the best conditions for observing BKT physics in superconducting films. However, in contrast to this paper, the gap opening below T_ρ is particularly emphasized here.

While Eq. (19) was derived under assumption $T_{BKT} \lesssim T_\rho$, in the general case when T_{BKT} can be substantially less than T_ρ one must solve the self-consistent equation (10) with $J(\epsilon_F, T_{BKT}, \rho(\epsilon_F, T_{BKT}))$ given by (18). Recall, however, that to make any quantitative estimates, the more realistic d-wave model has to be considered, and the inequality $\epsilon_F \tau_{tr} >> 1$ should not be assumed [12].

The value of the zero-temperature superfluid density is now given by $n_s(T=0) = \pi n_f \tau_{tr} \rho << n_f$, since $\tau_{tr}\rho << 1$. This does not contradict the results of [27] because the system is not translationally invariant in the presence of impurities [28]. Furthermore, as one can see, the low value of the superfluid density in HTSC [5] may be related to the impurities which are inevitably present in HTSC. Another factor that leads to lowering of the superfluid density is the presence of the lattice, which also destroys a continuous translational invariance. We note that, as was pointed out in [29], quantum fluctuations also lead to a decrease in the superfluid density.

4. The temperature dependence of the bare superfluid density

In this section we try to correlate the temperature dependence of the observed in-plane resistivity $\rho_{ab}(T)$ with the recently measured temperature dependence of the bare superfluid density [2].

For $T > T_{BKT}$ the expression for the bare superfluid density in the dirty limit (18) can be rewritten in terms of the in-plane conductivity, $\sigma = e^2 n_f \tau_{tr}/m$, where e is the charge of an electron:

$$J(\sigma(\epsilon_F, T), \rho(\epsilon_F, T)) = \frac{\pi}{4} \frac{\sigma\rho}{e^2} \tanh \frac{\rho}{2T} . \quad (20)$$

The in-plane resistivity $\rho_{ab} \sim \sigma^{-1}$ in cuprates has been extensively studied [1], and its temperature and concentration dependencies must reflect the pseudogap properties observed in other experiments. One can say that $\rho_{ab}(T)$ is linear above $T^* \simeq T_\rho$ and roughly linear between T_{BKT} and T_ρ but with a lower slope. Thus in the interval $T_{BKT} < T < T_\rho$ the resistivity can be approximately written as $\rho_{ab}(T) = aT + b$, where a and b are functions of ϵ_F but not of temperature.

Now, substituting $\sigma \sim \rho_{ab}^{-1}(T)$ into Eq. (20), one obtains

$$n_s(T) \sim \frac{\rho}{aT + b} \tanh \frac{\rho}{2T} . \quad (21)$$

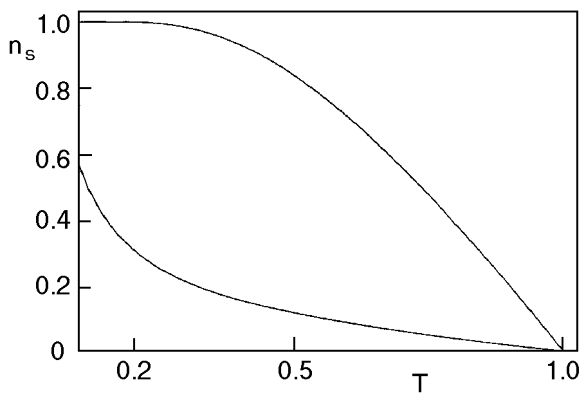

Fig. 1. The behavior of $n_s(T)$ in the clean (upper curve) and dirty (lower curve) limits. The value of $n_s(T)$ is normalized to $n_s(T = 0)$ for the clean system; T is given in units of T_ρ .

Our estimates based on Eq. (21) are shown in Fig. 1. One can see that, in contrast to the almost linear BCS dependence of $n_s(T)$, we have convex behavior, and the superfluid density becomes zero at T_ρ . We stress that the curvature of $n_s(T)$ is the result of both the temperature dependence of $\rho(T)$ and $\sigma(T)$ for $T_{BKT} < T < T_\rho$. More importantly, the slope of the curve $n_s(T)$ at T_ρ for the dirty metal is substantially less than for the clean one. The experiment [2] shows the same curvature for $n_s(T)$ but indicates that the bare superfluid density disappears at a lower temperature, $T_s < T^*$. Since the slope $dn_s(T)/dT$ at T_ρ is very small, as predicted by Eq. (21) and observed experimentally, the nonzero value of $n_s(T)$ between T_s and T_ρ may however simply be too small to be experimentally observed. A definitive answer to this question demands further experiments and theoretical studies. In particular, ρ fluctuations should be taken into account [9,10].

One can also comment on the experimentally observed change in the curvature of the total superfluid density, $N_s(T)$, with changing carrier density [3], even though $N_s(T)$ cannot be directly related to the bare superfluid density $n_s(T)$ discussed here. Although the total superfluid density disappears above T_{BKT} , the curvature present in the bare superfluid density $n_s(T)$ seems to be retained as a curvature in the total superfluid density, $N_s(T)$, below T_{BKT} [2,3]. For low carrier densities (the underdoped region) the pseudogap region, $T_{BKT} < T < T_\rho$, is larger, and therefore the curvature in $n_s(T)$ is more pronounced. This behavior seems to be reflected in the total superfluid density, $N_s(T)$ below T_{BKT} [3]. It is important, however, to study experimentally and theoretically the concentration dependence of the bare superfluid density $n_s(T)$ in order to make a full comparison with the results from [3] for $N_s(T)$.

The experimental data of [3] also show that $N_s(T)$ does not display the Nelson-Kosterlitz jump. This is probably related to the influence of the interlayer coupling, see the references cited in [10].

5. Conclusion

Since in HTSC the pairing scale T^* is different from the superconducting transition temperature, the role of nonmagnetic impurities is not traditional, and they in fact govern the superconducting properties of a «bad» metal. In particular, the presence of nonmagnetic impurities strongly increases the size of the pseudogap phase originating from the fluctuations of the phase of the order parameter. In addition, the behavior of the superfluid density in the presence of impurities is closer to that experimentally observed.

Our results are only qualitative, since we have considered a model with nonretarded s-wave attraction and an isotropic fermion spectrum. However, it is likely that the properties obtained will persist for d-wave pairing. There is, of course, the problem of why strong disorder does not destroy the d-wave superconductivity, when nonmagnetic impurities are pair-breaking. As was suggested by Sadovskii [30], even the d-wave pairing may persist if the coupling is strong enough. Further studies are necessary; for example, it is important to explain the concentration dependence of the superfluid slope, $dn_s(T)/dT$ at $T = 0$ [3,4]. Our results also indicate that it would be interesting to study the BCS-Bose crossover problem in the presence of impurities, especially in the d-wave case [30].

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