Fizika Nizkikh Temperatur, 2001, v. 27, Nos. 9/10, p. 1083–1091

Bound spin-excitons in two-dimensional electron gas

V. Fleurov

School of Physics and Astronomy, Beverly and Raymond Sackler Faculty of Exact Sciences
Tel Aviv University, Tel Aviv 69978, Israel
E-mail: fleurov@post.tau.ac.il

K. Kikoin

Physics Faculty, Ben-Gurion University of the Negev Beer-Sheva 84105, Israel

Received April 23, 2001

A theory of the spin exciton capture by a magnetic impurity in a 2D electron gas is developed. We consider a resonance model for the electron scattering by a transition metal impurity and calculate the binding potential for spin excitons. This potential is spin selective and is capable of binding a spin exciton with zero angular momentum. In order to trap an exciton with a nonzero angular momentum m, the potential must exceed a certain threshold value, depending on m.

PACS: 73.20.Dx

1. Introduction

Shubnikov–de Haas effect is a powerful tool for studies of a two-dimensional electron gas (2DEG) in a strong magnetic field. In particular, the oscillatory behavior of the electronic g factor in a 2DEG was investigated with the help of this effect (see [1] and references therein). Two important parameters which predetermine the properties of the 2DEG in a strong magnetic field B are the cyclotron frequency $\omega_B = eB/mc$ and the effective Coulomb energy $E_c = e^2/\kappa l_B$ ($l_B = (\hbar c/eB)^{1/2}$) is the magnetic length). In the limit of ultra high magnetic fields, when $E_c << \hbar\omega_B$ and only the lowest Landau sublevel is filled, the low-energy branches of the excitation spectrum are represented by the well separated bands of spin waves, magnetoplasmons, etc. These excitations were studied in details during recent decades (see, e.g., Refs. 2–7). Various forms of spin excitations can be observed experimentally using the inelastic light scattering method as described in Ref. 8 where features are observed, attributed to collective excitation in a 2DEG. Presence of magnetic impurities may account for additional features due to localized spin excitations.

The spin waves, formed by the electrons in the down-spin and holes in up-spin $n = 0$ Landau subbands, make the lowest branch of magnetic excitations. These states are separated by the Zeeman gap $\Delta_B = g\mu_B B$ from the ground state and form a band with the width of $\sim E_c$. The exciton dispersion law is quadratic at small wave numbers and saturates in a short-wave limit, where the excitons, in fact, transform into free electron–hole pairs [2].

This paper studies the interaction between spin waves and a magnetic impurity in a 2DEG. To be more specific, we consider doped heterojunctions GaAs/GaAlAs and related materials, so we refer below to a III–V semiconductor as a host material, in which some cation atoms are substituted for magnetic transition metal impurities. It is known [9], that transition metal atoms create deep levels in the forbidden energy gap of the host semiconductor, and the main mechanism of the electron–impurity scattering is the resonance scattering by the d-levels of the unfilled $3d$ shell of transition metal ions. The intraatomic exchange interaction leads to the Hund rule which governs the occupation of the deep levels. As a result, the transition atoms in a semiconductor are magnetic. We will show below that the interplay between the magnetic impurity scattering and attractive electron–hole interaction in excited 2DEG results in a bound spin exciton. The spectrum of these bound states is the subject of the present study.

2. Model and approximations

We start with the model of 2DEG doped by magnetic impurities which is discussed in detail in Ref. 10. This model is described by the Hamiltonian

$$H = H_b + H_i \qquad (1)$$

where

$$H_b = \sum_{nm\sigma} E_{n\sigma}\, a^+_{nm,\sigma}\, a_{nm,\sigma} \qquad (2)$$

is the Hamiltonian of a 2DEG, strongly quantized by a magnetic field. It is convenient to use the symmetric cylindrical gauge for the vector potential $A = ((-B/2)y,\ B/2,\ 0)$, so that the index m in (2) describes different orbital states in a given Landau level

$$E_n = \hbar\omega_B \left(n + \frac{1}{2}\right) \equiv \frac{\hbar^2}{2m}\,\frac{2n+1}{l_B^2}\,. \qquad (3)$$

Then, assuming a small impurity concentration, the impurity related part H_i of the Hamiltonian (1) can be written in a general form

$$H_i = \sum_{\Gamma} \langle i\Gamma\,|E_\Gamma|\,i\Gamma\rangle + \sum_{\gamma}\sum_{nm\sigma} (\langle i\gamma|V|nm\rangle\, d^+_{i\gamma\sigma}\, a_{nm\sigma} + \text{h.c.}) + \sum_{nm\sigma} \langle nm|\Delta V|n'm'\rangle\, a^+_{nm\sigma}\, a_{n'm'\sigma}\,. \qquad (4)$$

Here the state of an isolated impurity ion is characterized by a configuration d^n of its unfilled d-shell in a crystal field preserving the point symmetry of the bulk semiconductor (we assume that the potential, responsible for the confinement in the z direction does not perturb the crystalline environment of the impurity site). Then the electrons in the d shell are characterized by the representations $\gamma = t_2$, e of the tetrahedral point group, and the many–electron states $|\Gamma\rangle$ of the $3d$ shell may be represented as

$$d^n_\Gamma = \left(n e^{r_1}_\uparrow\, e^{r_2}_\downarrow\, t^{r_3}_{2\uparrow}\, t^{r_4}_{2\downarrow}\right)_{\sum_i r_i = n}$$

($\downarrow$ and $\uparrow$ are two projections of the electron spin). The scattering part of H_i consists of two components [9]: the second term in Eq. (4) describes the resonance part of the impurity scattering; the third one represents the short-range substitution potential $\Delta V = V_i\,(r - R_0) - V_{\text{host}}(r - R_0)$, where R_0 is the position of the substitution impurity in the host lattice. The resonance scattering arises together with the usual «potential» scattering due to the fact that the energy level $\varepsilon_{i\gamma} = E_\Gamma(d^n) - E_{\Gamma'}(d^{n-1})$ enters the fundamental energy gap of the host semiconductor or appears in the lowest conduction band of the topmost valence band. Here the configuration d^{n-1} misses one electron in a state $\gamma = e$ or t_2 in comparison with the state d^n.

The impurity problem with the two scattering mechanisms can be solved in the general case [11,12]. In our special case its solutions are essentially different for e and t_2 channels. As is shown in [10], the t_2-component of impurity potential results in deep levels in forbidden energy gap or resonances in the conduction band, but only weakly perturbs the Landau levels E_n. On the other hand, e-scattering results in an appearance of the bound Landau states with $m = 0$ between Landau levels with nonzero orbital quantum number and in a splitting of the lowest bound state from the Landau grid. The reason for this difference is in the *short-range* nature of both components of the scattering potential. The eventual reason for this difference is the orbital dependence of the matrix element

$$\langle nm|\Delta U|\gamma\mu\rangle \sim \left(\frac{\rho}{l_B}\right)^{|m|} << 1 \qquad (5)$$

for $m \neq 0$ [9]. Here ρ is the radial variable in the cylindrical coordinates (ρ, φ, z). When estimating this matrix element one should choose ρ of the order of the atomic radius. As a result only the orbitals $|e1\rangle \propto |r^2 - 3z^2\rangle \sim Y_{20}$ with $m = 0$ may be strongly hybridized with the Landau states.

According to [10,12], the localized eigenstates of the Hamiltonian (1), (4) are given by the following equation

$$E_{i\gamma\sigma} - \varepsilon_{i\gamma\sigma} - M_\gamma\,(E_{i\gamma\sigma}) = 0\,, \qquad (6)$$

where

$$M_\gamma\,(E_{i\gamma\sigma}) = \sum_\beta \frac{\langle\gamma\mu|\Delta U|\beta\rangle\,\langle\beta|\Delta U|\gamma\mu\rangle}{E_{i\gamma\sigma} - E_\beta}\,, \qquad (7)$$

where $|\beta\rangle \equiv |bn\sigma\rangle$ stand for the eigenfunctions of the magnetically quantized electrons captured by the local potential ΔU. The latter problem was solved in Ref. 13 for an attractive potential $\Delta U < 0$. It is shown that the short range potential also perturbs only the zero orbital states $m = 0$. The general

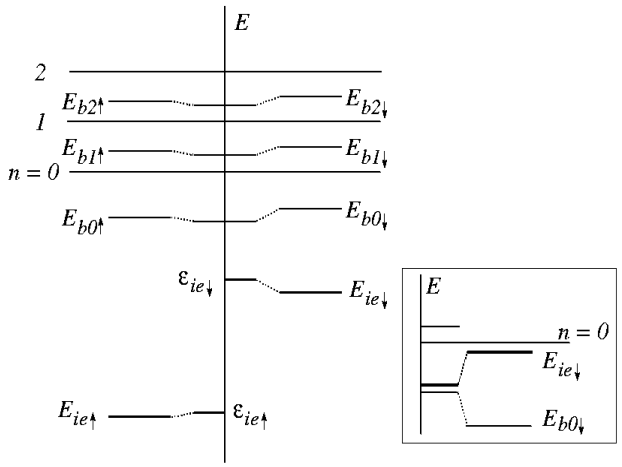

Fig. 1. A schematic representation of the interacting Landau and impurity levels. $E_{ie\sigma}$ are the impurity levels for up- and down-spin electrons. They result from the prime impurity levels $\varepsilon_{ie\sigma}$ shifted to new positions due to their interaction with the Landau levels with $m = 0$. The Landau levels with $m = 0$ are also shifted to their new positions $E_{be\sigma}$. The insert illustrates the case when an impurity prime spin-down state is nearly degenerate with the lowest Landau spin-down state.

structure of the electron spectrum of doped 2DEG is sketched in Fig. 1.

This observation allows us to divide the states in the Landau band (2) into two groups,

$$H_b = H_{b0} + H_b' \quad . \tag{8}$$

Here H_{b0} includes only the states with the zero angular moment $m = 0$, whereas H_b' includes all the remaining states. Only the states with $m = 0$ are involved in the formation of the bound Landau states given by the solutions of Eq. (6). Similar reduction can be made in the manifold $\{i\gamma\sigma\}$ due to the «selection rule» (5): when considering the renormalization of the states $|\beta\rangle$ due to the resonance scattering, we retain only the state $|e1\rangle$ in the corresponding sector of the secular matrix. As a result, one has the following equation

$$E_{bn\sigma} - E_{bn\sigma}^{(0)} = \frac{\langle bn\sigma|\Delta U|e1\sigma\rangle\,\langle e1\sigma|\Delta U|bn\sigma\rangle}{E_{bn\sigma} - E_{ie\sigma}} + \delta M(E_{bn\sigma}) \tag{9}$$

for the energy shift of a given level $E_{bn\sigma}^{(0)}$ which according to Ref. 13 appears in the gap between the bare Landau levels E_{n-1} and E_n. Here we have picked up the direct mutual repulsion of the nth Landau level and the d-level in the first term of the right-hand side of Eq. (9). The influence of the other bound Landau states is given by the second term,

$$\delta M(E) = \sum_{\beta' \neq \beta} \frac{\langle \beta'|\Delta U|e1\sigma\rangle\,\langle e1\sigma|\Delta U|\beta'\rangle\,(E - E_{\beta}^{(0)})}{(E - E_{ie\sigma})\,(E - E_{\beta'}^{(0)})} \quad .$$

The role of these states is to keep the renormalized levels in the same energy interval $E_{n-1,\sigma} < E_{bn\sigma} < E_{n\sigma}$.

An important feature of the resonant channel of impurity scattering is its *spin selectivity*, which stems from the spin structure of the transition metal d shell. It is known in Ref. 9 that the transition metal impurities follow in their main features the «Aufbau principle» of the quantum mechanics of isolated atoms. This means that the d-shell is usually filled in accordance with the Hund's rule or, in other words, the exchange interaction makes the level $E_{ie\uparrow}$ lie always below the level $E_{ie\downarrow}$. Then the splitting $\Delta_{es} = E_{ie\downarrow} - E_{ie\uparrow}$ generates the spin splitting $\Delta_{bs}^{(n)}$ of the bound Landau states via Eq. (9). This splitting can be estimated as

$$\Delta_{bs}^{(n)} \approx \frac{\Delta_{es}}{\Delta_{\uparrow}^{(n)}\,\Delta_{\downarrow}^{(n)}} \left(|V_{en}|^2 + \sum_{n' \neq n} \frac{|V_{en'}|^2 \delta_n}{E_{bn}^{(0)} - E_{bn'}^{(0)}} \right). \tag{10}$$

Here $\Delta_{\sigma}^{(n)} = E_{bn} - E_{ie\sigma}$, $V_{en} = \langle e1|\Delta U|bn\rangle$, $\delta_n = E_{bn} - E_{bn}^{(0)}$ (the spin dependence of the two last quantities is neglected). It is important for the further classification of the spin excitons that the sign of $\Delta_{bs}^{(n)}$, unlike the sign of Δ_{es}, can be both negative or positive, since the energy differences $\Delta_{\uparrow}^{(n)}$ and $\Delta_{\downarrow}^{(n)}$ can have either the same or the opposite signs depending on the type of the transition metal ion and the host matrix (see discussion in Ref. 10). According to Eq. (10) the resonance impurity scattering results in a «transfer» of the exchange splitting Δ_{es} of the impurity d-shell to the spectrum of the Landau electrons. This transfer is illustrated by the level renormalization in Fig. 1.

Now we know the general structure of the one-particle spectrum of a magnetically doped 2DEG, which should serve as a background when magnetic excitons and magnetoplasmons are formed. This spectrum consists of equidistant Landau levels with $m \neq 0$ which are unperturbed by the impurity scattering. The states with $m = 0$ form their own grid: a pair of spin split states $E_{bn\sigma}$ appears in each energy gap $E_n^{(0)} - E_{n-1}^{(0)}$, and the lowest pair of bound states $E_{bn\sigma}$ arises in the fundamental energy gap of the 2D semiconductor below the Landau level E_0.

3. Energy spectrum of localized spin excitons

According to the general classification of the multiparticle excitations from a filled Landau level in 2DEG [4], the lowest branch of the electron-hole excitations is that of the spin waves. These spin excitons arise as bound electron-hole pairs with parallel spins as a result of a spin-flip excitation from the lower filled sublevel $E_{0\uparrow}$ of the lowest Landau level to its higher empty sublevel $E_{0\downarrow}$. To calculate the spectrum of the *localized* spin excitons, one should keep only the states with $n = 0$ in the bare Hamiltonian H_b (2), take into account the Zeeman splitting of the Landau states explicitly, and then add the electron–hole Coulomb interaction $H_{\rm int}$,

$$H^0_{\rm ex} = \sum_p (E_{0\uparrow} a^+_p a_p + E_{0\downarrow} b^+_p b_p) + H_{\rm int} . \quad (11)$$

Operators $a^+(a)$ and $b^+(b)$ describe creation (annihilation) of electrons with the up or down spins σ, index $n = 0$ being omitted. Here the electron wave functions are written in the asymmetric Landau gauge $A = B(y, 0, 0)$. Then the total Hamiltonian of a doped 2DEG takes the form

$$H = H^0_{\rm ex} + H_i . \quad (12)$$

The impurity related term H_i was analyzed in the previous Section using the symmetric gauge. Now it should be re-derived in the Landau gauge.

Using the solution of the impurity problem found in the previous Section and taking into account that the Landau levels with $m = 0$ are included in $H_{\rm ex}$ contrary to the «extraction» principle formulated in Eq. (8), we can write H_i in the form (see Appendix),

$$H_i = (|A_{00}|^2 E_{b0\uparrow} - E_{0\uparrow})\, a^+_{i0} a_{i0} +$$

$$+ (|B_{00}|^2 E_{b0\downarrow} - E_{0\downarrow})\, b^+_{i0} b_{i0} + H_{i,nd} . \quad (13)$$

Here $E_{b0\sigma}$ are the solutions of Eq. (9) for the lowest bound Landau state, and the operators a^+_{i0} and b^+_{i0} create the corresponding eigenfunctions in the symmetric cylindrical gauge. The coefficients A_{00}, B_{00} are defined in Eq. (A.2). The off-diagonal term $H_{i,nd}$ contains the contribution from the higher Landau levels with $n > 0$ whose value is of the order of $\sim |A_{0n}|^2/E_n << 1$ and will be neglected in the further calculations.

In principle, the scattering potential contains also the terms corresponding to the spin-flip processes,

$$H_\perp = J\,(d^+_{ie\uparrow} d_{ie\downarrow} b^+_{i0} a_{i0} + d^+_{ie\downarrow} d_{ie\uparrow} a^+_{i0} b_{i0}) . \quad (14)$$

The spin-flip terms are inessential in comparison with the leading spin-diagonal terms (13) because each spin flip costs the Hund energy $\Delta_{es} = E_{ie\downarrow} - E_{ie\uparrow}$, so that $J \sim |V_{e0}|^2/\Delta_{ex}$. The spin-flip processes, in principle, result in a multiple creation of spin excitons, but the contribution of these processes to the spin-wave spectrum is at least $\sim J^2$, and we also neglect them in the further calculations.

Then, we turn from the cylindrical gauge to the Landau gauge,

$$a^+_{i0} = \sum_p A_p a^+_p , \quad b^+_{i0} = \sum_p B_p b^+_p , \quad (15)$$

where

$$A_p = \langle i0\uparrow | p\uparrow\rangle , \quad B_p = \langle i0\downarrow | p\downarrow\rangle . \quad (16)$$

As a result the impurity Hamiltonian in the Landau gauge has the form

$$H_i = \sum_{pp'} [\,U_\uparrow(p, p') a^+_p a_{p'} + U_\downarrow(p, p') b^+_p b_{p'}] \quad (17)$$

where

$$U_\uparrow(p, p') = K_\uparrow J_{p\uparrow} J^*_{p'\uparrow} , \quad U_\downarrow(p, p') = K_\downarrow J_{p\downarrow} P^*_{p'\downarrow} . \quad (18)$$

Here $J_{p\sigma} = \langle \psi_{b\sigma} | \psi_{p,0\sigma} \rangle$, and $\psi_{p,0\sigma}$ are the wave functions of the lowest Landau level,

$$\psi_{p,0\sigma}(x, y) = \frac{1}{(2\pi^{3/2})^{1/2}} \exp(ipy) \exp\left(-\frac{(x+p)^2}{2}\right). \quad (19)$$

Here x, y are Cartesian projections of the dimensionless vector $\mathbf{r}/l_B$. The coefficients K_σ are calculated in Appendix.

To derive the impurity Hamiltonian in the Landau gauge, we use the identity

$$a^+_p a_{p'} = \int \frac{dq_x}{2\pi} \exp\left[-iq_x \frac{p+p'}{2}\right] \rho_\uparrow(q_x , p' - p) ,$$
$$b^+_p b_{p'} = \int \frac{dq_x}{2\pi} \exp\left[-iq_x \frac{p+p'}{2}\right] \rho_\downarrow(q_x , p' - p) , \quad (20)$$

where the electron density operators are

$$\rho_{\uparrow}(\mathbf{q}) = \sum_p a_p^+ a_{p+q_y} \exp\left(iq_x\left(p + \frac{q_y}{2}\right)\right)$$
$$\rho_{\downarrow}(\mathbf{q}) = \sum_p b_p^+ b_{p+q_y} \exp\left(iq_x\left(p + \frac{q_y}{2}\right)\right) \quad (21)$$

with $\mathbf{q} = (q_x, q_y)$. Then inserting Eq. (20) into Eq. (17), we get after straightforward calculations

$$H_i = \sum_{\mathbf{q},\sigma} U_\sigma(\mathbf{q})\, e^{q^2/4}\, \rho_\sigma(\mathbf{q}) \quad (22)$$

where the matrix elements of the impurity potential are

$$U_\sigma(\mathbf{q}) = K_\sigma \int d^2r_1 d^2r_2 \psi_{b\sigma}\left(\frac{r_1^2}{2}\right)\psi_{b\sigma}\left(\frac{r_2^2}{2}\right)\exp\left(-\frac{|r_1 - r_2|^2}{4}\right)\times$$
$$\times \exp\left\{\frac{1}{2}\left[(x_1 - x_2)q_y + (y_1 - y_2)q_x - i(y_1 + y_2)q_y + i(x_1 + x_2)q_x - i(x_1 + x_2)(y_1 - y_2)\right]\right\}. \quad (23)$$

Here the coefficients K_σ determined in Appendix depend on the specific form of the scattering impurity potential acting on the electrons in the Landau subband 0σ [see Eqs. (16), (A.3), (A.6), (A.8)]. We discuss here the limit of a strong magnetic field when the cyclotron energy $\hbar\omega_B = \hbar^2/2ml_B$ is large compared to the Coulomb energy $e^2/\kappa l_B$. It is essential that the impurity potential is spin selective, i.e., its components acting on the electrons in the two Landau subbands can differ significantly in magnitude (see below).

We consider the case of the filling factor $\nu = 1$, when the spin-up Landau band is totally full and the spin-down Landau band is completely empty. Then the eigenfunctions of the Hamiltonian (11)

$$\Psi^{(0)}_{\text{ex},\,\mathbf{k}} = \sum_p b_p^+ a_{p+k_y} \exp\left(ik_x\left(p + \frac{k_y}{2}\right)\right)|0\rangle \quad (24)$$

correspond to the free spin–excitons with the energy spectrum $\varepsilon_{\text{ex}}(k)$,

$$H^{(0)}\Psi^{(0)}_{\text{ex}}(k) = \varepsilon_{\text{ex}}(k)\Psi^{(0)}_{\text{ex},\mathbf{k}}. \quad (25)$$

Here $\mathbf{k} = k_x, k_y$ is the wave vector of a spin exciton. The exciton dispersion law is

$$\varepsilon_{\text{ex}}(k) = \Delta_B + \left(\frac{e^2}{\kappa l_B}\right)\left(\frac{\pi}{2}\right)^{1/2}\left[1 - e^{-k^2/4} I_0(k^2/4)\right] \equiv$$
$$\equiv \Delta_B + \Omega(k^2) \quad (26)$$

(see [2–4,14]). Here $\Delta_B = |g\mu_B B|$ is the Zeeman energy; I_0 is a modified Bessel function.

The wave function of a bound exciton is looked for in the form

$$\Psi_{\text{ex}} = \sum_{\mathbf{k}} f(\mathbf{k})\, \Psi^{(0)}_{\text{ex},\mathbf{k}} \quad (27)$$

The function (27) must be an eigenfunction of the Hamiltonian (12). The standard procedure leads to the equation

$$[\varepsilon_{\text{ex}}(k) - \varepsilon]\, f(\mathbf{k}) +$$
$$+ 2i \sum_{\mathbf{k}'} \tilde{U}_+(\mathbf{k} - \mathbf{k}') \sin\frac{1}{2}[\mathbf{k}' \times \mathbf{k}]_z\, f(\mathbf{k}') -$$
$$- 2 \sum_{\mathbf{k}'} \tilde{U}_-(\mathbf{k} - \mathbf{k}') \cos\frac{1}{2}[\mathbf{k}' \times \mathbf{k}]_z\, f(\mathbf{k}') = 0, \quad (28)$$

for the envelope function $f(\mathbf{k})$. Here

$$\tilde{U}_\pm(\mathbf{q}) = \frac{1}{2}\left(U_\uparrow(\mathbf{q}) \pm U_\downarrow(\mathbf{q})\right) e^{-q^2/4} \equiv U_\pm(\mathbf{q})\, e^{-q^2/4}.$$

As is discussed in Appendix, the localization radius ρ_b of the impurity wave function is essentially smaller than the magnetic length. Then the q-dependence of the matrix elements (23) is insignificant, and they can be estimated as

$$U_\sigma \approx K_\sigma(2\pi^2)\left(\int_0^\infty d\xi \psi_{b\sigma}(\xi)\right)^2 \equiv K_\sigma I_b. \quad (29)$$

Taking into account the cylindrical symmetry of the problem, the solutions of Eq. (28) are looked for in the form

$$f(\mathbf{k}) = f_m(k)\, e^{im\varphi} \quad (30)$$

where the integer quantity m is the quantum number of the bound exciton.

Now we substitute the functions (30) in Eq. (28) and carry out the integration over the directions of the vector $\mathbf{k}'$. Then the term proportional to U_+ contains the integrals

$$\int_0^{2\pi} \frac{d\varphi}{2\pi} \exp\left(\frac{kk'}{2}\cos\varphi\right) \sin\left(\frac{kk'}{2}\sin\varphi\right) \sin m\varphi =$$

$$= \frac{(kk')^{|m|}}{2^{|m|+1}|m|} \operatorname{sign}(m) \qquad (31)$$

in which the sign function is defined as

$$\operatorname{sign}(m) = \begin{cases} 1\ , & m > 0\ , \\ 0\ , & m = 0\ , \\ -1\ , & m < 0\ . \end{cases} \qquad (32)$$

The term proportional to U_- contains the integrals

$$\int_0^{2\pi} \frac{d\varphi}{2\pi} \exp\left(\frac{kk'}{2}\cos\varphi\right) \sin\left(\frac{kk'}{2}\sin\varphi\right) \cos m\varphi =$$

$$= \frac{(kk')^{|m|}}{2^{|m|+1}|m|} \qquad (33)$$

for all values of the quantum number m.

The equations defining the radial parts $f_m(k)$ of the envelope functions (30) are

$$f_m(k) = W_n k^{|m|}\, \mathrm{e}^{-k^2/4}\, \frac{1}{\varepsilon - \varepsilon_{\mathrm{ex}}(k)}\, F_m \qquad (34)$$

where

$$W_m = \frac{1}{2^{|m|+1}|m|!} \left\{ U_\downarrow [1 + \operatorname{sign}(m)] - U_\uparrow [1 - \operatorname{sign}(m)] \right\}$$

and

$$F_m = \sum_{\mathbf{k}} \mathrm{e}^{-k^2/4}\, k^{|m|}\, f_m(k)\ .$$

The energy ε_m of the bound exciton with the quantum number m can be found as a solution of the equation

$$1 = W_m M_m(\varepsilon_m)\ , \qquad (35)$$

where

$$M_m(\varepsilon) = \sum_{\mathbf{k}} \frac{k^{2|m|}\, \mathrm{e}^{-k^2/2}}{\varepsilon - \varepsilon_{\mathrm{ex}}(k)}\ . \qquad (36)$$

We introduce now the new variable $\omega = k^2$ and convert summation in Eq. (36) in integration. Then the fact that at small ω the dispersion law $\Omega(\omega) \propto \propto \omega$ allows us to find the behavior of the quantity $M_m(\varepsilon)$ at $\varepsilon \to \Delta_B$.

$$M_m(\varepsilon) = \pi \int_0^D d\omega\, \frac{\omega^m\, \mathrm{e}^{-\omega/2}}{\varepsilon - \Delta_B - \Omega(\omega)} \approx$$

$$\approx \begin{cases} -2\pi m^*_{\mathrm{ex}} \ln m^*_{\mathrm{ex}} |\Delta_B - \varepsilon|\ , & m = 0 \\ M_m(\Delta_B) - M'_m(\Delta_B) |\varepsilon - \Delta_B|\ , \ \varepsilon \to \Delta_B - 0,\ m \neq 0\ . \end{cases} \qquad (37)$$

Here $m^*_{\mathrm{ex}} = 2\kappa\hbar^2/e^2\,(2eB/\pi\hbar c)^{1/2}$ is the effective mass of the free exciton at small momenta [2], $M_m(\Delta_B) < 0$, $M'(\Delta_B) = (dM(\varepsilon)/d\varepsilon)\,|_{\varepsilon - \Delta_B \to -0} > 0$.

Now using the normalization condition

$$\sum_{\mathbf{k}} |f_m(k)|^2 = 1$$

one finds that

$$F_m = |W_m|\, [M'(\varepsilon_m)]^{-1/2}\ . \qquad (38)$$

This equation closes the procedure. Now the spectrum of spin excitations in 2DEG pinned by the resonance impurity with its own localized spin is determined by Eqs. (35)–(37). The coupling constants are determined in Eqs. (29), (A.7), (A.8), and the wave function of the bound spin exciton is given by Eqs. (27), (34), and (38).

Starting the analysis of the bound exciton solutions with the case of $m = 0$, we see that in the weak scattering limit the magnitudes of both coupling constants (29) are determined by the coefficient (A.7). Then the potential in Eq. (35) is given by the following equation

$$W_0 = -U_- = -V_{eb} I_b (\beta_\uparrow - \beta_\downarrow) = V_{eb}^2 I_b\, \frac{\Delta_{es}}{\Delta_\uparrow \Delta_\downarrow} > 0\ . \qquad (39)$$

The scattering potential (39) is repulsive and the Eq. (35) for $m = 0$ has no bound solutions below the exciton band.

Such a solution appears when the resonance scattering for the spin down electrons is strong (see insert in Fig. 1). This is a realistic condition for transition metal impurities [8]. In that case we use Eq. (A.7), for the potential acting on the spin up electrons, and Eq. (A.8), for the potential acting on the spin down electrons. The coupling constant is

$$W_0 = - U_- \approx - \frac{1}{2} (V_{eb} - w_0) I_b . \qquad (40)$$

Due to the logarithmic divergence of the function $M_0(\varepsilon)$ near the bottom of the excitonic band (see Eq. (37)), the discrete solution of Eq. (35) appears, provided the resonance component of impurity scattering is stronger than the potential one, $V_{eb} > w_0$, i.e., $W_0 < 0$ (see Fig. 2).

The selectivity with respect to the orbital quantum number m of the bound spin exciton is intrinsically connected with the spin selectivity of the impurity potential. The orbital momentum is the sum of the electron and hole momenta, $m = m_e + m_h$. As is shown in Refs. 10,13, only the electrons or holes with $m_{e,h} = 0$ can be captured by the short-range impurity potential which is $U_\downarrow$ for electrons and $-U_\uparrow$ for holes. Therefore, in a bound exciton with $m \neq 0$ one of the carriers (electron or hole) must have zero momentum. Then the momentum of the whole exciton is, in fact, the momentum of the second carrier. This second carrier is bound by the combined action of the Coulomb attraction of the first carrier and the diamagnetic contribution of the Lorenz force. The sign of the orbital momentum and the charge of the carrier predetermines the sign, attractive or repulsive, of Lorenz force contribution to the total potential. As a result, only the electrons with $m_e > 0$ and the holes with $m_h < 0$ can be captured in the limit of a strong magnetic field. Hence, the sign of the exciton orbital momentum provides an information on which carrier is bound by the short-range potential. It is the hole, bound by the potential $-U_\downarrow$, in the case of $m > 0$ and the electron, bound by the potential $-U_\uparrow$, in the case of $m < 0$.

In the case of $m = 0$ both carriers are captured by their corresponding short-range potentials and then their total coupling strength, $U_\downarrow - U_\uparrow$, determines the binding energy of the exciton.

4. Conclusions

To conclude, we have found that a magnetic impurity can bind a spin exciton in a 2DEG. It turned out that the mechanism of the exciton capture is the *spin-selective resonance scattering* by the deep impurity levels. This spin selectivity stems from the Hund rule in particular case of transition metal impurities. The interaction in the second order in scattering potential can be described in terms of an indirect spin exchange J (see Eq. (14)), and only the longitudinal component of this exchange is essential for the formation of a bound spin exciton. The transversal components of this exchange give a contribution to the binding energy only in fourth order in the scattering potential.

It is found that the spin-selective impurity potential is always capable of binding the exciton in a state with the moment $m = 0$ due to the edge van Hove singularity of the density of states in a 2DEG. Excitons with $m \neq 0$ can be also trapped, but then the conditions for the capture are more severe.

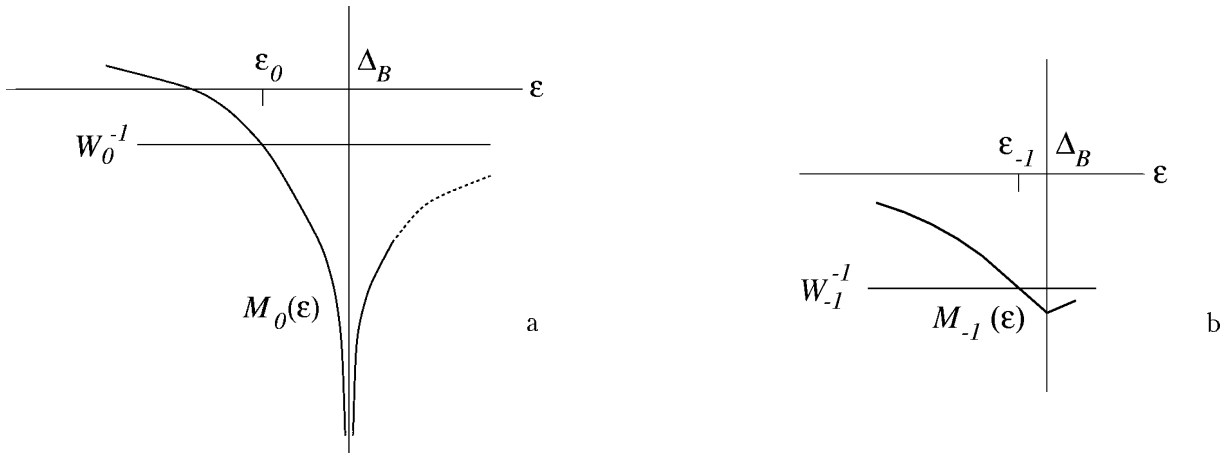

Fig. 2. A graphical solution of Eq. (35) for $m = 0$. Due to the logarithmic divergence of the function $M_0(\varepsilon)$ at $\varepsilon = \Delta_B$, a spin–exciton can be always bound with an energy ε_0 below the bottom of the spin–exciton band (*a*). A graphical solution of Eq. (35) for $m = -1$. The function $M_{-1}(\varepsilon)$ is now non-divergent. Hence, a spin–exciton can be bound with an energy ε_{-1} below the bottom of the spin–exciton band if the parameter W_{-1} is larger than the threshold value $\overline{W}_{-1} = 1/M_{-1}(\Delta_B)$ (*b*).

The analysis of the electronic structure of $3d$ transition metal impurities in GaAs-related systems shows that the strongest binding potential is created by the light elements (V, Cr).

This work was supported by the German–Israeli Foundation for Research and Development, Grant No. 0456–220.07195. K.K. thanks Israeli Science Foundation for support (grant «Nonlinear Current Response of Multilevel Quantum Systems»). The authors are indebted to Yu. Bychkov, S. Dickmann, T. Maniv, and I. Vagner for valuable discussions.

Appendix

In order to derive the impurity Hamiltonian for 2D electrons in the Landau gauge, we use the fact that the resonance scattering involves only the Landau states with $m = 0$. These states are included in the Hamiltonian (11), so one should subtract them from H_i which then acquires the form

$$H_i = E_{b0\uparrow}\, a_{i0}^{+}\, a_{i0} + E_{b0\downarrow}\, b_{i0}^{+} b_{i0} - $$

$$- E_{0\uparrow}\, a_0^{+}\, a_0 - E_{b0\downarrow}\, b_0^{+}\, b_0 \,. \qquad \text{(A.1)}$$

Then the bound Landau states with $m = 0$ can be re-expanded in free Landau states,

$$a_{i0} = \sum_n A_{0n}\, a_{n0} \,, \quad b_{i0} = \sum_n B_{0n}\, b_{n0} \qquad \text{(A.2)}$$

[13]. As a result we come to Eq. (13).

In order to calculate the expansion coefficients A_p , B_p in Eq. (15), one needs the wave functions of the bound electron in the lowest Landau level. The wavefunctions of the lowest localized Landau states correspond to the solutions $E_{b0\sigma}$ of Eq. (9). If these states are deep enough below the unperturbed Landau spectrum, one can neglect the contribution $\delta M(E)$ of the higher Landau levels, and the eigenfunctions $\psi_{i0\sigma}$ has the form [10]

$$\psi_{i0\sigma} = -\sin\theta_\sigma\, \psi_{ie\sigma} + \cos\theta_\sigma\, \psi_{b\sigma} \qquad \text{(A.3)}$$

with the mixing coefficient given by $\tan 2\theta_\sigma = 2V_{eb}/\Delta_\sigma$, and $V_{eb} = \langle e1|\Delta U|b0\rangle$. The wavefunctions $\psi_{b\sigma}$ describe the Landau state bound in a short range attractive potential [13],

$$\psi_{b\sigma}(\xi) = \frac{\Gamma(\xi)}{l_B\,[2\pi\psi'(\xi)]^{1/2}}\,\frac{W_{\alpha,0}(\xi)}{\xi^{1/2}} \,, \qquad \text{(A.4)}$$

where $2\xi = (\rho/l_B)^2$, Γ, ψ' and $W_{\alpha,m}$ are gamma function, trigamma function and Whittaker function, respectively. Index α is determined by the corresponding eigenstate, $\alpha = 2^{-1}(1 - \varepsilon_{b0\sigma} l_B^2)$ with $\varepsilon = = 2m^* E/\hbar^2$. When the level $\varepsilon_{b0\sigma}$ is deep enough or the magnetic field is weak enough, i.e. $|\varepsilon_{b0\sigma}| l_B^2 >> 1$, the wave function $\psi_{b\sigma}$ (A.4) has the standard asymptotic,

$$\psi_{b\sigma} \sim \frac{\mathrm{e}^{-\wp}}{\sqrt{\wp}} \,. \qquad \text{(A.5)}$$

Here $|\alpha| \approx l_B^2/2\rho_b^2$, $\wp = \rho/\rho_b$ and $\rho_b^{-1} = \varepsilon_b$. Thus, the localization radii of the d-electron (ρ_d), bound Landau electron (ρ_b) and free Landau electron (l_B) obey the hierarchy $\rho_d << \rho_b << l_B$. As a result one can safely neglect the contribution of the d-component $\psi_{ie\sigma}$ (A.3) in the overlap integral (16). Then in the case of a week scattering, $\beta_\sigma = V_{eb}/\Delta_\sigma << 1$, ($\theta << 1$) and the overlap integrals (16) can be approximated by the following equations

$$A_p = (1 - \beta_\uparrow^2)\, J_{p\uparrow} \,, \quad B_p = (1 - \beta_\downarrow^2)\, J_{p\downarrow} \,. \qquad \text{(A.6)}$$

Thus the spin dependence of the matrix elements (A.6) is determined by the energy differences Δ_σ in front of the integral and by the index α of the Whittaker function. Having in mind the difference in localization degrees of the wave functions (A.4) and (19), the magnitude of the overlap integrals $J_{p\sigma}$ can be estimated as $J_{p\sigma} \sim (\rho_{b\sigma}/l_B)$, i.e., these integrals are sensitive both to the spin splitting and to the magnetic field. The energy differences in the Hamiltonian (13) are determined by the short-range component of the impurity potential which in our theory enters as a phenomenological parameter ω_0 . In a weak scattering limit $E_{b0\sigma} - E_0 = -\omega_0 + \beta_\sigma V_{eb}$ (if the short-range potential is attractive) [10]. As a result the latter factor is dominant in this limit, so that

$$K_\sigma = -(\omega_0 - \beta_\sigma\, V_{eb}) \,. \qquad \text{(A.7)}$$

In accordance with the Hund's rule for $3d$-impurities $\Delta_\uparrow > \Delta_\downarrow$, so $K_\uparrow > K_\downarrow$.

Next we consider the situation where one of the resonance d-levels, namely $\varepsilon_{ie\downarrow}$, is above the lowest Landau level, and the scattering is strong for the down-spin electrons, $\theta_\downarrow \sim -\pi/4$. The inequality $\theta_\uparrow << 1$ still holds, since the estimates (A.6) and (A.7) for up spin states are still valid. As for the down spin states, the coefficient $\sin\theta_\downarrow$ is $\sim 1/2$ and

$$K_\downarrow = -\frac{1}{2}\,(\omega_0 + V_{eb}) \,. \qquad \text{(A.8)}$$

The magnitudes of the scattering potentials $U_\uparrow$ and $U_\downarrow$ differ noticeably in this case.

1. R. J. Nicolas, R. J. Haug, K. von Klitzing, and G. Weimann, *Phys. Rev.* **B37**, 1294 (1988).
2. I. V. Lerner and Yu. E. Lozovik, *Zh. Eksp. Teor. Fiz.* **78**,1167 (1980) [*Sov. Phys. JETP* **51**, 588 (1980)].
3. Yu. A. Bychkov, S. V. Iordanskii, and G. M. Eliashberg, *Pis'ma Zh. Exp. Teor. Fiz.* **33**, 152 (1981) [*JETP Lett.* **33**, 143 (1981)].
4. C. Kallin and B. I. Halperin, *Phys. Rev.* **B30**, 5655 (1984).
5. C. Kallin and B. I. Halperin, *Phys. Rev.* **B31**, 3635 (1985).
6. D. Antoniou and A. H. MacDonald, *Phys. Rev.* **B43**, 11686 (1991).
7. J. P. Longo and C. Kallin, *Phys. Rev.* **B47**, 4429 (1993).
8. A. Pinczuk, D. Heiman, S. Schmitt-Rink, C. Kallin, B. S. Dennis, L. N. Pfeiffer, and K. W. West, in: *Light Scattering in Semiconductor Structures and Superlattices*, D. J. Lockwood and J. F. Young (eds.), Plenum, New York (1991).
9. K. A. Kikoin and V. N. Fleurov, *Transition Metal Impurities in Semiconductors*, World Scientific, Singapore (1994), Chapt. 8.
10. P. Dahan, V. Fleurov, K. Kikoin, and I. Vagner, *Phys. Rev.* **B**, submitted; *cond.-mat/0106339*.
11. G. Picoli, A. Chomette, and M. Lannoo, *Phys. Rev.* **B30**, 7138 (1984).
12. V. N. Fleurov and K. A. Kikoin, *J. Phys.* **C19**, 887 (1986).
13. Y. Avishai, M. Ya. Azbel, and S. A. Gredeskul, *Phys. Rev.* **B48**, 17280 (1993).
14. Yu. A. Bychkov, T. Maniv, I. D. Vagner, and P. Wyder, *Phys. Rev. Lett.* **73**, 2911 (1994).