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Analytical solutions of equation for the order parameter of dense superfluid neutron matter with anisotropic spin-triplet *p*-wave pairing at finite temperatures

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The previously derived equations for the components of the order parameter (OP) of dense superfluid neutron matter (SNM) with anisotropic spin-triplet *p*-wave pairing and with taking into account the effects of magnetic field and finite temperatures are reduced to the single equation for the one-component OP in the limit of zero magnetic field. Here this equation is solved analytically for arbitrary parametrization of the effective Skyrme interaction in neutron matter and as the main results the energy gap (in the energy spectrum of neutrons in SNM) is obtained as nonlinear function of temperature T and density n in two limiting cases: for low temperatures near $T = 0$ and in the vicinity of phase transition temperature $T_{c0}(n)$ for dense neutron matter from normal to superfluid state. These solutions for the energy gap are specified for generalized BSk21 and BSk24 parametrizations of the Skyrme forces (with additional terms dependent on density n) and figures are plotted on the interval $0.1n_0 < n < 2.0n_0$, where $n_0 = 0.17\ \mathrm{fm}^{-3}$ is nuclear density.

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1. Introduction

This article is a continuation of our works [1,2] devoted to theoretical study of phase transitions in dense neutron matter with generalized Skyrme forces [3,4] and anisotropic spin-triplet *p*-wave pairing of the ^{3}He–*A* type [5,6] in strong magnetic field (see also [7]). Here we shall study the same dense superfluid neutron matter (SNM) in the limit of zero magnetic field ($H = 0$) and analytical solutions will be found at finite temperatures of the single equation for the order parameter (OP) which is a consequence (at $H = 0$) from the set of equations (see (9) in [2]) for the components of OP (at $H \neq 0$) of dense SNM.

Note that this study may be interesting in connection with investigation of thermodynamic properties of dense superfluid outer cores in a majority of ordinary isolated neutron stars (non-accreting pulsars) which magnetic fields are much less in comparison with extremely strong fields of magnetars (see, *e.g.*, [8–14] and also [15] and references therein).

Moreover, recent discovery with the aid of the NASA's Chandra X-Ray Observatory of unusually fast cooling of supernova remnant in Cassiopeia A (Cas A), which is the youngest known neutron star (NS) in the Milky Way Galaxy, has attracted great attention (see, *e.g.*, [16–30] and references therein). Several authors [18–23] explain such rapid cooling of NS in Cas A during last years (since August 1999, when Chandra found point x-ray source in the Cas A, up to 2014) due to the existence of spin-triplet superfluidity of neutrons inside high-density liquid outer core of this NS. But alternative explanations for the observed rapid cooling of Cas A have also been proposed (see, *e.g.*, [24–29] and the discussion of [24] in [20,21]). This NS in Cas A is the first one whose cooling has been observed in the real time. Note also that there is, to date, no evidence for the presence of a significant magnetic field in the Cas A neutron star [20,21].

This discovery has revived interest in the problem of the correct theoretical description of neutron spin-triplet superfluidity in cores of NSs and, in particular, in dense neutron matter within different alternative theoretical methods (see, *e.g.*, [30–33] and reviews [21,34–37], references therein and also the discussion at the end of [23]).

In the present work we follow the so-called generalized Fermi-liquid approach (see, *e.g.*, review [38] and also [39–41] and references therein) which has been already used in [42] and [1,2] to describe dense SNM with anisotropic spin-triplet p-wave pairing in steady and homogeneous strong magnetic field. Previously in [42] we applied conventional Skyrme forces (see, *e.g.*, [43,44]) with only one term dependent on density n and then in [1,2] we used generalized BSk18 [3] and BSk20, BSk21 [4] Skyrme forces (with additional density dependent terms which better take into account effects of three-body forces and other properties of nuclear matter important at high densities) as interaction in SNM at sub- ($n < n_0$) and supra-saturation ($n > n_0$) densities (where $n_0 = 0.17$ fm^{-3} is nuclear density). Here we apply generalized BSk21 and BSk24 Skyrme forces [47,48] which lead to sufficiently stiff equations of state of dense pure neutron matter (NM) and are consistent (see [45–47] for details) with the recently measured values $(1.97 \pm 0.04)M_{\text{Sun}}$ and $(2.01 \pm 0.04)M_{\text{Sun}}$ for the masses of the heaviest yet observed pulsars PSR J1614–2230 [49] and PSR J0348–0432 [50]. Note that selected here BSk21 and BSk24 are most likely the best parametrizations among other generalized parametrizations of the Skyrme forces (see Conclusions in [47]) which are sufficiently accurate in calculation of neutron effective mass (which is strongly density dependent). It is particularly important because the magnitude of the energy gap in SNM (in the energy spectrum of neutrons in SNM) is very sensitive not only to the strength of attractive forces but also to the effective mass of a neutron (see, *e.g.*, review [34] and references therein).

We write down below the equation for the OP which in the limit of zero magnetic field is a particular case of the set of equations for the components $\Delta_\downarrow \neq \Delta_\uparrow \neq 0$ (at $H \neq 0$; see Eqs. (9) from [2]). Then we shall solve this single equation (valid for arbitrary parametrization of the Skyrme forces) by analytical methods in two limiting cases: for low temperatures near $T = 0$ and in the vicinity of phase transition (PT) temperature $T_{c0}(n)$ for dense neutron matter from normal to superfluid state (with anisotropic spin-triplet p-wave pairing of the ^{3}He–A type). These solutions are specified then for generalized BSk21 and BSk24 parametrizations of the Skyrme forces and figures for the PT temperatures and energy gap in SNM are plotted on the interval $0.1n_0 < n < 2.0n_0$. In conclusion we shall briefly discuss our main results.

2. General equation for the OP for SNM with generalized Skyrme forces between neutrons and anisotropic spin-triplet pairing in zero magnetic field

It is evident that in the absence of magnetic field ($H = 0$) the effective magnetic field in SNM equals to zero, $\xi = 0$ (see notations in [2]). In this case the components of the OP $\Delta_{\uparrow(\downarrow)}(T, \xi = 0)$ for SNM with spin-triplet anisotropic p-wave pairing of the ^{3}He–A type coincide to each other:

$$\Delta_\uparrow(T, \xi = 0) = \Delta_\downarrow(T, \xi = 0) = \Delta(T).$$

For brevity, here and below we shall not write down density n explicitly as the second argument of the function $\Delta(T)$. It is obvious now that the set of two equations (see (9) from [2]) for the components of the OP is reduced to the following equation for determination of $\Delta(T)$:

$$\Delta(T) = -\Delta(T)\frac{c_3}{8\pi^2\hbar^3}J(T). \quad (1)$$

Here $c_3 \equiv t_2'(n)/\hbar^2 < 0$ is coupling constant leading to spin-triplet p-wave pairing of neutrons, which is expressed through the generalized parameters dependent on density:

$$t_2'(n) = t_2(1 + x_2) + t_5(1 + x_5)n^\gamma \quad (2)$$

(see (5) and details from [2] and also [3,4]) of the Skyrme interaction. Double integral $J(T)$ is defined as follows:

$$J(T) = \int_{p_{\min}}^{p_{\max}} dq q^4 \int_0^1 dx (1 - x^2) \frac{\tanh(E(q, x^2; T)/2T)}{E(q, x^2; T)}. \quad (3)$$

Here $p_{\max} = p_F\sqrt{1+a}$, $p_{\min} = p_F\sqrt{1-a}$ with cutoff parameter $0 < a < 1$, where $a = E_c/\varepsilon_F(n)$; E_c is the cutoff energy, $\varepsilon_F(n) = p_F^2/2m^*$ and p_F are the Fermi energy and momentum; m^* is the neutron effective mass dependent on density n of NM and on the generalized Skyrme parameters $t_1'(n)$ and $t_2'(n)$ according to general formula (10) from [2] (see also (29) and (30) here below). The function $E(q, x^2; T)$ is the energy of quasiparticles (neutrons) in SNM with anisotropic spin-triplet pairing of the ^{3}He–A type and it has the form

$$E(q, x^2; T) = \sqrt{q^2\Delta^2(T)(1 - x^2) + z^2(q)}, \quad (4)$$

where

$$z(q, T) = q^2/2m^* - \mu(T) \approx \varepsilon(q) - \varepsilon_F(n) = z(q)$$

($\mu(T)$ is the chemical potential which is substituted approximately by the Fermi energy at low temperatures in SNM, $0 < T < T_{c0}(n) \ll \varepsilon_F(n)$).

It will be more convenient to use another integral j which is related with $J(T)$ by the following formula:

$$J(T) \equiv m^* p_F^3 j(T, \delta(T); a). \quad (5)$$

Then Eq. (1) for the function $\Delta(T)$ with account of (5) gets the following final form:

$$1 = -c_3 \frac{3n}{8} m^*(n) j(T, \delta(T); a), \tag{6}$$

where

$$\delta(T) \equiv \frac{\varepsilon_F}{2m^* \Delta^2(T)} = \left(\frac{\varepsilon_F}{p_F \Delta(T)} \right)^2 >> 1. \tag{7}$$

Note that the function $p_F \Delta(T) \equiv G_F(T, n)$ is the maximal value of the anisotropic energy gap in the energy spectrum (4) of neutrons in SNM. In the next sections we shall solve this basic nonlinear integral Eq. (6) for determination of reduced energy gap $g(T, n) \equiv G_F(T, n) / \varepsilon_F(n) = = 1/\sqrt{\delta(T, n)}$ by analytical methods in two limiting cases: near $T = 0$ and close to the PT temperature T_{c0} without specifying of the parametrization of the generalized Skyrme forces. Then we shall select parametrization of the Skyrme forces in order to plot figures for obtained solutions.

3. Solution of equation for the OP for SNM near $T = 0$

Here we shall consider SNM at low temperatures, when $0 < T << T_{c0}(n) << \varepsilon_F(n)$. In this case integral $j(T, \delta(T); a)$ in (6) can be approximated as the difference:

$$j(T, \delta(T); a) \approx j_0(\delta(T); a) - 2 j_1(T, \delta(T); a), \tag{8}$$

where integrals $j_0(\delta(T); a)$ and $j_1(T, \delta(T); a)$ have the following explicit form:

$$j_0(\delta(T); a) \equiv \sqrt{\delta(T)} \int_{-a}^{a} dy (1+y) \int_0^1 dx \frac{1 - x^2}{\sqrt{b^2(y, \delta(T)) - x^2}}, \tag{9}$$

$$j_1(T, \delta(T); a) \equiv \sqrt{\delta(T)} \int_{-a}^{a} dy (1+y) \int_0^1 dx (1 - x^2) \times \frac{\exp\left[-A(y, T, \delta(T)) \sqrt{b^2(y, \delta(T)) - x^2} \right]}{\sqrt{b^2(y, \delta(T)) - x^2}}. \tag{10}$$

Here functions $b(y, \delta(T))$ and $A(y, T, \delta(T))$ are defined as follows:

$$b^2(y, \delta(T)) \equiv 1 + \frac{y^2}{1+y} \delta(T), \tag{11}$$

$$A(y, T, \delta(T)) \equiv \frac{\varepsilon_F}{T} \sqrt{\frac{1+y}{\delta(T)}} \equiv \frac{\sqrt{1+y}}{\eta(T)} >> 1. \tag{12}$$

As a result of analytical calculations we have obtained expressions for the integrals j_0 and j_1. Namely, for the j_0 the following exact formula is valid:

$$\begin{aligned} j_0(\delta; a) = \sqrt{\delta} \frac{a}{2} \Bigg[& \left(1 + \frac{a}{2} - \frac{a^2}{3} \delta \right) \arcsin\left(\frac{1}{b(a, \delta)} \right) + \left(1 - \frac{a}{2} - \frac{a^2}{3} \delta \right) \arcsin\left(\frac{1}{b(-a, \delta)} \right) \Bigg] + \frac{\delta}{6} a^2 (\sqrt{1+a} + \sqrt{1-a}) + \\ & + \frac{1}{9} \left[(11 + \frac{5}{4} a) \sqrt{1+a} + (11 - \frac{5}{4} a) \sqrt{1-a} - 22 \right] + \\ & + \frac{5}{12\delta} (2 - \sqrt{1+a} - \sqrt{1-a}) + \\ & + \frac{\frac{1}{3} - \frac{7}{24\delta} + \frac{5}{96\delta^2}}{\sqrt{1 - \frac{1}{4\delta}}} \ln \left[\left(\frac{1 + \sqrt{1 - \frac{1}{4\delta}}}{1 - \sqrt{1 - \frac{1}{4\delta}}} \right)^2 \times \right. \\ & \left. \times \left(\frac{1 + \frac{a}{2} - \sqrt{1+a} \sqrt{1 - \frac{1}{4\delta}}}{1 + \frac{a}{2} + \sqrt{1+a} \sqrt{1 - \frac{1}{4\delta}}} \right) \left(\frac{1 - \frac{a}{2} - \sqrt{1-a} \sqrt{1 - \frac{1}{4\delta}}}{1 - \frac{a}{2} + \sqrt{1-a} \sqrt{1 - \frac{1}{4\delta}}} \right) \right] + \\ & + \frac{1}{\sqrt{\delta}} \left(\frac{3}{4} - \frac{5}{24\delta} \right) \left[\arctan\left(\frac{\sqrt{1+a}}{a\sqrt{\delta}} \right) - \arctan\left(\frac{\sqrt{1-a}}{a\sqrt{\delta}} \right) \right]. \end{aligned} \tag{13}$$

The integral j_1 is closely approximated by the following formula which is valid at $0 < T << T_{c0} << \varepsilon_F$:

$$\begin{aligned} & j_1(T, \delta(T); a) \approx \\ & \sqrt{\delta(T)} \int_{-a}^{a} dy (1+y) \exp(-Ac) \frac{2}{A^2} \left[c + \frac{1}{A} + \left(\frac{2c^2}{A} + \frac{6}{A^2} \left(c + \frac{1}{A} \right) \right) \right] \approx \\ & \approx 8\eta^4(T) [1 + 8\eta^2(T)] + O(\eta^8), \end{aligned} \tag{14}$$

where

$$c(y, \delta(T)) \equiv \sqrt{b^2(y, \delta(T)) - 1} = \frac{|y| \sqrt{\delta(T)}}{\sqrt{1+y}} \geq 0. \tag{15}$$

Thus, with account (8) and (14) we can write down now general Eq. (6) in the following approximate form valid at low temperatures, $0 < T << T_{c0} << \varepsilon_F$:

$$1 \approx -c_3(y) \frac{3n_0}{8} y m^*(y) \left[j_0(\delta(T); a) - 16\eta^4(T)(1 + 8\eta^2(T)) \right]. \tag{16}$$

Here we have introduced reduced density $y \equiv n / n_0$ of SNM (where $n_0 = 0.17 \text{ fm}^{-3}$ is nuclear density, which plays role of the density scale factor). Function $j_0(\delta(T); a)$ (see (13)) determines the solution of Eq. (16) in the limit of zero temperature (at $H = 0$) for the required reduced energy gap $g(T, y; a) << 1$ (see after (7)). Thus, because $\eta(0) = 0$ (see definition (12)), we obtain at $T = 0$ from (16) the following expression for $g(0, y; a) \equiv G_F(0, y; a) / \varepsilon_F(y)$ (see also [42]):

$$g(0, y; a) = \exp\left[M^{(s)}(a) + \frac{2}{c_3(y) n_0 y m^*(y)} \right], \tag{17}$$

where $c_3(y) < 0$ (see note after (1) and also (2) for generalized parametrizations of the Skyrme forces in SNM). Function $M^{(s)}(a)$ which depends only on the cutoff parameter $a = E_c / \varepsilon_F(y) < 1$ is determined by the formula

$$M^{(s)}(a) \equiv 2\ln 2 - \frac{11}{6} + \frac{4+a}{3}\sqrt{1+a} + \frac{4-a}{3}\sqrt{1-a} + \\ + \frac{1}{2}\ln\left[\frac{(\sqrt{1+a}-1)(1-\sqrt{1-a})}{(\sqrt{1+a}+1)(1+\sqrt{1-a})}\right]. \quad (18)$$

In view of Eq. (17), we get from (16) the transcendental equation for the function $g(T, y; a) << 1$ (at $0 < T << T_{c0}$):

$$g(T, y; a) = \exp\left[M^{(s)}(a) + \frac{2}{c_3(y) n_0 y m^*(y)} - \right. \\ \left. -16\frac{\tau^4}{g^4(T, y; a)}\left(1 + 8\frac{\tau^2}{g^2(T, y; a)}\right)\right], \quad (19)$$

where $\tau \equiv T / \varepsilon_F(y) << 1$. Owing to the smallness of the temperature correction we get from (19) the following solution in the main approximation on the small T:

$$g(T, y; a) \approx g(0, y; a)\left\{1 - 16\left(\frac{T}{G_F(T, y; a)}\right)^4 \times \right. \\ \left. \times\left[1 + 8\left(\frac{T}{G_F(T, y; a)}\right)^2\right]\right\} \lesssim \\ \lesssim g(0, y; a)\left\{1 - 16\left(\frac{T}{G_F(0, y; a)}\right)^4\left[1 + 8\left(\frac{T}{G_F(0, y; a)}\right)^2\right]\right\}. \quad (20)$$

Note that obtained here in (20) leading power-law of temperature dependence $\sim T^4$ for the energy gap in SNM (with anisotropic spin-triplet p-wave pairing) near $T = 0$ is in qualitative accordance with the similar result obtained earlier for the superfluid ^{3}He–A (see, *e.g.*, review [51]) but it is quite different from the exponential temperature dependence of the isotropic energy gap near $T = 0$ in traditional superconductors with spin-singlet s-pairing [52,53].

4. Solution of equation for the OP for SNM near T_{c0}

Let us consider SNM in the region of temperatures close to T_{c0}, when $|T - T_{c0}| << T_{c0}$. But at the beginning we shall study the limiting case, $T \to T_{c0}$. It can be shown that the Eq. (6) in this limit is reduced to the following transcendental equation:

$$1 \approx -c_3(y)\frac{n_0 y m^*(y)}{2}\left[\ln\left(\frac{2E_c\gamma}{\pi T_{c0}}\right) + \right. \\ \left. + \frac{3}{16}\left(\frac{E_c}{\varepsilon_F(y)}\right)^2 + \frac{3}{512}\left(\frac{E_c}{\varepsilon_F(y)}\right)^4\right], \quad (21)$$

where $\gamma = e^C \approx 1.781072418$ ($C = 0.5772156649...$ is Euler's constant). Here in [...] we neglected by small terms $O(T_{c0} / \varepsilon_F)^2$. We get from (21) the following approximate solution for the PT temperature $T_{c0}(y; E_c)$ of SNM:

$$T_{c0}(y; E_c) \approx E_c\frac{2\gamma}{\pi}\exp\left[\frac{2}{c_3(y) n_0 y m^*(y)} + \right. \\ \left. + \frac{3}{16}\left(\frac{E_c}{\varepsilon_F(y)}\right)^2 + \frac{3}{512}\left(\frac{E_c}{\varepsilon_F(y)}\right)^4\right]. \quad (22)$$

Note that pre-exponential numerical factor here, $2\gamma / \pi \approx 1.134$, is somewhat more refined in comparison with analogous expressions [1,2] for PT temperature of SNM.

Now we define reduced PT temperature $t_{c0}(y; a)$ of SNM:

$$t_{c0}(y; a) \equiv \frac{T_{c0}(y; a)}{\varepsilon_F(y)} << 1$$

and then using obtained expression (17) for the reduced energy gap $g(0, y; a)$ we find as a result the following ratio for these functions:

$$\frac{g(0, y; a)}{t_{c0}(y; a)} \approx \frac{\pi}{2\gamma}\exp\left(\frac{5}{6}\right) = \frac{\pi}{2}\exp\left(\frac{5}{6} - C\right). \quad (23)$$

This ratio is "universal" because it does not depend neither on the cutoff parameter $a < 1$ nor on the nature of interaction in the Fermi superfluid with anisotropic spin-triplet p-wave pairing (in particular (23) is valid for arbitrary parametrizations of the Skyrme forces in SNM) and it exactly coincides with analogous ratio for the superfluid ^{3}He–A phase (see, *e.g.*, [5]). The ratio (23) depends only on the symmetry of the OP of superfluid system.

Now in order to solve Eq. (6) for SNM at temperatures $|T - T_{c0}(n)| << T_{c0}(n)$ we rewrite it as follows:

$$\ln\left(\frac{T_{c0}}{T}\right) = \frac{3}{4}\sqrt{\delta(T)}\int_{-a}^{a} dy(1+y)\int_0^1 dx(1-x^2) \times \\ \times\left[\frac{\sqrt{1+y}\tanh\left(\frac{\varepsilon_F}{2T}y\right)}{\sqrt{\delta(T)}y} - \frac{\tanh\left(\frac{A(y)}{2}\sqrt{b^2(y, \delta(T)) - x^2}\right)}{\sqrt{b^2(y, \delta(T)) - x^2}}\right], \quad (24)$$

where we have used (21) and neglected by the small terms $O(T_{c0} / \varepsilon_F)^2$. Functions $A(y, T, \delta(T))$, $b^2(y, \delta(T))$ and $\delta(T) >> 1$ are defined by formulas (12), (11) and (7), respectively. From (24) we obtain in the main approximation on small parameter $1/\delta(T) \equiv g^2(T) << 1$ the following approximate equation:

$$\ln\left(\frac{T_{c0}}{T}\right) \approx -\frac{1}{5\delta(T)}\int_0^a dy \frac{(1-y)^{5/2}+(1+y)^{5/2}}{y}\times$$
$$\times\frac{d}{dy}\left(\tanh\left(\frac{\varepsilon_F}{2T}y\right)\Big/ y\right). \quad (25)$$

Let us use the following expansion into a series [52,53]:

$$\tanh\left(\frac{x}{2}\right) = 4x\sum_{n=0}^{\infty}\frac{1}{[\pi^2(2n+1)^2+x^2]}. \quad (26)$$

Substituting this in Eq. (25) we obtain as a result of calculations the final approximate equation (see also note after (7)) valid at $|T-T_{c0}(n)| << T_{c0}(n)$:

$$\ln\left(\frac{T_{c0}(n)}{T}\right) \approx \frac{1}{\delta(T)}\left(\frac{\varepsilon_F}{T}\right)^2\frac{7\zeta(3)}{10\pi^2} = \left(\frac{G_F(T,n)}{T}\right)^2\frac{7\zeta(3)}{10\pi^2} \quad (27)$$

($\zeta(x)$ is the Riemann zeta function). It is obvious from (27) that the energy gap has the form

$$G_F(T,n) \approx T\sqrt{\frac{10\pi^2}{7\zeta(3)}\ln\left(\frac{T_{c0}(n)}{T}\right)}, \quad (28)$$

where $\sqrt{10\pi^2/[7\zeta(3)]} \approx 3.4248$. It is in accordance with analogous result [5] for ^{3}He–A but at the same time (28) is more accurate than in [5], where $T\sqrt{\ln(T_{c0}/T)}$ is approximated by $T_{c0}\sqrt{1-T/T_{c0}}$ (note here that such temperature dependence of the energy gap in the vicinity of T_{c0} is consistent with Landau's theory of second-order phase transitions; see, *e.g.*, Appendix II in [52]). Moreover, for SNM (with spin-triplet anisotropic p-wave pairing and with generalized parametrizations of the Skyrme forces) density profile of PT temperature $T_{c0}(n)$ is essentially different than in ^{3}He–A and it will be evident in the next section.

5. Solutions of equation for the OP for SNM with generalized Skyrme forces near $T = 0$ and close to T_{c0} and their density and temperature profiles

Formulas (17), (19), (20), (22), (28) contain the effective mass of neutron m_n^*, which depends on the density $n \equiv yn_0$ of NM as in [2]:

$$\frac{m}{m_n^*} = 1+\frac{myn_0}{4\hbar^2}[t_1'(n)+3t_2'(n)], \quad (29)$$

where $m \approx (m_p+m_n)/2 \approx 938.91897$ MeV / c^2 is mean value of free nucleon mass. Generalized parameters

$$t_1'(n) = t_1(1-x_1)+t_4(1-x_4)n^{\beta}, \quad (30)$$

and $t_2'(n)$ (see (2)) have specific numerical values for each Skyrme parametrization. For NM with the best BSk21 and BSk24 generalized parametrizations [4,47] of the Skyrme forces we have from (29) that

$$m^*_{n,BSk21}(y) \approx$$
$$\approx \frac{m}{1+y(3.97930y^{1/12}+0.0422618\sqrt{y}-3.89571)}, \quad (31)$$

$$m^*_{n,BSk24}(y) \approx$$
$$\approx \frac{m}{1+y(3.97930y^{1/12}+0.0422618\sqrt{y}-3.89025)}, \quad (32)$$

and the Fermi energies of NM for the BSk21 and BSk24 Skyrme forces have the following forms (which are close to each other because the parameters of the two forces are very similar; see Fig. 1):

$$\varepsilon_{F,BSk21}(y) \approx y^{2/3}[1+y(3.97930y^{1/12}+$$
$$+0.0422618\sqrt{y}-3.89571)]\cdot 60.902 \text{ (MeV)}, \quad (33)$$

$$\varepsilon_{F,BSk24}(y) \approx y^{2/3}[1+y(3.97930y^{1/12}+$$
$$+0.0422618\sqrt{y}-3.89025)]\cdot 60.902 \text{ (MeV)}. \quad (34)$$

In zero magnetic field $H = 0$ from general formula (22) (see also (29)–(34) and (2)) it follows as the particular results the expressions for PT temperatures of dense NM (with BSk21 and BSk24 Skyrme parametrizations) to SNM with anisotropic spin-triplet pairing of ^{3}He–A type:

$$T_{c0,BSk21}(E_c;y) \approx$$
$$\approx \frac{2\gamma}{\pi}E_c\exp\left[\frac{3}{16}\left(\frac{E_c}{\varepsilon_{F,BSk21}(y)}\right)^2+\frac{3}{512}\left(\frac{E_c}{\varepsilon_{F,BSk21}(y)}\right)^4\right]\times$$
$$\times\exp\left[\frac{1+y(3.97930y^{1/12}+0.0422618\sqrt{y}-3.89571)}{y(2.65286y^{1/12}-2.85028)}\right], \quad (35)$$

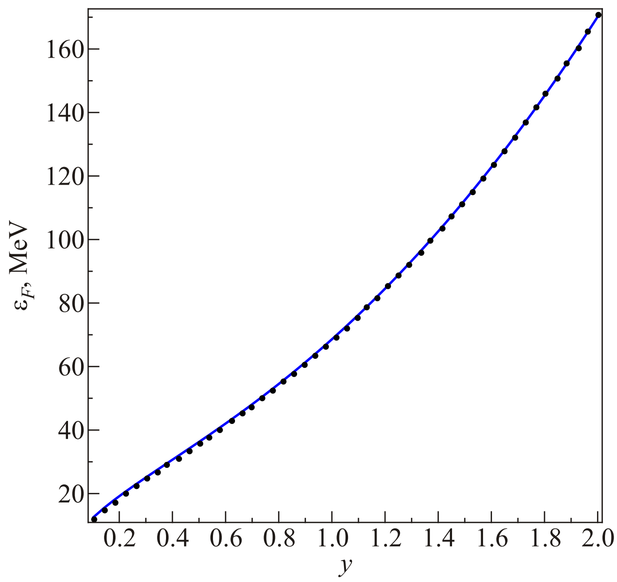

Fig. 1. Fermi energies for SNM (see (33) and (34)) with BSk21 (line) and BSk24 (points) Skyrme forces as the functions of reduced density $y = n/n_0$ are close to each other.

$$T_{c0,BSk24}(E_c;y) \approx \frac{2\gamma}{\pi} E_c \exp\left[\frac{3}{16}\left(\frac{E_c}{\varepsilon_{F,BSk24}(y)}\right)^2 + \right.$$
$$\left. + \frac{3}{512}\left(\frac{E_c}{\varepsilon_{F,BSk24}(y)}\right)^4\right] \times$$
$$\times \exp\left[\frac{1+y(3.97930y^{1/12}+0.0422618\sqrt{y}-3.89025)}{y(2.65286y^{1/12}-2.84870)}\right] \quad (36)$$

(here E_c is the cutoff energy which is less than Fermi energies, $E_c < \varepsilon_{F,BSk24}(y)$ and $E_c < \varepsilon_{F,BSk21}(y)$). Compare improved formula (35) (see note after (22)) with analogous formula (16) from [2] for $T_{c0,BSk21}(E_c;y)$.

If for the definiteness, we select cutoff energy $E_c = 10$ MeV (so that $E_c < \varepsilon_{F,BSk21}(y)$ and $E_c < \varepsilon_{F,BSk24}(y)$, see Fig. 1) it is easy to plot figures (see Figs. 2 and 3) for the PT temperatures (35), (36) of NM at sub- and supra-saturation densities on the interval $0.1n_0 < n < 2.0n_0$.

6. Conclusion

Thus, we can conclude that temperature dependence $(\sim (T/T_{c0})^4 \ll 1$, see (20)) of the energy gap in superfluid of the ^{3}He–A type near $T = 0$ and close to $T_{c0}(n)$ (see (28)) is determined only by the symmetry of the OP and doesn't depend on the nature of interactions which lead to the spin-triplet Cooper pairing in the system. But as we can see from Figs. 2–7 the density dependences of the PT temperature $T_{c0,BSk}(E_c;y)$ and the energy gap in SNM are significantly different than in the superfluid ^{3}He–A [5].

Note also that obtained here general formula (22) for PT temperature $T_{c0,BSk}(E_c;y)$ of dense NM (in zero magnetic field) to superfluid state with anisotropic p-wave pairing of ^{3}He–A type and with generalized Skyrme interactions [4,47] depends on density in nonmonotone way ((35) and (36) exhibit a bell-shaped density profile, see Fig. 2). Such behavior of these PT temperatures $T_{c0,BSk21}(10;y)$ and $T_{c0,BSk24}(10;y)$ and their maximal values are in qualitative agreement with results of recent articles [18,19,30] and are of the same order in magnitude at $E_c = 10$ MeV (namely, max $(T_{c0,BSk21}(10;y)) \approx 0.063$ MeV and max$(T_{c0,BSk24}(10;y)) \approx 0.060$ MeV, see Fig. 3).

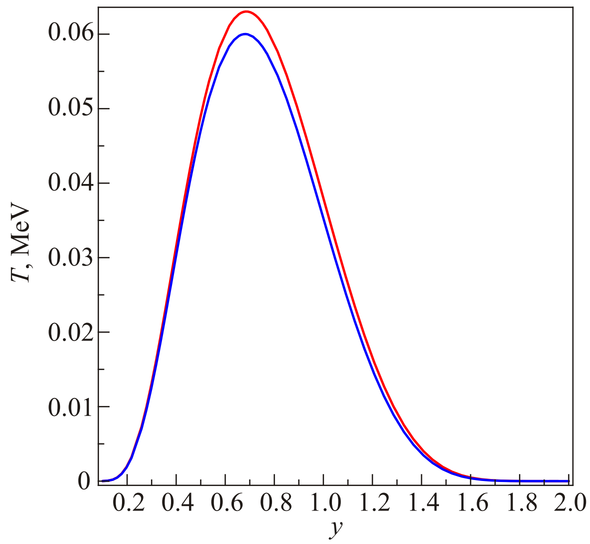

Fig. 2. PT temperatures of SNM with generalized BSk21 and BSk24 Skyrme forces (see (35) and (36) at $E_c = 10$ MeV): $T_{c0;BSk21}(10;y)$ (upper curve); $T_{c0;BSk24}(10;y)$ (lower curve).

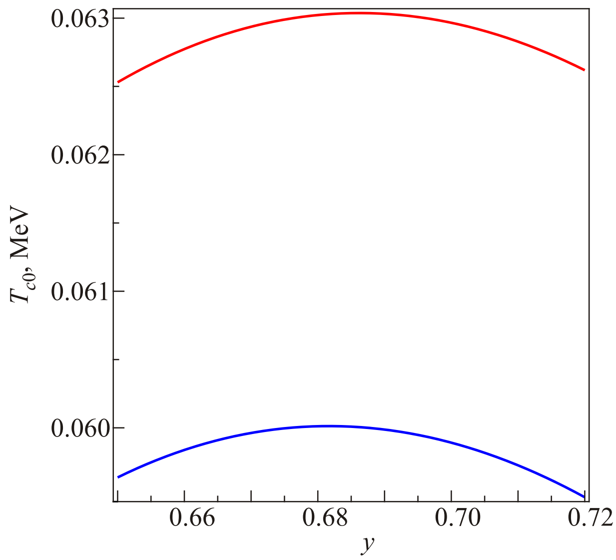

Fig. 3. The same PT temperatures of SNM as in Fig. 2 with BSk21 and BSk24 forces at $E_c = 10$ MeV near their maxima: $T_{c0;BSk21}(10;y)$ (upper curve); $T_{c0;BSk24}(10;y)$ (lower curve).

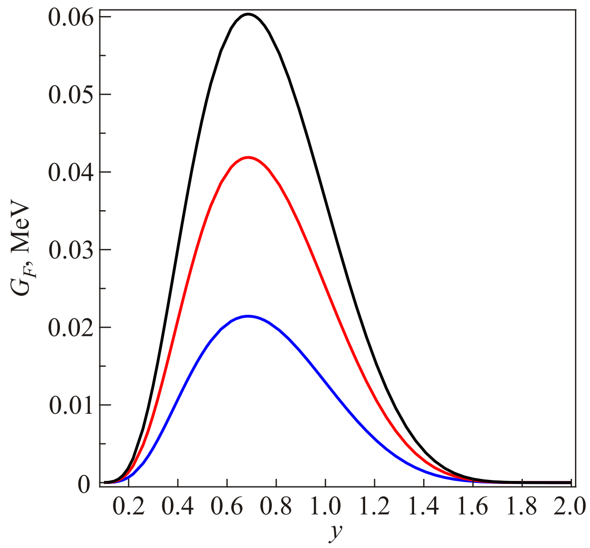

Fig. 4. Energy gap of SNM $G_F(T,y;10)$ (see (28)) with BSk21 Skyrme force and with anisotropic spin-triplet p-wave pairing (in zero magnetic field, $H = 0$) as a function of reduced density $y = n/n_0$ at three temperatures near $T_{c0,BSk21}(10;y)$ (see (35) with cutoff energy $E_c = 10$ MeV): at $T = 0.91T_{c0,BSk21}(10;y)$ (upper curve), at $T = 0.96T_{c0,BSk21}(10;y)$ (middle curve) and at $T = 0.99T_{c0,BSk21}(10;y)$ (bottom curve).

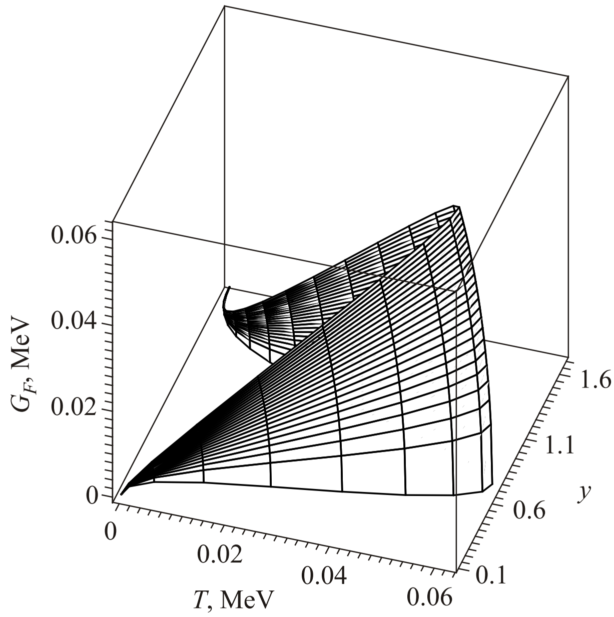

Fig. 5. The same as in Fig. 4 energy gap $G_F(T, y;10)$ of SNM (see (28)) with BSk21 Skyrme force (at $E_c = 10$ MeV and $H = 0$) but as 3d function of temperature (near $T_{c0,BSk21}(10; y)$) and reduced density $y = n / n_0$.

Note finally, that results (20), (28) for energy gap are general and figures similar to Figs. 4–7 are valid also for SNM with BSk24 parametrization of the generalized Skyrme forces (proposed recently in [47,48]) and they are close with BSk21 (it is clear from (20), (28) and from Figs. 1, 2 for Fermi energies (33), (34) and PT temperatures (35), (36) for BSk21 and BSk24 which are close to each other).

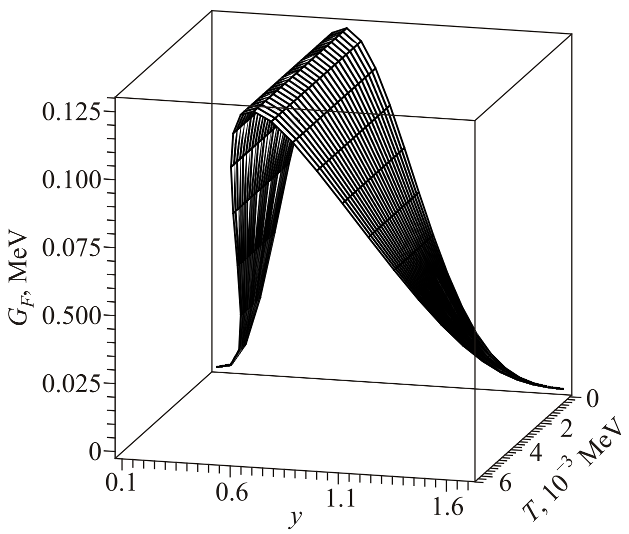

Fig. 6. Energy gap $G_F(T, y;10)$ of SNM (see (20)) with BSk21 Skyrme force (at $E_c = 10$ MeV and $H = 0$) as 3d function of temperature (near $T = 0$) and reduced density $y = n / n_0$.

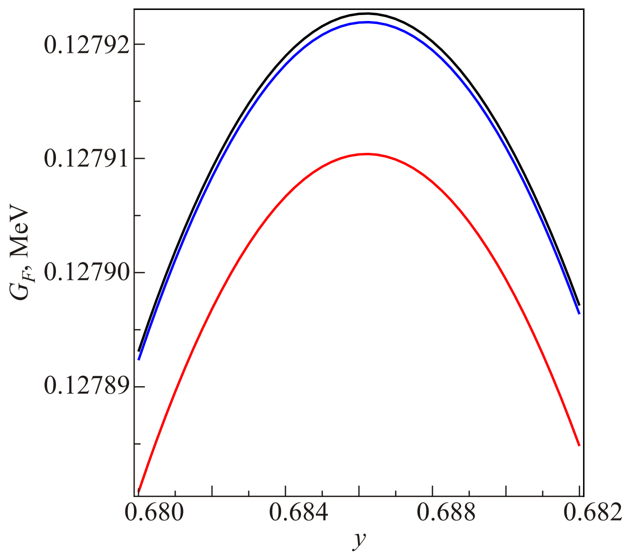

Fig. 7. Energy gap $G_F(T, y;10)$ of SNM (see (20)) with BSk21 Skyrme force and with anisotropic spin-triplet *p*-wave pairing (in zero magnetic field, $H = 0$) as a function of reduced density $y = n / n_0$ at three temperatures near $T = 0$ (see (23) and (35) with cutoff energy $E_c = 10$ MeV): at $T = 0$ (upper curve), at $T = 0.05T_{c0,BSk21}(10; y)$ (second curve) and $T = 0.1T_{c0,BSk21}(10; y)$ (bottom curve) in the vicinity of their maxima.

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