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Quantum Hall effect in p-Ge/Ge$_{1-x}$Si$_x$ heterostructures with low hole mobility

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The apparent insulator—quantum Hall—insulator (I—QH—I) transition for filling factor $\nu = 1$ has been investigated in p-type Ge/Ge$_{1-x}$Si$_x$ heterostructures with $\varepsilon_F \tau / \hbar \approx 1$. Scaling analysis is carried out for both the low- and high-field transition point. In low magnetic fields $\omega_c \tau < 1$ pronounced QH-like peculiarities for $\nu = 1$ are also observed in both the longitudinal and Hall resistivities. Such behavior may be evidence of a localization effect in the mixing region of Landau levels and is inherent for two-dimensional structures in a vicinity of the metal—insulator transition.

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Introduction

A magnetic-field-induced transition from an Anderson insulator to quantum Hall effect (QHE) conductor has been reportedly observed both for low-electron-mobility GaAs/AlGaAs heterostructures [1–4] and low-hole-mobility Ge/SiGe quantum wells [5,6], which at magnetic field $B = 0$ exhibit insulating behavior with a divergent resistance $\rho(T \to 0) >> h/e^2$. An initial very large decrease of diagonal resistivity ρ_{xx} (giant negative magnetoresistence [7]) is followed by a clear critical point at $B = B_C$ where the ρ_{xx} value is temperature independent. At higher fields the QHE minima for filling factor either $\nu = 2$ or $\nu = 1$ are developed. The insulator to QHE boundary points at $B = B_C$ are characterized by the equality of the diagonal and Hall resistivities, $\rho_{xxc} = \rho_{xyc}$, within experimental uncertainty [5]. Just the T-independent point B_C is identified by the authors of [1–6] as the quantum phase transition point between the insulator and QHE conductor.

In contrast to that, Huckestein [8] identifies the apparent low-field insulator—QHE transition as a crossover due to weak localization and a strong reduction of the conductivity when Landau quantization becomes dominant at $\omega_c \tau \geq 1$, ω_c being the cyclotron frequency and τ being the elastic mean free time.

On the other hand, for well-conducting 2D systems with $k_F l >> 1$ (k_F is Fermi quasimomentum and l is the mean free path) the interplay of classical cyclotron motion and the quantum correction $\Delta\sigma_{ee}$ due to electron—electron interaction (EEI) to the Drude conductivity $\sigma_D = (e^2/h)(k_F l)$ leads to a parabolic negative magnetoresistance [9–11]:

$$\rho_{xx}(B,T) = \frac{1}{\sigma_D} + [1 - (\omega_c \tau)^2] \frac{\Delta\sigma_{ee}(T)}{\sigma_D^2} . \quad (1)$$

The temperature independent point at $\omega_c\tau = 1$ (for $\rho_{xx} \cong \rho_{xy}$) predicted by Eq. (1) has been observed in various experiments and used for the estimation of the σ_D value (see, for example, [12–15]).

It seems for us that the results of the paper [16] of C.F. Huang et al. are an especially beautiful experimental demonstration just of this (EEI) physical picture in a gated GaAs/AlGaAs heterostructure (our estimations give $4 \le k_F l \le 13$ for five V_g values on your Fig. 2), but the authors of [16] treated the low-field T-independent point as a kind of quantum phase transition (see also [17]).

Here we report and analyze the results of magnetotransport measurements for low-mobility p-Ge/Ge$_{1-x}$Si$_x$ heterostructures, where the low-field temperature-independent point on the $\rho_{xx}(B)$ dependence is clearly observed.

Experimental results and discussion

Experimental data are presented for two samples A and B of a multilayered Ge/Ge$_{1-x}$Si$_x$ p-type heterostructures. The hole density and Hall mobility, as obtained from zero field resistivity ρ_0 and low field Hall coefficient at $T = 4.2$ K, are $p = 1.3(1.1) \cdot 10^{11}\,\mathrm{cm}^{-2}$ and $\mu = 3.6(4.0) \cdot 10^3\,\mathrm{cm}^2/(\mathrm{V}\cdot\mathrm{s})$ ($\rho_0 = 16(15)$ k$\Omega/\square$). From the relation $\rho_0^{-1} = (e^2/\pi\hbar) \times (\varepsilon_F\tau/\hbar)$ the important parameter, connecting the Fermi energy ε_F and elastic mean free time τ may be estimated: $\varepsilon_F\tau/\hbar = 0.8(0.85)$. Thus for the samples investigated $\varepsilon_F\tau/\hbar \approx 1$, and we are in a region of conjectural metal-insulator transition, which is seen experimentally in a variety of two-dimensional semiconductor systems [18].

The dependencies of longitudinal ρ_{xx} and Hall ρ_{xy} resistivities on magnetic field B at $T = 1.7$–4.2 K up to $B = 12$ T for sample A are shown in Fig. 1. The quantum Hall effect (QHE) plateau number one with corresponding ρ_{xx} minimum at $B \approx 3.5$ T are well seen in the pictures. The estimation of the hole mobility from the condition $\mu B_{C1} = 1$, where B_{C1}(= 2.7 T) is the field where $\rho_{xx} = \rho_{xy}$ (see Fig. 1,*a*), gives $\mu = 3.7 \cdot 10^3\,\mathrm{cm}^2/(\mathrm{V}\cdot\mathrm{s})$ in reasonable accordance with the low-field estimate.

We take notice that at $B < 0.5$ T positive magnetoresistance due to the effect of Zeeman splitting [19] is observed for all temperatures. At fields $B > 0.5$ T up to QHE ρ_{xx} minimum a background negative magnetoresistance takes place with the following peculiarities observed: i) Shubnikov–de Haas (SdH) oscillation structure with maximum at $B \approx 2$ T, and ii) the ρ_{xx} temperature-independent point at $B \approx B_{C1}$ (Fig. 1,*b*). In the high-field region the transition from the QHE regime to the insulator takes place in the vicinity of $B_{C2} \approx 7.5$ T (Fig. 1,*a*).

In a great deal of work [1–6,16,17] the low-field temperature-independent point at $B = B_C$ on the $\rho_{xx}(B)$ dependence is interpreted as a point of insulator–QHE quantum phase transition. A criterion of existence of a phase transition is a scaling dependence of $\rho_{xx}(B,T) = f((B - B_C)/T^{\kappa})$ in the vicinity of B_C with κ being a critical exponent [20]. By plotting $\ln\ (d\rho_{xx}/dB)_{B=B_C}$ versus $\ln T$, one could obtain κ. Such a situation may be realized in a system with genuine (strong) localization, e.g., with variable range hopping conduction at $B = 0$.

But for a system with weak localization we think that it is not the case. The weak localization regime at $k_F l >> 1$ ($\varepsilon_F\tau/\hbar >> 1$) is in fact the regime of the electron diffusion from one scattering event on an impurity to another, with some mean free path l. Here the notion of insulating behavior is valid only in the sense that $d\rho/dT < 0$. For such a system there exists another reason for a temperature-independent point on the

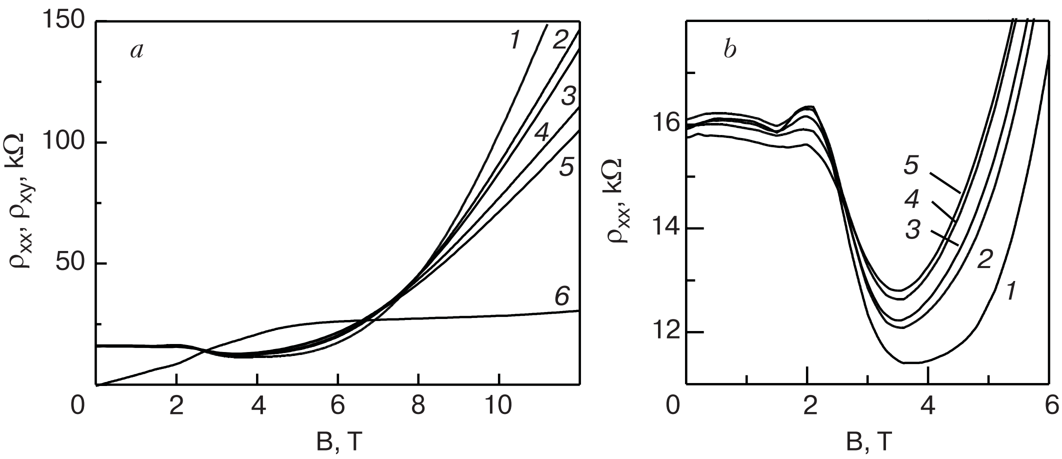

Fig. 1. Longitudinal resistivity (*1–5*) and Hall resistivity (*6*) as functions of magnetic field for sample A. T, K: *1*, *6* — 1.7; *2* — 2.3; *3* — 2.9; *4* — 3.7; *5* — 4.2.

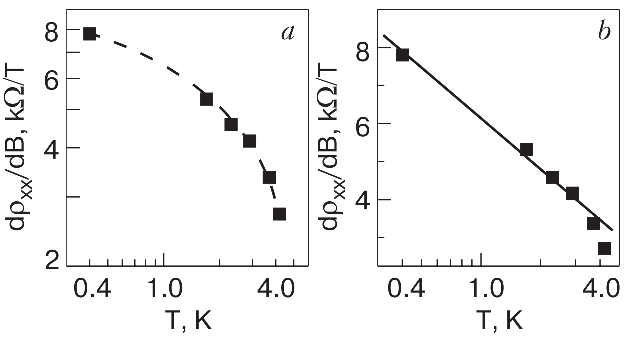

Fig. 2. The first derivative $d\rho_{xx}/dB$ as a function of temperature in a vicinity of low-field critical point in log–log scale (*a*) and linear–log scale (*b*). Dashed line on Fig. 2,*a* is a guide for eye.

$\rho_{xx}(B)$ dependence at $\omega_c\tau = 1$ ($B_{C1} = mc/e\tau$): it is a consequence of the interplay of classical cyclotron motion and the EEI correction $\Delta\sigma_{ee}$ to the Drude conductivity (see Eq. (1)). According to Eq. (1) the derivative $(d\rho/dT)_{B=B_C}$ should be proportional to $\ln T$ as $\Delta\sigma_{ee}$ is proportional to $\ln(kT\tau/\hbar)$.

To distinguish between the two cases in our samples with $\varepsilon_F\tau/\hbar \cong 1$ an analysis of dependence $(d\rho_{xx}/dB)_{B=B_C}$ on T has been carried out. Figure 2,*a* shows the nonscaling behavior of $\rho_{xx}(B,T)$ near the low-field critical point B_{C1}: it is not possible to extract consistently any power law from the temperature dependence of derivative $(d\rho_{xx}/dB)_{B=B_{C1}}$. On the other hand, rather good linear dependence of $(d\rho/dB)_{B=B_{C1}}$ on $\ln T$ is observed up to $T \approx 3$ K that is an argument in favor of the EEI version. In contrast to it, real scaling behavior of $\rho_{xx}(B,T)$ with critical exponent $\kappa = 0.38$ (compare with theoretical

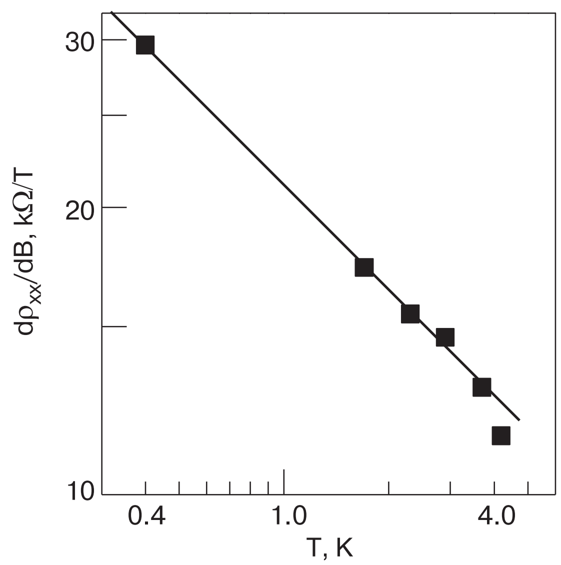

Fig. 3. The first derivative $d\rho_{xx}/dB$ as a function of temperature in a vicinity of high-field critical point (log–log scale).

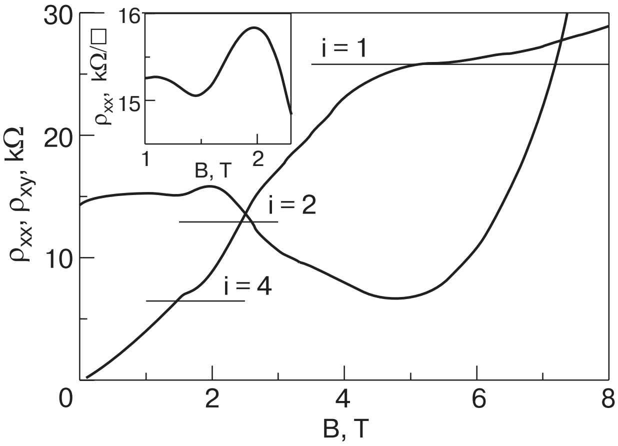

Fig. 4. Longitudinal and Hall resistivities as functions of magnetic field for sample B at $T = 0.4$ K.

value $\kappa = 0.42$ for the spin-split case [21]) takes place in a vicinity of high-field critical point B_{C2} (Fig. 3).

The experimental data for sample B at $T = 0.4$ K are presented on Fig. 4. The QHE plateau number one and corresponding minimum at $B = 5.6$ T are clearly seen on $\rho_{xy}(B)$ and $\rho_{xx}(B)$ dependencies. The estimation of the hole mobility from the $\rho_{xx} = \rho_{xy}$ point $B_{C1} = 2.5$ T gives $\mu = 4.0\cdot 10^3\,\mathrm{cm}^2/(\mathrm{V\cdot s})$. The condition for the field of QHE $\rho_{xx}(B)$ minima, $p = i(e/hc)B_i$, where i is the number of the plateau, gives $p = 1.2\cdot 10^{11}\,\mathrm{cm}^{-2}$.

It is seen from Fig. 4 that in low-field region $B < B_{C1}$ ($\omega_c\tau \approx 0.7$) minimum in $\rho_{xx}(B)$ at $B_4 = 1.4$ T (see inset of this figure) and precursor of $\rho_{xy}(B)$ plateau number four are observed. Really, Fig. 5 shows pronounced QHE-like structures on the dependence of first derivative $d\rho_{xy}/dB$ on filling factor for $\nu = 1$, 2, and 4.

In complete QHE regime at $\omega_c\tau >> 1$ the appearance of quantized plateaus in the $\rho_{xy}(B)$ dependences with vanishing values of ρ_{xx} is commonly accepted to be caused by the existence of disorder-induced mobility gaps (stripes of localized states) between the nar-

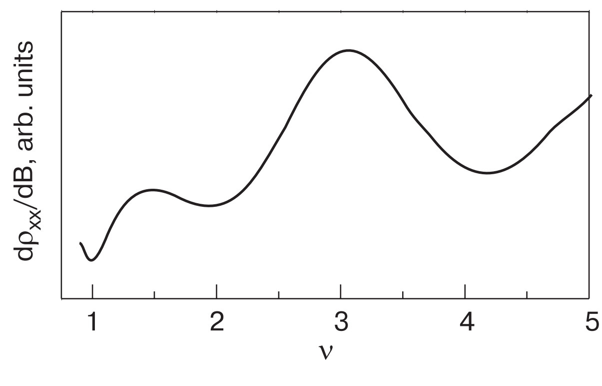

Fig. 5. The first derivative $d\rho_{xy}/dB$ as a function of filling factor ν for sample B at $T = 0.4$ K.

row bands of extended states of width Γ presented close to the center of each of the Landau subbands [22]. The existence of QHE-like structures at $\omega_c\tau < 1$ then should be a manifestation of localization of electron states in mixing regions for adjacent Landau subbands so that the width of extended state bands is less than the collision broadening of Landau level: $\Gamma < \hbar/\tau$. We think that realization of such a situation is more preferable just for $\varepsilon_F\tau/\hbar \cong 1$ when the localization effect is more essential than for $\varepsilon_F\tau/\hbar >> 1$ but is not yet too strong as for $\varepsilon_F\tau/\hbar << 1$.

Conclusions

Both low-field (B_{C1}) and high-field (B_{C2}) T-independent points on $\rho_{xx}(B)$ dependence with the $\nu = 1$ QHE state between them have been observed for p-type $Ge/Ge_{1-x}Si_x$ heterostructures with low hole mobility ($k_F l \approx 1.6$). In contrast to series of works [1–6] and [16,17] where the low-field point is treated as the critical point of an insulator → QHE phase transition, we speculate that in our 2D systems with $k_F l \geq 1$ such a point at $\omega_c\tau = 1$ is a manifestation of quantum $e-e$ interaction correction in the diagonal component of the magnetoresistivity tensor.

On the other hand, in accordance with [1–6] the high-field B_{C2} point is a point of genuine quantum phase transition between the $\nu = 1$ QHE phase and the high-field insulator and corresponds to passing of the Fermi level through the lowest Landau level.

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1. H.W. Jiang, C.E. Johnson, K.L. Wang, and S.T. Hannahs, *Phys. Rev. Lett*. **71**, 1439 (1993).
2. T. Wang, K.P. Clark, G.F. Spencer, A.M. Mack, and W.P. Kirk, *Phys. Rev. Lett.* **72**, 709 (1994).
3. R.J.F. Hughes, J.T. Nicholls, J.E.F. Frost, E.H. Linfield, M. Pepper, C.J.B. Ford, D.A. Ritchie, G.A.C. Jones, E. Kogan, and M. Kaveh, *J. Phys: Condens. Matter* **6**, 4763 (1994).
4. D. Shahar, D.C. Tsui, and J.E. Cunningham, *Phys. Rev*. **B52**, R14372 (1995).
5. S.-H. Song, D. Shahar, D.C. Tsui, Y.H. Xie, and Don Monroe, *Phys. Rev. Lett*. **78**, 2200 (1997).
6. M. Hilke, D. Shahar, S.H. Song, D.C. Tsui, Y.H. Xie, and Don Monroe, *Phys. Rev*. **B56**, R15545 (1977).
7. H.W. Jiang, C.E. Johnson, and K.L. Wang, *Phys. Rev*. **B46**, 12830 (1992).
8. B. Huckestein, *Phys. Rev. Lett*. **84**, 3141 (2000).
9. A. Houghton, J.R. Senna, and S.C. Ying, *Phys. Rev*. **B25**, 2196, 6468 (1982).
10. S.M. Girvin, M. Jonson, and P.A. Lee, *Phys. Rev*. **B26**, 1651 (1982).
11. K.K. Choi, D.C. Tsui, and S.C. Palmateer, *Phys. Rev*. **B33**, 8216 (1986).
12. Yu.G. Arapov, G.I. Harus, O.A. Kuznetsov, V.N. Neverov, and N.G. Shelushinina, *Semiconductors* **32**, 643 (1998), *cond-mat/0203435*.
13. G.M. Minkov, O.E. Rut, A.V. Germanenko, A.A. Sherstobitov, B.N. Zvonkov, E.A. Uskova, and A.A. Birukov, *Phys. Rev*. **B65**, 235322 (2002); G.M. Minkov, O.E. Rut, A.V. Germanenko, A.A. Sherstobitov, V.I. Shashkin, O.I. Khzykin, and B.N. Zvonkov, *Phys. Rev*. **B67**, 205306 (2003).
14. L. Li, Y.Y. Proskuryakov, A.K. Savchenko, E.H. Linfield, and D.A. Ritchie, *Phys. Rev. Lett.* **90**, 076802 (2003).
15. V.T. Renard, I.V. Gornyi, O.A. Tkachenko, V.A. Tkachenko, Z.D. Kvon, E.B. Olshanetsky, A.I. Toropov, and J.-C. Portal, *Phys. Rev*. **B72**, 075313 (2005).
16. C.F. Huang, Y.H. Chang, C.H. Lee, H.T. Chou, H.D. Yeh, C.-T. Liang, Y.F. Chen, H.H. Lin, H.H. Cheng, and G.J. Hwang, *Phys. Rev.* **B65**, 045303 (2001).
17. T.Y. Huang, J.R. Juang, C.F. Huang, G.-H. Kim, C.-P. Huang, C.-T. Liang, Y.H. Chang, Y.F. Chen, Y. Lee, and D.A. Ritchie, *Physica* **E22**, 240 (2004); T.-Y. Huang, C.-T. Liang, G.-H. Kim, C.F. Huang, C.-P. Huang, J.-Y. Lin, H.-S. Goan, and D.A. Ritchie, *cond-mat/0601490*.
18. E.A. Abrahams, S.V. Kravchenko, and M.P. Sarachik, *Rev. Mod. Phys*. **73**, 251 (2001).
19. Yu.G. Arapov, V.N. Neverov, G.I. Harus, N.G Shelushinina, M.V. Yakunin, O.A. Kuznetsov, A. de Visser, and L. Ponomarenko, *Low Temp. Phys.* **30**, 1157 (2004); *cond-mat/0404355*.
20. H.P. Wei, A.M. Chang, D.C. Tsui, and M. Razeghi, *Phys. Rev.* **B32**, 7016 (1985).
21. S.W. Hwang, H.P. Wei, L.W. Engel, D.C. Tsui, and A.M.M. Pruisken, *Phys. Rev*. **B48**, 11416 (1993).
22. *The Quantum Hall Effect*, R.E. Prange and S.H. Girvin (eds.), Springer-Verlag (1987).