Fizika Nizkikh Temperatur, 2003, v. 29, No. 8, p. 840–841

Unstable states of the superfluid confined between rotating spheres*

A. Gongadze, L. Kiknadze, Yu. Mamaladze, and S. Tsakadze

E.A. Andronikashvili Institute of Physics of the Georgian Academy of Sciences
6 Tamarashvili Str., Tbilisi 380077, Georgia
E-mail: yum@iph.hepi.edu.ge; yum270629@yahoo.com

Received December 19, 2002

The unstable states (including those related to self-accelerations of pulsars) in which the mutual friction causes an irreversible motion of vortices is considered.

PACS: 67.40.Vs

Introduction

The concentric spheres with radii R_1 and R_2 rotating with the constant or variable angular velocity ω are considered. Both equilibrium and metastable states of this system are the solutions of the equations of vortex dynamics with the given velocity and zero mutual friction between vortices and the normal component. This force realizes the transitions from one equilibrium or metastable state to another at the change of angular velocity. But there also exist such configurations of vortices, which cannot stay stable even if the variation of the velocity of rotation was interrupted.

In this publication of our report at CWS-2002 the part devoted to equilibrium and metastable rotation (see [1]) is omitted. This paper is dedicated to the mechanism of unstable processes and to the difference between double-cylinder and double-sphere devices.

Breaking and connection of vortices

The clear example of the difference between the coaxial cylinders and spheres is the generation of the first vortices. Fetter [2] had shown that the vortex generation begins at the outer cylinder at $\omega > \hbar \ln(2C)/2mRd$ $(d \equiv R_2 - R_1,\ R \equiv (R_1 + R_2)/2,\ C \approx 1)$, but the generated vortex has no equilibrium position in the space between the cylinders until ω exceeds the value $\hbar \ln(d/a)/md^2$. Therefore, the vortices move to the inner cylinder, annihilate on it, and leave there the circulation. In contrast to this, in the case of spheres the part of the axis of rotation is placed in liquid, a vortex has the equilibrium position there if $\omega > \hbar \ln(2R/a)/2mR^2$ [1], and that is less than the critical velocity of vortex generation at the equator of the outer sphere: $\hbar \ln(2C)/2mR_2 d$. In this situation all generated vortices have their equilibrium positions in the vicinity of the axis of rotation and move to them being broken in two parts.

The same processes of vortex breaking and the opposite processes of two vortex connection happen when vortices displace to their equilibrium positions or when the metastable vortex cluster [3] expand or compress according to variations of the angular velocity.

The mechanism of these processes is shown in Fig. 1. Its left part shows what happens when a vortex approaches the inner sphere from the area $r > R_1$. At first the interaction of the vortex with the sphere («with the own image») manifests itself in the nearest part of the vortex to the equator which begins to distort. The prominent part of the vortex line and its image make «the leading pair». The sequence of the following events is represented by Fig. 1 (left): the above-mentioned part of the vortex interacting with its image stretches along the equator, moves faster and faster, approaches the wall nearer and nearer, and an-

* The report was submitted at the 3rd International Workshop on Low Temperature Microgravity Physics (CWS-2002).

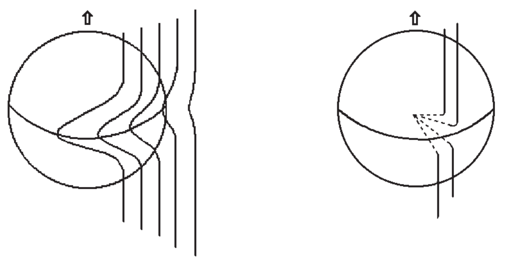

Fig. 1. The sequence of events during the breaking of a vortex moving to the axis of rotation (left), and the beginning of the connection of two vortices outgoing from the inner sphere (right).

nihilates in it. So the vortex breaks in two parts, and the edges of two remaining vortices find their equilibrium positions, not shown in Fig. 1.

The second picture of Fig. 1 (right) shows what happens when a vortex approaches the equator of the inner sphere from the area $r < R_1$. Then a vortex approaches the equator simultaneously with its continuation situated on the other side of the equator. Here, being perpendicular to the surface, the ends of the vortices form the leading pair. They move along the equator, approach each other and annihilate. The remaining parts of vortices join and form one vortex outside the inner sphere.

Thus, opposite processes of vortex breaking and connection at the equator of the inner sphere do not represent the sequence of similar events observed in reverse order. The leading pairs, their orientation, and the directions of their gathering are different.

Annihilation of outgoing vortices

The position near the equator of the outer sphere is also where the equilibrium rotation of a vortex with the vessel and the normal component is impossible. The vortex may appear here, e.g., as a result of deceleration of almost freely rotating double-sphere. Then the interaction with its own image becomes decisive, and a vortex leaves the vessel moving along and to the equator. It is known that the mutual friction results in time dependence $(t_{\rm an} - t)^{1/2}$ for compressing linear dimensions: the distance from the vortex to the wall of the cylinder [4], and the radius of the ring [5] ($t_{\rm an}$ is the moment of annihilation). Freely rotating vessel responds to the changes of liquid angular momentum (which is proportional to the area outlined by the vortex and the wall) by the change of rotation velocity. The result is the self-accelerations superimposed on general deceleration of the vessel. In the case of a cylinder $d\omega/dt \propto (t_{\rm an} - t)^{-1/2}$ [4] in accordance with Packard's idea that the annihilation of vortices may be the reason of pulsar self-accelerations (starquakes) [6]. But a pulsar is a sphere, and in this case it is possible to represent the annihilating vortex as something like a small compressing ring. Then the self-acceleration also would happen but the derivative $d\omega/dt$ would be finite. However, this statement requires more detailed consideration.

Conclusions

More detailed observations of the pulsars self-accelerations, modeling experiments (like [7]), and more detailed theoretical studies are desirable to compare the date, to discuss the similarity and difference between the cylindrical and spherical models and pulsars, and to distinguish between the processes taking place in the pulsar solid crust and the neutron liquid pierced by vortices.

Acknowledgments

This work was partly supported by INTAS (Network OPEN 97-1643) and by Grant 2.17.02 of Georgian Academy of Sciences. We thank the Organizing Committee of CWS-2002 for the support of attendance of one of us (Yu.M.) at the workshop, Prof. W. Glaberson for discussions about experimental possibilities connected with the creation of freely rotating device and Prof. G. Kharadze for his interest and attention.

1. L. Kiknadze and Yu. Mamaladze, *J. Low Temp. Phys.* **127**, 271 (2002).
2. A.L. Fetter, *Phys. Rev.* **153,** 285 (1967).
3. Y. Kondo, J. Korhonen, and M. Krusius, *Proceedings of LT-19, PG.G10* (1991); Y. Kondo, A. Gongadze, Y. Pattz, Y. Korhonen, M. Krusius, and O. Lounasmaa, *Physica* **B178**, 90 (1992); A.D. Gongadze and G.E. Vachnadze, *Zh. Eksp. Teor. Fiz.* **109**, 471 (1996).
4. L.V. Kiknadze and Yu.G. Mamaladze, *Zh. Eksp. Teor. Fiz.* **75**, 607 (1978).
5. C.F. Barenghi, R.J. Donnelly, and W.F. Vinen, *J. Low Temp. Phys.* **52,** 189 (1983).
6. B.E. Packard, *Phys. Rev. Lett.* **28**, 1080 (1972).
7. J.S. Tsakadze and S.J. Tsakadze, *Usp. Fiz. Nauk* **115,** 503 (1975); *J. Low Temp. Phys.* **39,** 649 (1980); S.J. Tsakadze, *Fiz. Nizk. Temp.* **6,** 674 (1980).