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The classical two-sublattice Landau–Lifshitz–Bloch equation for all temperatures

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Micromagnetic modeling has proved itself as a widely used tool, complimentary in many respects to experimental measurements. The Landau–Lifshitz equation provides a basis for this modeling, especially where the dynamical behaviour is concerned. However, this approach is strictly valid only for zero temperature and for high temperatures must be replaced by a more thermodynamically consistent approach such as the the Landau–Lifshitz–Bloch (LLB) equation. Here we review the recently derived LLB equation for two-sublattice systems and extend its derivation for temperatures above the Curie temperature. We present comparison with many-body atomistic simulations and show that this equation can describe the ultra-fast switching in ferrimagnets, observed experimentally.

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1 Introduction

The importance of the Landau–Lifshitz (LL) equation for modelling of magnetic materials is difficult to overestimate since it is the benchmark equation for static and dynamical simulations both in fundamental and applied nanomagnetism. Recent advances in synchrotron measurement techniques and Kerr magneto-optics [1] have allowed to measure the magnetization dynamics which is almost always modeled by the LL equation [2]. Magnetic recording simulations is also based on the LL approach to simulate the recording dynamics. However, the standard micromagnetics is essentially a zero-temperature approach although the temperature dependence can be included in macroscopic parameters such as the saturation magnetization or anisotropy. Temperature fluctuations can be also included in LL-based micromagnetics, following W.F. Brown [3], leading to the stochastic LL equation. However, this approach is known to seriously overestimate the Curie temperature [4].

Several recent technological applications such as heat-assisted magnetic recording, thermally assisted MRAM or spincaloritronics have shown the need to generalize the micromagnetic approach for high temperatures. This need was especially stimulated by the discovery of the ultra-fast magnetization dynamics [5] and its pure heat-driven origin [6,7].

The standard LL equation cannot be used at the micromagnetic level for temperatures close to the Curie temperature basically due to the fact that it conserves the magnetization magnitude, effectively truncating the high-frequency spin waves, thereby neglecting the fluctuations of the magnetization length at high temperatures. A more thermodynamically consistent approach was introduced by D. Garanin [8,9] who derived the Landau–Lifshitz–Bloch (LLB) equation for ferromagnets. The derivation has two counterparts: (i) the classical derivation assuming atomic localised spins governed by the stochastic LL equation-based dynamics and the Fokker–Planck equation [8] (ii) the quantum derivation assuming paramagnetic spin interacting with a phonon environment within the density matrix approach [9]. In both cases the ferromagnetic character was taken into account within the mean-field approximation (MFA). The LLB equation essentially interpolates between

the dynamics governed by the LL equation at low temperatures and by the Landau–Ginzburg free energy near the critical temperature. In comparison to the LL equation it contains an additional term, responsible for longitudinal relaxation of the magnetization magnitude, which is infinitely fast at low temperatures (and thus the LLB equation reduces to the LL one) and slows down at high temperatures, especially approaching the Curie temperature. LLB-based micromagnetics has been successfully tested via the corresponding atomistic large-scale simulations [10,11] and was successfully used to model the ultra-fast magnetization dynamics in ferromagnets such as Ni [12], Gd [13], and FePt [14].

One should also note an alternative macroscopic approach suggested by V. Bar'yakhtar [15], based on the symmetry principle which was recently also used to model ultra-fast dynamics [16]. Also the M3TM model [17], proposed by Koopmans as well as the self-consistent Bloch (SCB) equation [18] were used for this purpose. It should be noted that from the quantum LLB equation with spin $S = 1/2$ one can recover the SCB and M3TM equations [19]. The LLB equation could be further generalized to take into account additional terms like the inhomogeneous exchange field and, thus, it could be put in the form similar to the Bar'yakhtar one.

From the point of view of ultra-fast dynamics, two-component materials such as ferrimagnetic alloys like GdFeCo [20], TbFeCo [21], TbFe [22] are especially important, since an all optical switching (AOS) has been observed in these materials using an intense ultrashort pulse of circularly polarized light [23] and linearly polarized light [20]. Although initially it was thought that the inverse Faraday effect is solely responsible for the magnetization reversal in these materials [23], later it was shown that the reversal also occurs without its presence [20]. The reversal was explained [6,7] by pure thermodynamical reasons, involving the angular momentum transfer between the two sublattices [24,25]. More recently, T. Ostler *et al.* [6] used a multi-spin atomistic approach based on the Heisenberg model showing that the switching occurs without any applied field or even with the field up to 40 T applied in the opposite direction. The predictions for the heat-driven reversal were confirmed in several experiments in magnetic thin films and dots using linearly polarized pulses. [26–28] Furthermore, I. Radu *et al.* [20] used the same atomistic model for the magnetization dynamics to simulate GdFeCo and compared the simulation results to the experimental data measured by the element-specific x-ray magnetic circular dichroism (XMCD). They unexpectedly found that the ultrafast magnetization reversal in this material, where spins are coupled antiferromagnetically, occurs by way of a transient ferromagnetic-like state. The different ultra-fast spin dynamics in multi-component alloys has been also observed in ferromagnetic materials such as permalloy FeNi [29].

The latter experiments demonstrated the necessity to have a macroscopic equation where the different magnetization dynamics of the sublattices and their ultra-fast angular momentum transfer and relaxation can be taken into account separately. One of the first modeling of ultra-fast angular momentum transfer has been done by Mentink *et al.* [24] using the Bar'yakhtar equation for ferrimagnets. Initially this equation took into account only the longitudinal relaxation processes but more recently a more complete treatment to include the precessional motion was also presented [30].

Simultaneously, the LLB equation for ferrimagnetic alloys have been derived by U. Atxitia *et al.* [31]. Here we present a review of this classical equation and generalise it for two-component magnets (including the ferromagnetic two-sublattice systems). Additionally, in the present work we extend its derivation for temperatures above the Curie temperature. In order to show the viability of the equation we present its comparison with many-body atomistic simulation. We also show that the equation successfully describes the switching process in ferrimagnetic alloys.

2. The LLB equation for two-component magnets

2.1. General derivation

The derivation of the classical LLB for two sublattice magnets has the same starting point as for the ferromagnetic classical LLB equation [8]. Namely, we start with the usual atomistic approach where it is assumed that the dynamics of the atomic magnetic moment $\boldsymbol{\mu}_i = \mu_i \mu \mathbf{s}_i$ is governed by the Langevin dynamics based on the stochastic LL equation, which in terms of the unitary vector $\mathbf{s}_i$ reads

$$\frac{d\mathbf{s}_i}{dt} = \gamma\left[\mathbf{s}_i \times (\mathbf{H}_i + \boldsymbol{\zeta}_i)\right] - \gamma\lambda[\mathbf{s}_i \times [\mathbf{s}_i \times (\mathbf{H}_i + \boldsymbol{\zeta}_i)]], \quad (1)$$

where λ is the coupling to the bath parameter, γ is the giromagnetic ratio, $\mathbf{H}_i$ is the external magnetic field and the components of the stochastic Langevin field $\boldsymbol{\zeta}(t)$ are given by

$$\langle \zeta_i^{\alpha}(t)\zeta_j^{\sigma}(t')\rangle = \frac{2\lambda T}{\gamma\mu}\delta_{\alpha\sigma}\delta_{ij}\delta(t-t') \quad (2)$$

where the indices i and j stand for spin numbers and the indices α and σ for their components x, y, z and T is the bath's temperature.

The Fokker–Planck equation [32] corresponding to many-spin Eq. (1) was calculated in Ref. 8. Using the evolution of the probability function, governed by this equation, one obtains an equation for thermal average of the spin polarisation, i.e. the reduced magnetization $\mathbf{m}_{\nu} = \langle \mathbf{s}_i \rangle_{i\in\nu}$ (where ν denotes the sublattice) in a paramagnetic state. For the treatment of ferro (ferri) magnet, the external field is substituted by the mean field. A detailed discussion about the mean-field approximation (MFA) for a disordered ferrimagnet can be found in Ref. 33.

The corresponding set of coupled LLB equations for each sublattice magnetization $\mathbf{m}_\nu$ has the following form, see details in Ref. 31:

$$\frac{d\mathbf{m}_\nu}{dt} = -\gamma_\nu[\mathbf{m}_\nu \times \mathbf{H}_\nu^{MFA}] - \Gamma_{\nu,\|}\left(1 - \frac{\mathbf{m}_\nu \mathbf{m}_{0,\nu}}{m_\nu^2}\right)\mathbf{m}_\nu - \\ - \Gamma_{\nu,\perp}\frac{[\mathbf{m}_\nu \times [\mathbf{m}_\nu \times \mathbf{m}_{0,\nu}]]}{m_\nu^2} \quad (3)$$

where

$$\mathbf{m}_{0,\nu} = L(\xi_\nu)\frac{\boldsymbol{\xi}_\nu}{\xi_\nu}, \quad \boldsymbol{\xi}_\nu \equiv \beta\mu_\nu \mathbf{H}_\nu^{MFA}. \quad (4)$$

Here $\xi_\nu \equiv |\boldsymbol{\xi}_\nu|$, $L(\xi) = \coth(\xi) - 1/\xi$ is the Langevin function, $\beta = 1/k_B T$, k_B is the Boltzmann constant and

$$\Gamma_{\nu,\|} = \Lambda_{\nu,N}\frac{L(\xi_\nu)}{\xi_\nu L'(\xi_\nu)}, \quad \Gamma_{\nu,\perp} = \frac{\Lambda_{\nu,N}}{2}\left(\frac{\xi_\nu}{L(\xi_\nu)} - 1\right) \quad (5)$$

describe parallel and perpendicular relaxation, respectively, $\Lambda_{\nu,N} = 2\gamma_\nu \lambda_\nu k_B T/\mu_\nu$ is the characteristic diffusion relaxation rate or, for the thermo-activation escape problem, the Néel attempt frequency. $L'(\xi) = dL/d\xi$ is the derivative of the Langevin function. The mean fields have the following forms:

$$\mathbf{H}_\nu^{MFA} = \frac{J_{0,\nu}}{\mu_\nu}\mathbf{m}_\nu + \frac{J_{0,\nu\kappa}}{\mu_\nu}\mathbf{m}_\kappa + \mathbf{h}_\nu \quad (6)$$

where $J_{0,\nu} = x_\nu z J_{\nu\nu}$, $J_{0,\nu\kappa} = x_\kappa z J_{\nu\kappa}$, z is the number of nearest neighbours in the ordered lattice, $J_{\nu\nu}$ and $J_{\nu\kappa}$ are the Heisenberg intra and inter-sublattice exchange interaction parameters, x_ν and $x_\kappa = 1 - x_\nu$ are the concentrations of the sublattices ν and κ, respectively. The field $\mathbf{h}_\nu = \mathbf{H} + \mathbf{H}_{a,\nu}$ contains the external applied field ($\mathbf{H}$) and the anisotropy field ($\mathbf{H}_{a,\nu}$).

In Eq. (3) the first (precession) and the last (transverse relaxation) terms have forms, similar to the LL equation and turn to it if $m_\nu = 1$ and $\mathbf{m}_{0,\nu}$ is proportional to the effective field, acting on the sublattice. The latter is true if the external (such as the field) perturbations are not varied too fast.

On the other hand, if only longitudinal processes are considered, Eq. (3) becomes

$$\frac{dm_\nu}{dt} = -\Gamma_{\nu,\|}(m_\nu - m_{0,\nu}), \quad (7)$$

which coincides with the self-consistent Bloch equation [18]. In spite of the fact that the form of Eq. (7) is similar to the well known Bloch equation, the quantity $m_{0,\nu}$ is not the equilibrium magnetization and it changes dynamically through the dependence of the mean field given by Eq. (6). Moreover, the rate parameter $\Gamma_{\nu,\|}$ contains highly nonlinear terms in $m_{0,\nu}$ and $m_{0,\kappa}$. The MFA field Eq. (6) contains the homogeneous exchange field only ($k = 0$ mode). The inhomogeneous exchange field could be also taken into account, however, normally we include it within the many-body micromagnetic approach, based on the LLB.

Finally, the equilibrium solution of Eq. (3), $m_{e,\nu}$, coincides with the self-consistent solution of the Curie–Weiss equations with the MFA field

$$\mathbf{m}_\nu = L(\xi_\nu)\frac{\boldsymbol{\xi}_\nu}{\xi_\nu}. \quad (8)$$

In the absence of external magnetic field and anisotropy field ($\mathbf{h}_\nu = 0$) one finds from Eq. (8) that the Curie temperature (T_c) of the system is given by

$$T_c = \frac{J_{0,\nu} + J_{0,\kappa} + \sqrt{|J_{0,\nu} - J_{0,\kappa}|^2 + 4J_{0,\nu\kappa}J_{0,\kappa\nu}}}{6k_B}. \quad (9)$$

Equation (3) is ready for modeling. However, for analytical estimations a more closed form of the longitudinal relaxation in the LLB equation is convenient. Particularly, it is convenient to express the final form of the LLB equation in terms of physically measurable quantities such as the longitudinal susceptibilities $\tilde{\chi}_{\nu,\|} = (\partial m_\nu/\partial H)_{H\to 0}$ which can be evaluated independently or even measured. To this end, further approximations are made. Namely, assuming that the longitudinal homogeneous exchange field is large in comparison to the other fields, h_ν, in Eq. (6) and adiabatic (quasi stationary) processes and after an expansion around the equilibrium up to the fourth order, one arrives from Eq. (3) to (see Ref. 31)

$$\frac{d\mathbf{m}_\nu}{dt} = -\gamma_\nu[\mathbf{m}_\nu \times \mathbf{H}_{\mathrm{eff},\nu}] + \gamma_\nu\alpha_\|^\nu\frac{\mathbf{m}_\nu \cdot \mathbf{H}_{\mathrm{eff},\nu}}{m_\nu^2}\mathbf{m}_\nu - \\ -\gamma_\nu\alpha_\perp^\nu\frac{[\mathbf{m}_\nu \times [\mathbf{m}_\nu \times \mathbf{H}_{\mathrm{eff},\nu}]]}{m_\nu^2}, \quad (10)$$

where the effective field is given by

$$\mathbf{H}_{\mathrm{eff},\nu} = \mathbf{H} + \mathbf{H}_{a,\nu} + \frac{J_{0,\nu\kappa}}{\mu_\nu}\boldsymbol{\Pi}_\kappa - \\ -\left[\frac{1}{\Lambda_{\nu\nu}}(m_\nu - m_{e,\nu}) - \frac{1}{\Lambda_{\nu\kappa}}(|\tau_\kappa| - |\tau_{e,\kappa}|)\right]\frac{\mathbf{m}_\nu}{m_\nu} \quad (11)$$

where $\mathbf{H}_{a,\nu}$ is the anisotropy field, $\mathbf{H}$ is the applied field,

$$\boldsymbol{\Pi}_\kappa = -[\mathbf{m}_\nu \times [\mathbf{m}_\nu \times \mathbf{m}_\kappa]]/m_\nu^2, \ \tau_\nu = (\mathbf{m}_\nu \cdot \mathbf{m}_\kappa)/m_\kappa, \\ \tau_{e,\nu} = (\mathbf{m}_{e,\nu} \cdot \mathbf{m}_{e,\kappa})/m_{e,\kappa} \text{ and}$$

$$\Lambda_{\nu\nu}^{-1} = \frac{1}{\tilde{\chi}_{\nu,\|}}\left(1 + \frac{J_{0,\nu\kappa}}{\mu_\nu}\tilde{\chi}_{\kappa,\|}\right), \quad \Lambda_{\nu\kappa}^{-1} = \frac{|J_{0,\nu\kappa}|}{\mu_\nu}. \quad (12)$$

The longitudinal susceptibility can be calculated in the MFA as [34]

$$\tilde{\chi}_{\nu,\|} = \frac{\mu_\kappa \beta L'_\nu J_{0,\nu\kappa}\beta L'_\kappa + \mu_\nu \beta L'_\nu(1 - J_{0,\kappa}\beta L'_\kappa)}{(1 - J_{0,\nu}\beta L'_\nu)(1 - J_{0,\kappa}\beta L'_\kappa) - J_{0,\kappa\nu}\beta L'_\nu J_{0,\nu\kappa}\beta L'_\kappa}. \quad (13)$$

where $L'_\nu = L'(\xi_{e,\nu})$ with $\xi_{e,\nu} = \beta(J_{0,\nu}m_{e,\nu} + |J_{0,\nu\kappa}|\, m_{e,\kappa})$. The damping parameters are

$$\alpha_{\parallel}^{\nu} = \frac{2\lambda_\nu}{\beta\tilde{J}_{0,\nu,e}}, \quad \alpha_{\perp}^{\nu} = \lambda_\nu\left(1 - \frac{1}{\beta\tilde{J}_{0,\nu,e}}\right), \tag{14}$$

where $\tilde{J}_{0,\nu,e} = J_{0,\nu} + |J_{0,\nu\kappa}|\,(m_{e,\kappa}/m_{e,\nu})$. Note that we have checked that all the expressions above are the same for two-sublattice ferrimagnetic and ferromagnetic alloys.

The LLB equation in the form given by Eq. (10) is very useful for analytic estimations and predictions since the relaxation terms have closed expressions. Particularly, one can estimate several asymptotic behaviours of the relaxation times. Namely, for relatively low temperatures and not too strongly coupled alloy we can estimate the longitudinal relaxation time as

$$\tau_\nu^{\parallel} \approx \frac{1}{2\gamma_\nu\lambda_\nu m_{\nu,e}H_{\nu,e}^{\mathrm{ex}}}, \tag{15}$$

where

$$H_{\nu,e}^{\mathrm{ex}} = \frac{\tilde{J}_{0,\nu,e}}{\mu_\nu}\, m_{e,\nu}, \tag{16}$$

is the homogeneous exchange field evaluated at the equilibrium. Note that in this approximation, the relaxation time is independent on the sign of the coupling between sublattices (ferro or antiferro).

Close to the Curie temperatures one can prove that the susceptibilities diverge, namely $\tilde{\chi}_{\nu,\parallel} \propto 1/|T - T_c|$ which is the main source for the critical slowing down of the longitudinal relaxation times near the Curie temperature. However, in weakly coupled ferrimagnets, only the material with the largest exchange value slows down at the common Curie temperature [35].

2.2. Classical LLB equation for two-component magnets above T_c

In the previous work [31] the derivation of the LLB equation given by Eq. (10) was presented for temperatures below the critical temperature T_c. However, in ultrafast magnetization dynamics the electronic temperature can easily exceed T_c. Therefore, it is interesting to analyze the applicability of Eq. (10) for temperatures above T_c. When $T \to T_c$ the longitudinal susceptibility goes to infinity ($\tilde{\chi}_{\nu,\parallel} \to \infty$) and the equilibrium magnetization to zero ($m_{e,\nu} \to 0$), as a consequence at $T = T_c$ the quantities $\Lambda_{\nu\nu}$ and the damping parameters $\alpha_{\parallel}^{\nu}$ and $\alpha_{\perp}^{\nu}$ are undefined *a priori*. However, the quantities $\Lambda_{\nu\nu}$, $\alpha_{\parallel}^{\nu}$ and $\alpha_{\perp}^{\nu}$ are continuous functions at $T = T_c$. In order to see this fact we can rewrite these quantities in a form suitable for temperatures very close to T_c. To this end we notice that

$$\tilde{\chi}_{\nu,\parallel} \simeq \begin{cases} \dfrac{\mu_\nu(3k_BT_c - J_{0,\kappa}) + \mu_\kappa J_{0,\nu\kappa}}{6k_BT_c(6k_BT_c - J_{0,\nu} - J_{0,\kappa})\varepsilon}, & T \lesssim T_c \\ \dfrac{\mu_\nu(3k_BT_c - J_{0,\kappa}) + \mu_\kappa J_{0,\nu\kappa}}{3k_BT_c(6k_BT_c - J_{0,\nu} - J_{0,\kappa})(-\varepsilon)}, & T \gtrsim T_c \end{cases} \tag{17}$$

and

$$\frac{m_{e,\kappa}}{m_{e,\nu}} \simeq \frac{3k_BT_c - J_{0,\nu}}{|J_{0,\nu\kappa}|} + \mathcal{O}(\varepsilon), \quad T \approx T_c \tag{18}$$

where $\varepsilon = (T_c - T)/T_c$. Note that it follows from Eq. (9) that $J_{0,\nu} < 3k_BT_c$.

With the help of Eqs. (17) and (18) we can rewrite Eqs. (12) and (14) as

$$\Lambda_{\nu\nu}^{-1}(T) \simeq \begin{cases} \dfrac{6k_B(T_c - T)(6k_BT_c - J_{0,\nu} - J_{0,\kappa})}{\mu_\nu(3k_BT_c - J_{0,\kappa}) + \mu_\kappa J_{0,\nu\kappa}} + \dfrac{3k_BT_c - J_{0,\nu}}{\mu_\nu}, & T \lesssim T_c \\ \dfrac{3k_BT_c - J_{0,\nu}}{\mu_\nu}, & T = T_c \\ \dfrac{3k_B(T - T_c)(6k_BT_c - J_{0,\nu} - J_{0,\kappa})}{\mu_\nu(3k_BT_c - J_{0,\kappa}) + \mu_\kappa J_{0,\nu\kappa}} + \dfrac{3k_BT_c - J_{0,\nu}}{\mu_\nu}, & T \gtrsim T_c \end{cases} \tag{19}$$

and

$$\alpha_{\parallel}^{\nu} = \alpha_{\perp}^{\nu} \simeq \frac{2\lambda_\nu T}{3T_c} + \mathcal{O}(\varepsilon), \quad T \gtrsim T_c. \tag{20}$$

Thus, above T_c the longitudinal and transverse damping parameters are equal and coincide with the expression [8] for the classical LLB equation of a ferromagnet above T_c, since at these temperatures the system becomes paramagnetic at the equilibrium.

In order to check the validity of Eq. (10) for temperatures above T_c we consider a disordered ferrimagnet like GdFeCo with parameters given in Table 1 and a rare-earth concen-

Table 1. Table with the parameters of GdFeCo used in the classical LLB equation. The exchange parameters are obtained through a renormalization of the atomistic exchange parameters given in Ref. 7 in order to obtain the same T_c as in the atomistic approach.

	FeCo	Gd	Gd–FeCo
$zJ_{\nu\kappa}$ (Joule)	$2.99\cdot10^{-20}$	$1.19\cdot10^{-20}$	$-1.04\cdot10^{-20}$
λ_ν	0.02	0.02	–
$\mu_\nu(\mu_B)$	1.92	7.63	–
d_ν (Joule)	$8.0\cdot10^{-24}$	$8.0\cdot10^{-24}$	–
γ_ν (rad·s^{-1}·Oe^{-1})	$1.76\cdot10^{7}$	$1.76\cdot10^{7}$	–

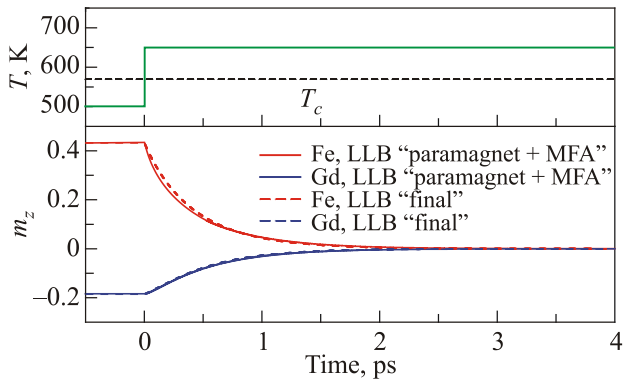

Fig. 1. (Color online) Magnetization dynamics obtained from LLB equation given by Eq. (3) (solid lines) called "paramagnet+MFA" and LLB equation given Eq. (10) (dash lines) called "final" for a temperature-step where the initial temperature is $T_0 = 500$ K which is below $T_c = 565$ K and the final temperature is $T_f = 650$ K which is above T_c.

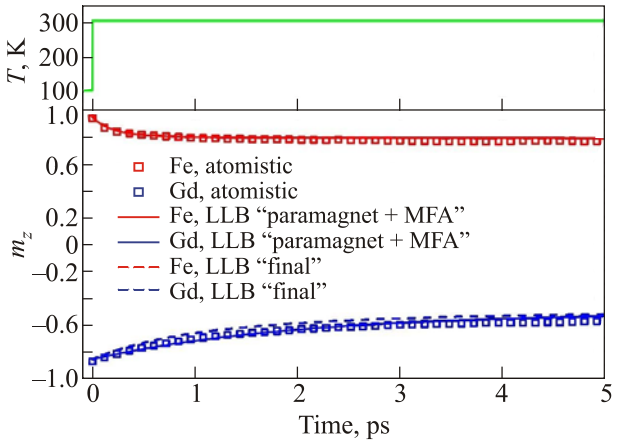

Fig. 2. (Color online) (Top) Low electronic temperature step where the initial temperature is $T_0 = 100$ K and the final temperature is $T_f = 300$ K. (Bottom) Magnetization dynamics obtained using atomistic simulation (squares), LLB equation given by Eq. (3) (solid lines) called "paramagnet+MFA" and LLB equation given Eq. (10) (dash lines) called "final" for a low temperature-step.

tration of 25% ($x_{Gd} = 0.25$). We consider zero external magnetic field ($\mathbf{H} = 0$) and uniaxial anisotropy in each sublattice given by $\mathbf{H}_{A,\nu} = 2(d_\nu / \mu_\nu) m_{\nu,z} \mathbf{e}_z$ where d_ν is the atomistic anisotropy constant. In Fig. (1) we present the calculated magnetization dynamics under a temperature-step with initial temperature $T_0 = 500$ K, below $T_c = 565$ K and the final temperature $T_f = 650$ K, above T_c. We integrate both "paramagnetic" LLB equation with the MFA expression for ferrimagnet, given by Eq. (3) and the "final" approximate ferrimagnetic LLB equation, given by Eq. (10), obtaining a good agreement. Above T_c the quantity $\alpha_{\|}^{\nu}$ was evaluated using Eq. (20), while since the final temperature was far from T_c the quantity $\Lambda_{\nu\nu}$ was evaluated using Eq. (12) with $L'_\nu = 1/3$, which comes immediately from the fact that at this temperature $m_{e,\nu} = 0$, instead of Eq. (19).

2.3. Comparison between the classical LLB equation and atomistic simulations

In this subsection we compare the classical LLB equation for two-component magnets with many-body atomistic simulations based on the stochastic LL equation (1). For this task, we perform atomistic simulations for GdFeCo ferrimagnetic compound, where the ultra-fast switching has been observed. The parameters are taken from Ref. 7. On the other hand, in the LLB model, as usual, we rescale exchange parameters in order to obtain the same critical temperatures T_c as in atomistic model. These parameters are presented in Table 1.

Firstly, we compare the magnetization relaxation dynamics under a temperature step using both approaches. In Fig. 2 we show the magnetization dynamics obtained with atomistic simulations (square), LLB equation given by Eq. (3) called "paramagnet+MFA" (solid line) and LLB equation given Eq. (10) called "final" (dash line) for an electronic temperature-step where the temperature is varied in step from $T_0 = 100$ K to $T_f = 300$ K. Both LLB equations and atomistic simulation produce the same relaxatinal dynamics. In Fig. 3 we show the same comparison but for a higher electronic temperature-step with the final temperature $T_f = 500$ K which is higher than the Curie temperature for Gd. We observe that in this case the time evolution of the magnetization obtained using atomistic simulation and the LLB equation given by Eq. (3) is very similar, however the magnetization dynamics obtained using the LLB equation given by Eq. (10) leads to a faster longitudinal relaxation of Gd. This discrepancy may be related to a large deviation from the equilibrium achieved due to

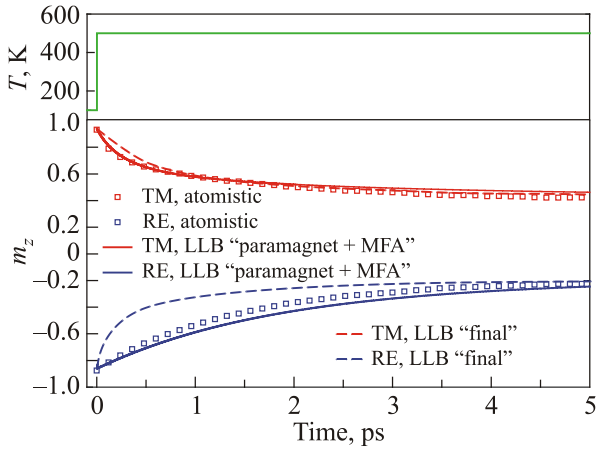

Fig. 3. (Color online) (Top) High electronic temperature step where the initial temperature is $T_0 = 100$ K and the final temperature is $T_f = 500$ K. (Bottom) Magnetization dynamics obtained using atomistic simulation (squares), LLB equation given by Eq. (3) (solid lines) called "paramagnet+MFA" and LLB equation given Eq. (10) (dash lines) called "final" for a high temperature-step.

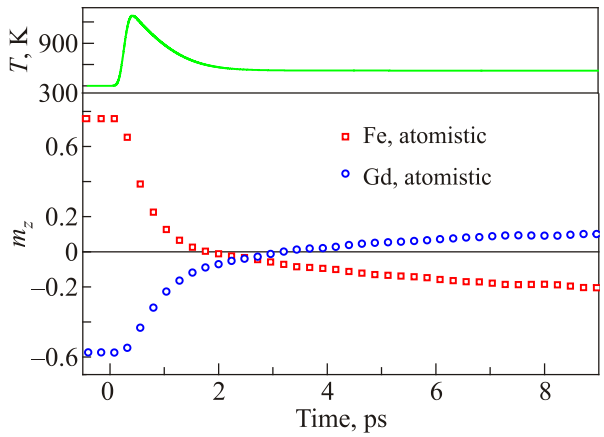

Fig. 4. (Color online) (Top) Electronic temperature dynamics (green line). (Bottom) Magnetization response to this electronic temperature dynamic obtained using atomistic simulation.

the high electronic temperature-step, in such situations the accuracy of some approximations in the derivation of Eq. (10) may be not sufficient and higher order corrections should be included.

Finally, we show that the LLB equation describes switching in ferrimagnetic FeCoGd under the laser pulse heating. The electronic temperature dynamics is modelled using the two-temperature model [36] with parameters taken from Ref. 7 and the laser pulse fluence 40 mJ/cm^2. The electronic temperature dynamics is presented in the upper panel in Fig. 4 and the switching of GdFeCo obtained by atomistic simulations is presented in the bottom panel in Fig. 4. In the LLB simulations, however, a pure longitudinal motion does not produce any torque on magnetization. This motion becomes unstable close to the point at which the magnetization of one of the sublattices is zero [25]. However, even in this unstable situation numerically there is no torque acting on the magnetization. This situation is similar to the integration of the LL equation with field parallel to

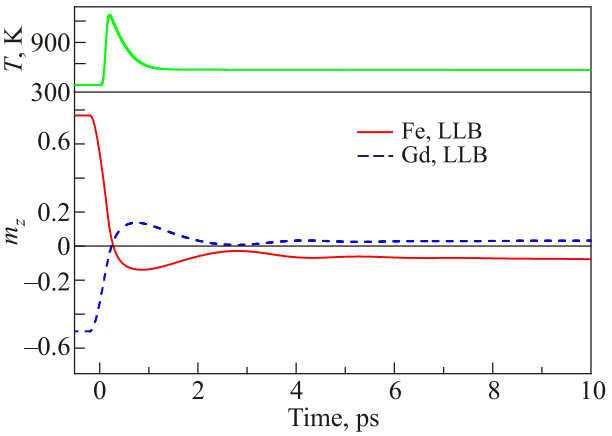

Fig. 5. (Color online) (Top) Electronic temperature dynamics (green line). (Bottom) Magnetization response to this electronic temperature dynamic obtained using the LLB equation given Eq. (3) where the lattices magnetizations are deviated by 10° from the antiparallel state.

the anisotropy where a small angle between them should be used in order to move the system from the point where the torque is zero. Similar to this, for the LLB equation one needs to use a small angle between sublattice magnetizations (or alternatively the stochastic LLB equation should be used). The results are presented in Fig. 5 where we see that a switching of the sublattices occurs using the same electronic temperature dynamics as in the atomistic approach showed in Fig. 4. Note that the stochastic nature of the switching and/or a small angle between the sublattice magnetizations naturally occur in the atomistic modelling. As was pointed out in Ref. 25, in order to switch the magnetization with a ultrashort laser pulse it is necessary that angular momentum is transferred from the longitudinal to the transverse magnetization components. This conclusion is also supported by atomistic modelling.

3. Conclusions

We presented a novel LLB equation for two-component alloys which can model the separate dynamics of their components in the whole temperature range. The new equation constitutes an important step forward in the description of the dynamics of two-coomponent alloys, such as ferrimagnets which are traditionally modelled using two coupled macroscopic LL equations. The two-component LLB equation has been already successfully applied to model FeCoGd [31] and more recently to FeNi [37] showing that sublattices have distinct dynamics, in agreement with experimental findings. Also the FMR and exchange modes in ferrimagnets and their temperature dependence are better understood within this approach [38].

These equations can serve in the future as a basis for multiscale modeling in two-component systems at high temperatures and/or ultrafast timescales, in the same way as the LLB equation for ferromagnets [39]. This also opens a possibility for novel micromagnetic modeling of ultrafast and/or temperature-driven dynamics in large structures, such as sub-micron and micron-size dots, stripes, nanowires etc., made of two-components ferro or ferrimagnetic alloys.

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